Two-level Method Based on Newton Iteration for the Stationary Navier-Stokes Equations

WenTao Cai[1] and XiaoHua Liu[2]

[1]Department of Science
Southwest Petroleum University
Chengdu, Sichuan 610500 China

[2] Department of Science
Southwest Petroleum University
Chengdu, Sichuan 610500 China

ABSTRACT—***This paper propose and analyze the two-level stabilized finite element method for the stationary Navier-Stokes equations based on Newon iteration. This algorithm involves solving one small, nonlinear coarse mesh with mesh size H and two linear problems on the fine mesh with mesh size h. Based on local Gauss integration and the quadratic equal-order triangular element ,the two-level stabilized method our study provide an approximate solution (u^h, p^h) with convergence rate of same order as the approximate solution (u_h, p_h) of one-level method,which involves solving one large Navier-Stokes problem on a fine mesh with mesh size h. Hence,our method can save a large amount of computational time. Finally,some numerical tests confirm the theoretical expectations.***

Keywords—Navier-Stokes equations; equal-order pair; Newton iteration; Local Gauss integration; Two-level strategy

---

1. INTRODUCTION

The Navier-Stokes equations are very important because they describe the physics of many things of academic and economic interest. These equations in their full and simplified forms help with the design of aircraft and cars, the study of blood flow, the design of power stations. the analysis of pollution, and many other things. In the various of works of studying Navier-Stokes equations, the mixed finite element method has been the subject of a very intense research activity over the past 30 years.

Recently,Xu [1,2] proposes a general two-grid method on nonlinear problems. This method makes the coarse mesh extremely coarse (in contrast to fine mesh) which still maintains the optimal approximation. This means that solving a nonlinear equation (include the N-S equations) is not much more difficult than solving one linear equation, and the work for solving a nonlinear problem on coarse mesh is relatively negligible.

Moreover, Dai and Cheng [7] simple state the general two-grid method as follows: first they solve one small, nonlinear system on a coarse mesh, then solve one linearized problem on fine mesh based on Newton's method and at last solve one linear correction problem on the coarse mesh.

In works of Xu [1,2],Dai and Cheng [7],the mixed finite element methods all satisfy the inf-sup condition.However ,due to computational convenience and efficiency in practice, some mixed finite element pairs which do not satisfy the inf-sup condition are also popular. Recent studies have focused on stabilization of the lowest equal-order finite element pair $P_1 - P_1$ (linear function) or $Q_1 - Q_1$ (bilinear function) using the projection of the pressure on the piecewise constant space [6,12]. This stabilization technique is free of stabilization parameters and does not require any calculation of high-order derivatives or edge-based data structures. Therefore, this method is gaining more and more popularity in computational fluid dynamics.

The method we study in this paper mainly concentrates on a two-level stabilized method based on Newton iteration for the stationary Navier-Stokes equations, which uses the conforming piecewise quadratic polynomial approximations for the velocity and pressure based on local Gauss integration, the present pair is shown to be more computationally efficient without a loss of accuracy. Hence, this paper complements the results in Dai and Cheng [7].

The outline of this paper is organized as follows: In Section 2,we introduce the notations, an abstract functional setting of the Navier-Stokes problem and some well-known results used throughout this paper. A quadratic equal-order stabilized method is proposed in Section 3.Moreover,two-level stabilized method based on Newton iteration is given in

Section 4. In Section 5,we use some numerical experiments to verify the result of theoretical analysis. Finally, we end with a short conclusion in Section 6.

2. PRELIMINARIES

In this paper,we consider the equilibrium,incompressible Navier-Stokes equations:

$$-\nu\Delta u+(u\cdot\Delta)u+\nabla p=f,\qquad \text{in}\quad \Omega \tag{1}$$

$$\text{div u=0}\qquad \text{in}\quad \Omega \tag{2}$$

$$\text{u=0}\qquad \text{on}\quad \partial\Omega \tag{3}$$

$$\int_\Omega pdx=0\qquad \text{in}\quad \Omega \tag{4}$$

which models a steady flow of the incompressible viscous Newtonian fluid in a bounded domain. Here, Ω be a bounded convex and open subset of R^n with a Lipschitz continuous boundary $\partial\Omega$ .We define that $u=(u_1,u_2)$ and p are the velocity and pressure,respectively. $\nu>0$ is the viscosity and f is the body force per unit mass.

