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# Comparative Analysis of SFA and DEA Models: An Application to Health System Efficiency in SSA

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**ABSTRACT----**
***Introduction***
***The Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA) are the two major approaches in estimating technical efficiency. However, the two approaches have different underlying assumption and present different efficiency estimates when used in similar situation. In this study we compare the two approaches with an application to health system efficiency across countries in Sub-Saharan Africa (SSA).***
***Methods***
***The study used cross section data for 45 countries in SSA, sourced from the World Bank world development indicators. Both the DEA and SFA were used to estimate health system efficiency using per capita health expenditure as input while under-five and infant mortality was used as health outcomes. Scatter and Kernel density plots were used to supplement the comparison of estimates from the two models.***
***Results***
***The findings suggest that there exist disparities between estimates from the DEA and SFA models. Estimates from the SFA models were relatively higher than those from the DEA models. However, there was not much difference in the ranking of individual countries in terms of efficiency performance. The findings of the various model specifications show average health system efficiency scores of approximately 0.44 and 0.50 for the DEA specifications while 0.70 and 0.72 was estimated for the SFA specifications***
***Conclusion***
***The efficiency scores suggest that there is room for improvement in terms health system performance. The choice between the two models should be based on the availability of data and the limitations posed by the data requirements of each of the models.***

**Keywords---** DEA, SFA, health system efficiency, health expenditure, SSA

## 1. INTRODUCTION

An important step to improve health status in sub-Saharan Africa (SSA) and other developing regions is to strengthen health systems and increase equitable access to effective health care. This can be achieved by ensuring adequate and efficient expenditure in the health sector. Improving health status and other aspects of human capital does not only improve the welfare of the population but also raise productivity levels in any region. Empirical evidence on the relationship has been unanimous in the sense that higher health care spending improves productivity and economic growth (Bukenya, 2009, Heshmati, 2001, Bloom *et al.*, 2004). For developing regions like SSA, such investments are inevitable considering the persistent high levels of poverty and inequality (World Bank, 2010).

However, most SSA countries face a major challenge of determining whether or not their expenditure on health translate into improvements in health status of the population. Some researchers have noted that increasing public expenditure on health may not mitigate the health challenges in SSA (Gupta *et al.*, 2002). For instance the World Health Organization (WHO) (2012) noted that high or low levels of health funding might not translate into improved health outcomes but rather efficiency and equity in the use of these resources. Significant inefficiencies in public expenditure on health have been recorded not only in advanced economies but also emerging and developing ones (Gupta et al., 2002, Herrera and Pang, 2005, Jayasuriya and Wodon, 2003). Grigoli and Levy (2012) argued that reducing the considerable waste that emerge from the inefficiencies will be crucial in improving health indicators. This is even more important in resource poor regions such as SSA.

Identifying and improving health system inefficiencies will not only improve population health but also create additional fiscal space for health. Achieving high levels of efficiency is therefore a major priority in any economic. Two main approaches have dominated the literature in estimating efficiency levels. These are the Data Envelopment Analysis (DEA) and the Stochastic Frontier Analysis (SFA). The distinction between the two family of methods is well documented with some authors favouring the SFA ahead of DEA method (Hollingsworth and Wildman, 2003, Chirikos and Sear, 2000, Jacobs, 2000). This is largely because the SFA model is able to statistically control for several variables that influence health outcomes in the estimation of efficiency scores. The non-parametric DEA method has been found to be very sensitive in the case of heterogeneous decision making units since efficiency estimates are derived from production frontiers that are influenced by outliers (Fiorentino *et al.*, 2006). While we do not attempt to rank these models in this paper, we provide a simple comparison of these approaches with particular focus on the efficiency of health systems in SSA.

The rest of the paper proceeds as follows; section two presents a brief review of the literature. Section three presents methods including data used in the analysis while section four presents the results. In section five, the results are discussed with various policy implications. The summary and conclusions are briefly presented in section six.

## 2. LITERATURE REVIEW

With regards to estimating health system efficiency, two key methodologies have been used in the literature. These are the non-parametric and parametric approaches. The non-parametric approach include the data envelopment analysis (DEA) and the free disposable hull (FDH). The parametric approach includes the stochastic frontier analysis (SFA). It must be noted that the DEA approach have been relatively dominant in the health system efficiency literature.

Examples of studies that have used the nonparametric methods include Afonso and Aubyn (2005) who used both the DEA and FDH approaches to estimate efficiency using infant survival and life expectancy at birth as outputs while availability of doctors, availability of nurses and hospital beds were used as inputs. Other studies that used the DEA approach include Alexander et al. (2003) who also used gender specific disability adjusted life expectancy (DALE) and infant mortality rate as outputs while per capita expenditure adjusted for price differences across countries was used as inputs. Herrera and Pang (2005) employed both the DEA and FDH methods using life expectancy at birth, immunization and DALE as outputs while aggregate public spending on health was used as input. In terms of the parametric methods, Hernandez de Cos and Moral-Benito (2011) used the SFA approach with life expectancy at birth as output and per capita health expenditure as inputs. Grigoli and Kapsoli (2013) also employed the SFA in analysing the efficiency of health expenditure in emerging and developing economies.