Next,we introduce the following Hilbert spaces

$$X=(H_0^1(\Omega))^2\quad Y=(L^2(\Omega))^2\quad M=\{q\in L^2(\Omega):\int_\Omega qdx=0\}$$

The space of $L^2(\Omega)$ is equipped with the L^2 -scalar product $(\cdot,\cdot)$ and L^2 -norm $\|\cdot\|_0$ .The space $H_0^1(\Omega)$ and X are endowed with their usual scalar product $(\nabla u,\nabla v)$ and norm $\|\nabla u\|_0$,respectively. Also, we denote by $\|\cdot\|_{L^q}$ the norm on space $L^q(\Omega)$ or $(L^q(\Omega))^2$ with $1<q\le\infty$.

Assumption 1. For a given $g\in L^2(\Omega)^d$ and Stokes problem

$$-\Delta v+\nabla q=g\qquad \text{in}\quad \Omega$$

$$divv=0\qquad \text{in}\quad \Omega$$

$$v=0\qquad \text{on}\quad \partial\Omega$$

we assume that (v,q) satisfies the following result:

$$\|v\|_2+\|q\|_1\le c\|g\|_0$$

where c is a positive constant and depend only on Ω . c will denote a positive constant which stand for different value at its different occurrences.

We define the continuous bilinear forms $a(u,v)$ and $d(u,v)$ respectively by

$$a(u,v)=\nu(\nabla u,\nabla v)\qquad \forall u,v\in X$$

and

$$d(v,p)=(divv,p)\qquad \forall v\in X,p\in M$$

Meanwhile,we define the trilinear form

$$b(u,v,w)=((u\cdot\nabla)v,w)+\frac{1}{2}((divu)v,w)$$

$$=\frac{1}{2}((u\cdot\nabla)v,w)-\frac{1}{2}((u\cdot\nabla)w,v)\qquad \forall \text{u, v, w}\in \text{X}$$

It is well known that $b(\cdot,\cdot,\cdot)$ satisfies the following properties

$$b(u,v,w)=-b(u,w,v)\qquad \forall \text{u, v, w}\in \text{X} \tag{5}$$

$$|b(u,v,w)|\le N\|\nabla u\|_0\|\nabla v\|_0\|\nabla w\|_0\qquad \forall \text{u, v, w}\in \text{X} \tag{6}$$

where

$$N=\sup_{0\ne u,v,w\in X}\frac{|b(u,v,w)|}{\|\nabla u\|_0\|\nabla v\|_0\|\nabla w\|_0}$$

is a positive constant depending only on the domain Ω.

We define a generalized bilinear form on $(X\times M)\times(X\times M)$ by

$$B((u,p);(v,q))=a(u,v)-d(v,p)+d(u,q) \quad \forall(u,p),(v,q)\in X\times M$$

Then, the variational formulation of problem (1)-(4) as follow: find a pair $(u,p)\in(X,M)$ such that for all $(v,q)\in X\times M$

$$B((u,p),(v,q))+b(u,u,v)=(f,v) \tag{7}$$

Meanwhile, we remark that $B(\cdot,\cdot,\cdot)$ and $b(\cdot,\cdot,\cdot)$ satisfy the following important properties[8]

$$|B((u,p),(v,q))|\le C(\nu\|\nabla u\|_0+\|p\|_0)(\|\nabla v\|_0+\|q\|_0) \quad \forall(u,p),(v,q)\in X\times M$$

$$\sup_{(v,q)\in X\times M}\frac{|b((u,p),(v,q))|}{\|\nabla v\|_0\|\nabla q\|_0}\ge\beta_0(\nu\|\nabla u\|_0+\|p\|_0) \quad \forall(u,p)\in X\times M$$

where β_0 is a positive constant and depends only on Ω.

The following existence and uniqueness of solution of (7) are classical results [4,6,8].