While the DEA and FDH are the most used in the estimation of health system efficiency, they are weak in the sense that they are extremely sensitive to the presence of outliers, which define the frontier. Their nonparametric nature also imply that they are unable to address random variations in the data which are then captured as inefficiency. While the SFA addresses these weaknesses, it is also limited in the imposition of some functional form on the production function which, in some cases, become difficult to estimate. A critical advantage of the SFA over nonparametric methods lies in its ability to control for large number of variables that can influence health outcomes. Efficiency scores from the nonparametric methods become biased when large number of inputs are used with small sample size, making it difficult to rank countries in terms of efficiency. While the second stage regression analysis have been employed to resolve this problem, it does not allow one to derive efficiency scores in a way that incorporates the influence of these factors (Burgess, 2006).

Comparative studies on these methods have been inconclusive as to which is most preferred (Chirikos and Sear, 2000, Hollingsworth and Wildman, 2003). However econometric studies favour the SFA due to its ability to control for randomness in the data and a wide range of variables that influence health outcomes. In the current study, both the DEA and SFA methods were employed to allow for comparison and robustness check. The study also deviates from studies that have used the SFA method by controlling for unobserved heterogeneity that may bias the inefficiency estimates (Greene, 2004). This aspect of the parametric methods, even though critical, has been missing in empirical studies that used the SFA.

## 3. METHODOLOGY

### 3.1 The DEA model

The methodology adopted in the study follows Fare et al. (1994) and Alexander et al. (2003) using non-parametric linear programming techniques. The analysis starts with an optimization problem which determines the available population health outcome of other health systems. A 'best practice' frontier based on a piece-wise linear envelopment of the health expenditure - health outcome data for the sample countries, was used to solve the optimization problem.

Efficiency in the production of population health is measured relative to such a frontier for each country. The health systems of countries that operate on (and determine) the frontier are termed efficient (with efficiency score of 1.00), while countries operating off the frontier are considered inefficient (with efficiency scores less than 1.00). Inefficiency in this case should be understood to mean that better population health outcomes could have been attained from the observed health expenditure, were performance similar to that of 'best practice' countries.

To better understand the procedures described above, let $S^t$ be the technology that transforms health sector expenditure into population health outcomes. This technology can be modelled by the output possibility set

$$P^t(x^t) = \left\{ y^t : (x^t, y^t) \in S^t \right\} t = 1, \ldots, T \tag{1}$$

where $P^t(x^t)$ denotes the collection of population health output vectors that consume no more that the bundle of resources indicated by the resource vector $x^t$, during period $t$.

The best practice frontier can be empirically estimated as the upper bound of the output possibility set, $P^t(x^t)$. The output possibility set, $P^t(x^t)$, can be estimated empirically by assuming that the sample set is made up of observations on $j=1,...J$ countries' health systems, each using $n=1,...N$ resources, $x^t_{jn}$, during period $t$, to generate $m=1,...,M$ population health outcomes, $y^t_{jm}$, in period $t$. Accordingly, $P^t(x^t)$ is estimated from the observed set of health expenditures, and population health outcomes for all the countries of the sample.

The empirical construction of the piece-wise linear envelopment of the input possibility set is given by

$$\begin{aligned} P^t(x^t) = \{ y^t : x^t_n \le \sum_{j=1}^{J} z_j x^t_{jn}, n = 1, \ldots, N \\ \sum_{j=1}^{J} z_j y^t_{jm} \ge y^t_m, m = 1, \ldots, M \\ \sum_{j=1}^{J} z_j = 1 \\ z_j \ge 0, j = 1, \ldots, J \} \end{aligned} \tag{2}$$

where $z_j$ is a variable indicating the weighting of each of the health systems.

The output-based efficiency score for each country's health system for period $t$ can be derived as

$$F^t_o(x^t_j, y^t_i) = \max\{\theta \text{ such that } \theta y^t \in P^t(x^t)\} \text{ where } F^t_o(x^t_j, y^t_i) \ge 1. \tag{3}$$

This suggests that a county's health outcomes vector, $y^t$, will be located on the efficiency frontier when equation (3) has a value of one. However, if equation (3) produces a value less than one, the health system must be classified as inefficient relative to best-observed practice. This measure can be computed for country $j$ as the solution to the linear programming problem

$$F^t_o(x^t_j, y^t_i) = \max \theta \tag{4}$$

with θ, z such that

$$\sum_{j=1}^{J} z_j y_{jm}^t \geq \theta y_{jm}^t, m = 1, \ldots, M,$$
$$\sum_{j=1}^{J} z_j x_{jn}^t \leq x_{jn}^t, n = 1, \ldots, N,$$
$$\sum_{j=1}^{J} z_j = 1,$$
$$z_j \geq 0, j =, \ldots, J, \tag{5}$$

where the restrictions on the weighting variables, $z_j$, imply a variable returns to scale assumption in regard to the underlying technology of health production.

**3.2 The SFA model**

Drawing from (Belotti *et al.*, 2012), a simple cross sectional SFA model was used in the analysis. The model basically generates stochastic error and inefficiency term from the residuals obtained from an estimated production function expressed as follows:

$$y_i = \alpha + x_i^{'} \beta + \varepsilon_i$$
$$\varepsilon_i = v_i - u_i$$
$$v_i \sim N(0, \delta_v^2)$$
$$u_i \sim f \qquad \text{i = 1,..,N} \tag{6}$$

where $y_i$ represents the logarithm of output of the *ith* decision making unit (DMU), $x_i$ is a vector of inputs and $\beta$ is the vector of technology parameters. The error term $\mathcal{E}_i$ is composed of a sum of normally distributed disturbance ($v_i$) which accounts for measurement and specification error and a one-sided disturbance ($u_i$) which measures inefficiency. Both $v_i$ and $u_i$ are assumed to be independent and identically distributed (*i.i.d*) across observations. An exponential assumption $[u_i \sim \varepsilon(\delta_u)]$ proposed by Meensen and VanBroeck (1977), was made about the distribution of the inefficiency term[1]. This assumption is necessary to make the model estimable because the efficiency term is assumed to be a stochastic variable, with a specific distribution function.

**3.3 Choice of inputs and outputs**

The choice of inputs in estimating the health production function is not straight forward as there exist several factors that influence population health status both directly and indirectly. As noted by Afonso and Aubyn (2005), efficiency results may be sensitive to the type of input used. The current study used monetary inputs measured as health expenditure per capita expressed in purchasing power parity (PPP) terms.

An important aspect of estimating health system efficiency is the choice of indirect inputs that influence health status but are not directly controlled by the health system (Tandon *et al.*, 2003). This is intuitively appealing since two countries that spend the same amounts on health may not necessarily have the same health outcomes if they operate in different environments. The current study employed education (measured by average years of schooling) as an environmental variable which, even though, not directly controlled by the health system, is highly likely to influence health status (Caldwell and Caldwell, 1985).

In terms of health system outputs used in the efficiency analysis, we employed infant and under five mortality rates. However, as noted by Afonso and Aubyn (2005), efficiency measurement techniques suggest that outputs are measured in such a way that "more is better". Therefore consistent with practice in the literature, various transformations were performed on the mortality variables so that they are measured in survival rates. For instance, infant mortality rate (IMR) is measured as [(number of children who died before 12 months)/(number of children born)] X 1000. This implies that an infant survival rate (ISR) can be computed as follows;

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[1] For further details on SFA models, see Belotti, F., S. Daidone, G. Ilardi and V. Atella (2012) 'Stochastic frontier analysis using Stata', *Center for Economic and International Studies Tor Vergata Research Paper Series,* 10(12), 1-48.

$$ISR = \frac{1000 - IMR}{IMR} \tag{7}$$

This shows the ratio of children that survived the first year to the number of children that died and this increases with better health status. In a similar measure, transformations were performed for the under five mortality rate variable.

### 3.4 Choice of orientation for efficiency measurement

In estimating health system efficiency, the choice of orientation is usually neglected. It is however important to note that the choice of orientation can have direct implications for policy recommendations based on the estimated efficiency scores. The output orientation estimates the potential for changing outputs without changing the inputs of the production system. The input orientation on the other hand estimates the potential for changing inputs without changing the output quantities produced.

The choice between input or output orientation is usually straight forward when decision units such as firms are considered. This is because the primary objective of these DMUs is to minimize inputs or maximize outputs as much as possible. In the case of health systems, the output orientation is more intuitively appealing. For instance, Alexander et al. (2003) argues that the output orientation is preferred because it helps to understand the potential for improvement in health outcomes rather than the potential for saving in health expenditure or reducing health related resources in general. Moreover, it may be impractical to recommend a reduction in health resources in a country while maintaining a fixed population health status. Following this argument, the current study reports efficiency estimates from the output orientation. This implies that in this context, the potential of improving health status using the same levels of health resources is explored.

## 4. RESULTS

The figure below shows a scatter plot of the correlation that exists between the DEA and SFA models employed in the study. The figure suggests the presence of a very strong correlation between efficiency estimates from the various SFA specifications. For instance Graph 1b shows that efficiency scores form the SFA models, using under five and infant mortality as outputs, are strongly correlated. Similarly, there was evidence of strong correlation established between the DEA models. In Graph 1c, the DEA models using under five and infant survival rates as outputs show close resemblance in terms of efficiency estimates. However, the DEA and SFA models showed significantly weak correlation in terms of efficiency estimates. The Graphs 1a and 1d present evidence of such weak correlation between the DEA and SFA models.