Theorem 2.1 Let $f\in X'$,and ν satisfy the following condition

$$\nu^2>N\|f\|_{-1}$$

where

$$\|f\|_{-1}=\sup_{v\in X}\frac{(f,v)}{\|\nabla v\|_0}$$

Then the solution of (7) is uniqueness and satisfies the following property:

$$\|\nabla u\|_0\le\nu^{-1}\|f\|_{-1}$$

3. A QUADRATIC EQUAL-ORDER STABILIZED METHOD

From now on, H is a real positive parameter tending to 0. Also, K_H is a uniformly regular partition of Ω into triangles with diameters bounded by the mesh size H, assumed to be uniformly regular in the usual sense. The finite element subspace $X_H\times M_H$ of $X\times M$ is characterized by K_H. Meanwhile, the fine partition K_h can be thought of as generated from K_H by a mesh refinement process. Also, we introduce finite element space $X_h\times M_h$ based on K_h .We shall assume them nested since it will simplify our analysis substantially ,i.e. $X_H\times M_H\subset X_h\times M_h\subset X\times M$ .Furthermore, let K_μ be a finite element partition with mesh size μ .Here, $\mu=h$ or H and $H \gg h$.

Then we define

$$X_\mu=\{u\in C^0(\bar\Omega)^2\cap X: u|_k\in P_2(K)^2,\forall K\in K_\mu\}$$
$$M_\mu=\{q\in C^0(\bar\Omega)^2\cap M: q|_k\in P_2(K),\forall K\in K_\mu\}$$

where $P_2(K)$ represents the space of quadratic polynomials on the set K. Note that this equal-order pair $X_\mu\times M_\mu$ does not satisfy the discrete inf-sup condition [9]:

$$\sup_{v_\mu\in X_\mu}\frac{d(v_\mu,q_\mu)}{\|\nabla v_\mu\|_0}\ge\beta\|q_\mu\|_0 \qquad \forall q_\mu\in M_\mu$$

where the constant $\beta>0$ is independent of μ .In order to fulfill this condition,a stabilized bilinear term[10,15,16] is used:

$$B_\mu((u_\mu,p_\mu),(v,q))=B((u_\mu,p_\mu),(v,q))+G_\mu(p_\mu,q)$$

where $G_\mu(p_\mu,q)$ can be defined by

$$G_\mu(p_\mu,q)=(\nabla p_\mu-\Pi_\mu\nabla p_\mu,\nabla q-\Pi_\mu\nabla q)$$

and Π_μ is an L^2-projection operator, which is defined by

$$(\zeta,\varphi_\mu)=(\Pi_\mu\zeta,\varphi_\mu), \qquad \forall\zeta\in L^2(\Omega),\varphi_\mu\in R_\mu$$

Here $R_\mu \subset L^2(\omega)$ denote the piecewise constant space associated with triangulation K_μ.The following properties of the projection operator Π_μ can be found [6,14,17]

$$\|\Pi_\mu p\|_0 \le c\|p\|_0, \qquad \forall p \in L^2(\Omega) \tag{8}$$

$$\|p-\Pi_\mu p\|_0 \le c\mu\|p\|_1, \qquad \forall p \in H^1(\Omega) \tag{9}$$

Then,we consider the finite element approximation of problem (7) is to find a pair $(u_\mu, p_\mu) \in X_\mu \times M_\mu$ such that

$$B_\mu((u_\mu, p_\mu),(v,q)) + b_\mu(u_\mu,u_\mu,v) = (f,v) \quad , \quad \forall (v,q) \in X_\mu \times M_\mu \tag{10}$$

where

$$B_\mu((u_\mu, p_\mu),(v,q)) = a(u_\mu,v) - d(v,p_\mu) + d(u_\mu,q) + G_\mu(p_\mu,q)$$

is a bilinear form defined on $\{X_\mu \times M_\mu\} \times \{X_\mu \times M_\mu\}$.