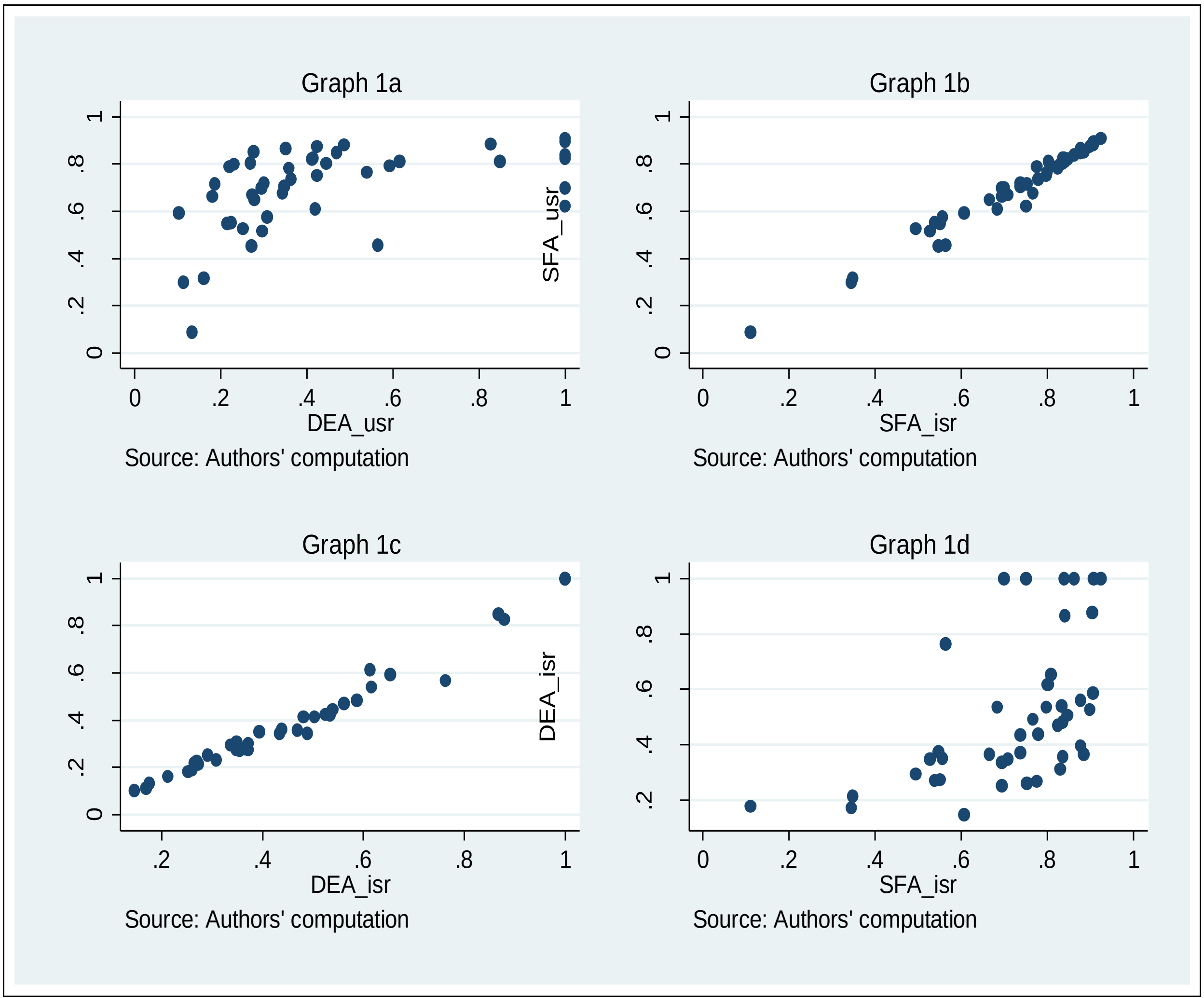


Figure 1: Scatter plot of DEA and SFA efficiency estimates

To further understand the similarity between the DEA and SFA models, the mean and variation in the distribution of estimated efficiency scores for the various model specifications were estimated and reported in the Kernel density plots in Figure 2. Evidence from the Kernel density plots suggest that efficiency scores from the SFA specifications generally had higher means relative to the DEA specifications. The variation in the distribution of efficiency estimates seem to be similar for both the SFA and DEA model specifications. The plots of the SFA kernel density estimates are generally skewed to the right while that of the DEA specifications are skewed to the left. This suggests that, on average efficiency scores are higher in the SFA estimates compared to the DEA estimates. Some researchers have attributed the difference in the mean to the fact that the SFA model includes other control variables that directly or indirectly influence the performance of the health system (Grigoli and Kapsoli, 2013, Danqua and Ouattara, 2012).

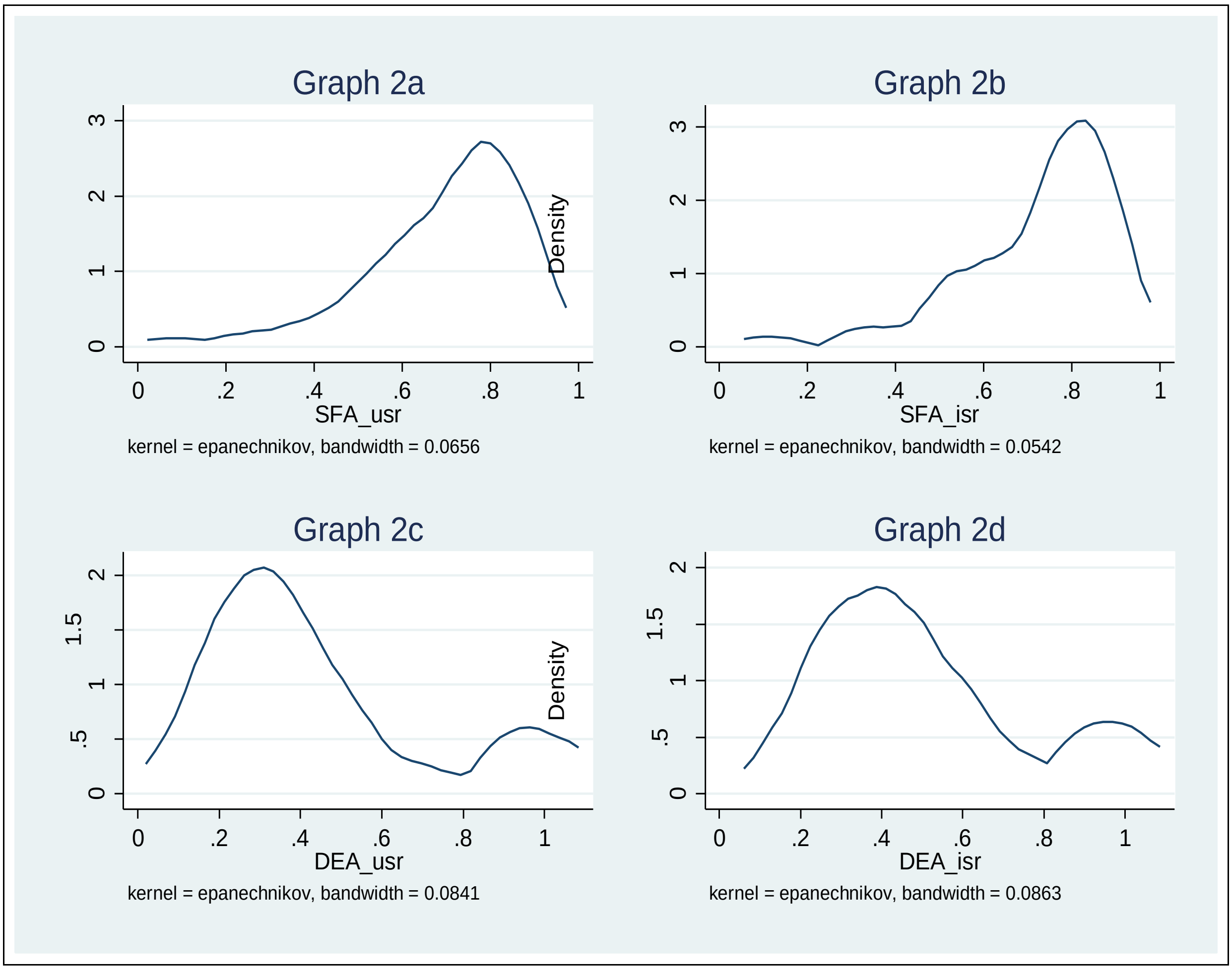


Figure 2: Kernel density plots of DEA and SFA efficiency estimates

The relationship between efficiency scores from the various model specifications is further examined in the correlation matrix below. The simple correlation matrix reported in Table 1 suggest that there exist a weak resemblance between the various SFA and DEA specifications. The simple correlation between the SFA models with under-five survival rate as output variable and that with infant survival rate as outcome variable was about 98%. On the contrary, the correlation between the SFA and DEA both with under-five survival rate as outcome was about 48%. Also, the DEA and SFA model specifications with infant survival rate as outcome variable showed a correlation of 53%. Similar relationship was established in the spearman rank correlation (Table 2). The matrix also showed strong similarity between the SFA and DEA model specifications separately but dissimilar when the SFA and DEA models are compared.

Table 1: Correlation matrix for DEA and SFA efficiency estimates

| | SFA_usr | SFA_isr | DEA_usr | DEA_isr |
|---|---|---|---|---|
| SFA_usr | 1 | | | |
| SFA_isr | 0.9838 | 1 | | |
| DEA_usr | 0.4781 | 0.5068 | 1 | |
| DEA_isr | 0.4936 | 0.5384 | 0.9896 | 1 |

**Source:** Authors' computation
**Note:** SFA_usr: Efficiency scores from cross section SFA model with under five survival as outcome variable. SFA_isr: Efficiency scores from cross section SFA model with infant survival as outcome variable. DEA_usr: Efficiency scores from DEA model with under five survival as outcome variable. DEA_isr: Efficiency scores from DEA model with infant survival as outcome variable.

Table 2. Spearman rank correlation matrix for DEA and SFA efficiency estimates

| | SFA_usr | SFA_isr | DEA_usr | DEA_isr |
|---|---|---|---|---|
| SFA_usr | 1 | | | |
| SFA_isr | 0.9825 | 1 | | |
| DEA_usr | 0.5943 | 0.6292 | 1 | |
| DEA_isr | 0.5849 | 0.6364 | 0.9856 | 1 |

**Source:** Authors' computation
**Note:** Variables as defined under Table 1

Table 3 shows a summary of the efficiency scores from the various model specifications. Consistent with the Kernel density estimates, the SFA model specifications have higher average efficiency scores compared to the DEA model specifications. For instance, average efficiency scores from the SFA model specification (with under five survival rate as output) was about 70% compared to the DEA specification (with under five survival rate as output) which recorded an average efficiency score of about 44%. Similarly, efficiency scores from infant survival specification was high in the SFA model (72%) relative to the DEA model (50%).

Table 3: Summary of efficiency estimates from DEA and SFA models

| Variable | Mean | Standard deviation | Minimum | Maximum |
|---|---|---|---|---|
| SFA_usr | 0.69519 | 0.17585 | 0.08702 | 0.90648 |
| SFA_isr | 0.72338 | 0.17307 | 0.11065 | 0.92386 |
| DEA_usr | 0.44024 | 0.27342 | 0.10300 | 1.00000 |
| DEA_isr | 0.50478 | 0.25667 | 0.14700 | 1.00000 |

**Source:** Authors' computation
**Note:** Variables as defined under Table 1

While the average efficiency scores are reported in Table 3, it is important to compare the performance of individual countries. Table 4 shows the performance of individual countries across the SFA and DEA models. The table shows that only two countries (Cape Verde and Mauritius) were consistently efficient in both the DEA and SFA models, irrespective of output variable used. Similarly, four countries were consistently inefficient across the DEA and SFA models, irrespective of the outcome variable used. Some countries were inconsistent, in terms of ranking, across the two models. Kenya, Tanzania and Madagascar were efficient in the SFA model specification but were inefficient in the DEA model specification. The situation was evident irrespective of the outcome measure used. Similarly, four countries were found to be efficient in the DEA model specifications but were inefficient in the SFA model. This emphasises the inconsistency showed between the two models.

Table 4: comparing efficient and inefficient countries in the DEA and SFA models

<table>
<tr><td rowspan="8">SFA</td><td colspan="3">DEA (under five survival as output)</td></tr>
<tr><td></td><td>Efficient</td><td>Inefficient</td></tr>
<tr><td>Efficient*</td><td>Cape Verde, Mauritius</td><td>Kenya, Tanzania, Madagascar</td></tr>
<tr><td>Inefficient**</td><td>Eritrea, Mozambique, Niger, Seychelles</td><td>Angola, Equatorial Guinea, Swaziland</td></tr>
<tr><td colspan="3">DEA (infant survival as output)</td></tr>
<tr><td>Efficient*</td><td>Cape Verde, Mauritius</td><td>Kenya, Tanzania, Madagascar</td></tr>
<tr><td>Inefficient**</td><td>Eritrea, Mozambique, Niger, Seychelles</td><td>Angola, Equatorial Guinea, Swaziland</td></tr>
</table>

**Source:** Authors' computation

**Note:** * Efficient countries were those with score of 1.00 in the DEA model while the top five ranked countries in the SFA model were considered to be efficient. ** The bottom five ranked countries were considered inefficient in both models.

In Table 5, individual country efficiency scores from DEA and SFA models are presented using the output orientation. Under-five and infant survival rates were used as the output variables while per capita heath expenditure and average years of schooling were used as input variables. It can be observed from the table that most of the countries located on the estimated frontier were consistent across the various models. Again, Cape Verde, Eritrea, Mauritius, Madagascar and Seychelles were estimated to have the most efficient health systems in terms of the use of health expenditure. These countries therefore form the basis for comparison with other health systems in the region. This implies that a majority of countries in the SSA region have potential for improvement in the performance of the health system.

In the DEA model, for instance, Sierra Leone emerged the least efficient country with an estimated health system efficiency score of 0.10 and 0.15 depending on the model specification. Angola recorded an efficiency score of 0.11 when under-five survival is used as the outcome variable and 0.17 when infant survival is used as the outcome variable. Other countries worth mentioning include South Africa and Nigeria. South Africa recorded efficiency scores between 0.30 and 0.35 while Nigeria recorded estimated efficiency score between 0.18 and 0.25.

Individual country efficiency estimates from the stochastic frontier model are presented in the last four columns of Table 5. The table shows a cross section (2011) analysis using both under-five and infant survival rates as measures of health outcome. The results show mean efficiency of approximately 0.70 for the two models used in the analyses. A close observation of the results also shows strong similarity in the individual country efficiency scores and rankings.

The best performing countries from the cross-section SFA analysis include Mauritius and Cape Verde with efficiency estimate of about 0.90. This suggests that, relative to best practice, the health system in Mauritius and Cape Verde can be improved by about 10%. Other countries that also performed relatively well under the SFA model include Madagascar, Kenya, Tanzania and Eritrea.

Countries with relatively high potential for improvement in the performance of the health system with efficiency score way below the regional average include Angola, Equatorial Guinea, South Africa, Sierra Leone, Swaziland, Mauritania and Nigeria. The estimated efficiency scores for these countries suggest that, given the current level of health expenditure, it is possible to significantly improve population health status if best practices are followed in the production process.

Table 5: Country specific efficiency scores from DEA and SFA models

| | DEA models | | | | Cross section SFA models | | | |
|---|---|---|---|---|---|---|---|---|
| **Country** | **Rank1** | **DEA 1** | **Rank2** | **DEA 2** | **Rank1** | **SFA 1** | **Rank2** | **SFA 2** |
| Angola | 44 | 0.11 | 44 | 0.17 | 44 | 0.30 | 44 | 0.35 |
| Benin | 24 | 0.35 | 24 | 0.43 | 26 | 0.71 | 27 | 0.74 |
| Botswana | 10 | 0.59 | 10 | 0.65 | 18 | 0.79 | 18 | 0.81 |
| Burkina Faso | 11 | 0.57 | 9 | 0.76 | 41 | 0.46 | 36 | 0.56 |
| Burundi | 21 | 0.36 | 23 | 0.44 | 23 | 0.74 | 22 | 0.78 |
| Cameroon | 40 | 0.19 | 40 | 0.26 | 25 | 0.72 | 25 | 0.75 |
| Cape Verde | 1 | 1.00 | 1 | 1.00 | 2 | 0.89 | 2 | 0.91 |
| Central African Republic | 26 | 0.31 | 31 | 0.35 | 36 | 0.57 | 37 | 0.56 |
| Chad | 18 | 0.42 | 16 | 0.54 | 34 | 0.61 | 33 | 0.68 |
| Comoros | 9 | 0.61 | 12 | 0.61 | 13 | 0.81 | 19 | 0.80 |
| Congo, Dem. Rep. | 29 | 0.29 | 34 | 0.34 | 27 | 0.70 | 31 | 0.69 |
| Congo, Rep. | 30 | 0.28 | 29 | 0.36 | 32 | 0.65 | 34 | 0.67 |
| Cote d'Ivoire | 38 | 0.22 | 39 | 0.27 | 19 | 0.79 | 23 | 0.77 |
| Equatorial Guinea | 43 | 0.13 | 43 | 0.18 | 45 | 0.09 | 45 | 0.11 |
| Eritrea | 2 | 1.00 | 2 | 1.00 | 9 | 0.84 | 9 | 0.86 |
| Ethiopia | 7 | 0.85 | 8 | 0.87 | 14 | 0.81 | 11 | 0.84 |
| Gabon | 35 | 0.25 | 36 | 0.29 | 39 | 0.52 | 42 | 0.49 |
| Gambia, The | 25 | 0.34 | 20 | 0.49 | 29 | 0.67 | 24 | 0.77 |
| Ghana | 20 | 0.41 | 19 | 0.50 | 12 | 0.82 | 10 | 0.85 |
| Guinea | 12 | 0.54 | 11 | 0.62 | 21 | 0.76 | 20 | 0.80 |
| Guinea-Bissau | 32 | 0.27 | 32 | 0.35 | 30 | 0.67 | 29 | 0.71 |
| Kenya | 13 | 0.49 | 13 | 0.59 | 4 | 0.88 | 3 | 0.90 |
| Lesotho | 37 | 0.22 | 38 | 0.27 | 37 | 0.55 | 40 | 0.54 |
| Liberia | 23 | 0.35 | 25 | 0.39 | 6 | 0.87 | 7 | 0.88 |
| Madagascar | 8 | 0.83 | 7 | 0.88 | 3 | 0.88 | 4 | 0.90 |
| Malawi | 16 | 0.42 | 17 | 0.53 | 22 | 0.75 | 21 | 0.80 |
| Mali | 33 | 0.27 | 26 | 0.37 | 42 | 0.45 | 39 | 0.55 |
| Mauritania | 39 | 0.21 | 37 | 0.27 | 38 | 0.55 | 38 | 0.55 |
| Mauritius | 3 | 1.00 | 3 | 1.00 | 1 | 0.91 | 1 | 0.92 |
| Mozambique | 4 | 1.00 | 4 | 1.00 | 28 | 0.70 | 30 | 0.70 |
| Namibia | 15 | 0.44 | 15 | 0.54 | 16 | 0.80 | 15 | 0.83 |
| Niger | 5 | 1.00 | 5 | 1.00 | 33 | 0.62 | 26 | 0.75 |
| Nigeria | 41 | 0.18 | 41 | 0.25 | 31 | 0.66 | 32 | 0.69 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Rwanda | 14 | 0.47 | 14 | 0.56 | 8 | 0.85 | 8 | 0.88 |
| Sao Tome and Principe | 36 | 0.23 | 35 | 0.31 | 17 | 0.80 | 16 | 0.83 |
| Senegal | 19 | 0.41 | 21 | 0.48 | 10 | 0.82 | 13 | 0.84 |
| Seychelles | 6 | 1.00 | 6 | 1.00 | 11 | 0.82 | 12 | 0.84 |
| Sierra Leone | 45 | 0.10 | 45 | 0.15 | 35 | 0.59 | 35 | 0.61 |
| South Africa | 28 | 0.30 | 33 | 0.35 | 40 | 0.51 | 41 | 0.53 |
| Sudan | 34 | 0.27 | 30 | 0.36 | 15 | 0.80 | 14 | 0.84 |
| Swaziland | 42 | 0.16 | 42 | 0.21 | 43 | 0.32 | 43 | 0.35 |
| Tanzania | 17 | 0.42 | 18 | 0.52 | 5 | 0.87 | 5 | 0.90 |
| Togo | 27 | 0.30 | 27 | 0.37 | 24 | 0.72 | 28 | 0.74 |
| Uganda | 31 | 0.28 | 28 | 0.37 | 7 | 0.85 | 6 | 0.88 |
| Zambia | 22 | 0.36 | 22 | 0.47 | 20 | 0.78 | 17 | 0.83 |
| **Mean** | | **0.44** | | **0.50** | | **0.70** | | **0.72** |

**Source:** Authors' computation
**Note:** SFA1: Efficiency scores from cross section SFA model with under five survival as outcome variable. SFA2: Efficiency scores from cross section SFA model with infant survival as outcome variable. DEA1: Efficiency scores from DEA model with under five survival as outcome variable. DEA2: Efficiency scores from DEA model with infant survival as outcome variable.

## 5. DISCUSSION

The findings of the study suggest that there exist little similarity between efficiency estimates from the SFA and DEA models. A general observation suggest that the SFA models produced relatively higher efficiency estimates, compared to the estimates from the DEA models. The various specifications of the SFA and DEA models separately show strong similarities however, when the two models are compared they show weak correlation. This has implication that, even though the two models aim at evaluating the performance of decision making units the estimates may differ and resulting interpretations may not be the same.

This finding is not surprising as a number of empirical studies have reached similar conclusion. A common reason attributed to this difference is the fact that the DEA model only accounts for variables that serve as direct inputs to the health system. Unlike the DEA, the SFA is capable of accounting for both direct inputs and indirect inputs. This is particularly important for the analysis of health system performance because there are several factors that indirectly affect the outcome of the health system. These factors are usually not under the control of the health system. It is therefore important for such factors to be accounted for in efficiency analysis of this nature.

Grigoli and Kapsoli (2013) noted that efficiency estimates are generally higher when more control variables are included in the analysis. This is because including more control variables reduces the size of the error term and results in a better fit for the model. Other studies have favoured the SFA model on the grounds that results from the non-parametric models (including DEA) are influenced by the presence of outliers to create the production function and are very sensitive when DMUs are heterogeneous (Fiorentino *et al.*, 2006). This argument is a crucial concern in the context of the current study where different economies were used as DMUs. In this case, it can be expected that both direct and indirect determinants of health can vary widely. Danqua and Ouattara (2012) also showed that relative to the SFA, estimates from the DEA model specifications are not only lower but also they are more likely to be mis-specified.

It is worth mentioning that while the efficiency scores vary significantly, there was some consistency in the ranking of individual countries across the DEA and SFA models. There were some countries that were found to be efficient in the DEA model that were also efficient when the SFA models were specified. Similarly, some countries were ranked inefficient irrespective of the model specification used. In spite of this, there were still some inconsistencies in the ranking of countries. For instance, there were some countries that ranked relatively efficient in the SFA model specifications but inefficient in the DEA model specifications.

The individual country efficiency scores suggest that there exist some level of inefficiencies across health systems in SSA. The findings of the various model specifications show average health system efficiency scores of approximately 0.44 and 0.50 for the DEA specifications while 0.70 and 0.72 was estimated for the SFA specifications. This implies that there exist estimated inefficiency ranging between approximately 0.56 and 0.50 for the DEA specifications and 0.30 and 0.28 for the SFA specifications. This shows that there exist significant potential for health systems in SSA to improve population health status without any further increase in health inputs. Enhancing the efficiency of health resource use should therefore be an important aspect of health system reforms across these countries.

## 6. CONCLUSION

The study set out to provide a comparison between the DEA and SFA models with an application to the estimation of health system efficiency in SSA. The study used cross section data for 45 countries in SSA, sourced from the WDI. The findings suggest that there exist disparities between estimates from the DEA and SFA models. Estimates from the SFA models were relatively higher than those from the DEA models. However there was not much difference in the ranking of individual countries in terms of efficiency performance. This suggests that the choice of model should be based on available data. The efficiency scores suggest that there is room for improvement in terms health system performance. Improving the performance of health systems in the regions will be a step in the right direction.

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