Theorem 3.1.[6,16,17] The bilinear form $B_\mu((\cdot,\cdot)(\cdot,\cdot))$ satisfies the continuous property

$$|B_\mu((u_\mu, p_\mu),(v,q))| \le c(\nu\|\nabla u_\mu\|_0 + \|p_\mu\|_0)(\|\nabla v\|_0 + \|q\|_0) \quad \forall (u_\mu, p_\mu),(v,q) \in X_\mu \times M_\mu$$

and the coercive property

$$\sup_{0\ne(v,q)\in X_\mu\times M_\mu} \frac{|B_\mu((u_\mu, p_\mu),(v,q)|}{\|\nabla v\|_0 + \|q\|_0} \ge \beta(\nu\|\nabla u_\mu\|_0 + \|p_\mu\|_0) \quad \forall (u_\mu, p_\mu) \in X_\mu \times M_\mu$$

As in [6,11,16],we have the following results:

Theorem 3.2. Assume that Assumption 1 and the uniqueness condition are valid. Let the exact solution (u,p) be in $(H^3(\Omega)^2 \cap X) \times H^2(\Omega) \cap M$ .Then, (u_μ, p_μ) of problem (10) satisfies the following stability and error estimate:

$$\nu\|\nabla u_\mu\|_0 \le \|f\|_{-1} \tag{11}$$

$$\nu\|u-u_\mu\|_0 + \mu(\nu\|\nabla(u-u_\mu)\|_0 + \|p-p_\mu\|_0) \le \mu^3\|f\|_1 \tag{12}$$

4. TWO-LEVEL STABILIZED FINITE ELEMENT METHOD

As in [4,14],we define

$$\tilde{B}_\mu((u,p);(v_\mu,q_\mu)) = B_\mu((u,p);(v_\mu,q_\mu)) - G_\mu(p,q_\mu), \quad \forall (v_\mu,q_\mu) \in X_\mu \times M_\mu$$

and introduce the projection operators (R_μ, Q_μ): $X \times M \to X_\mu \times M_\mu$ through

$$B_\mu((R_\mu(v,q),Q_\mu(v,q));(v_\mu,q_\mu)) = \tilde{B}_\mu((v,q);(v_\mu,q_\mu)) \qquad \forall (v_\mu,q_\mu) \in X_\mu \times M_\mu \tag{13}$$

which is well defined and satisfies the following approximation property (see [4,14]):

$$\nu\|R_\mu(u,p)-u\|_0 + \mu(\nu\|\nabla(R_\mu(u,p)-u)\|_0 + \|Q_\mu(u,p)-p\|_0) \le \mu^3\|f\|_1 \tag{14}$$

for all $(u,p) \in (H^3(\Omega)^2 \cap X) \times H^2(\Omega) \cap M$

Algorithm 4.1.

Step 1. Solve nonlinear system on coarse mesh, i.e., find $(u_H, p_H) \in X_H \times M_H$ such that for all $(v,q) \in X_H \times M_H$

$$B_H((u_H, p_H);(v,q)) + b(u_H;u_H;v) = (f,v) \tag{15}$$

Step 2. Solve nonlinear system on fine mesh with one Newton iteration, i.e., find $(u_h^*, p_h^*) \in X_h \times M_h$ such that for all $(v,q) \in X_h \times M_h$

$$B_h((u_h^*, p_h^*);(v,q)) + b(u_H;u_h^*,v) + b(u_h^*;u_H,v) = (f,v) + b(u_H;u_H,v) \tag{16}$$

Step 3. Update on fine mesh based on Newton iteration, i.e., find $(u_h, p_h) \in X_h \times M_h$ such that for all $(v,q) \in X_h \times M_h$

$$B_h((u^h, p^h);(v,q)) + b(u_H;u^h,v) + b(u^h;u_H,v) = (f,v) + b(u_H;u_h^*,v) + b(u_h^*;u_H - u_h^*,v) \tag{17}$$

Theorem 4.1. Under the assumptions of Theorem 3.2, u_h and u_h^* defined by Algorithm 4.1,satisfy

$$\|\nabla u_h^*\|_0 \le (\nu - N\nu^{-1}\|f\|_{-1})^{-1}(N\nu^{-1}\|f\|_{-1} + N\nu^{-2}\|f\|_{-1}^2 + \|f\|_{-1})$$

and

$$\|\nabla u^h\|_0 \le (\nu - N\nu^{-1}\|f\|_{-1})^{-1}(\|f\|_{-1} + N\nu^{-1}\|f\|_{-1}\|\nabla u_h^*\|_0 + N\|\nabla u_h^*\|_0^2)$$

Proof. Taking $(v,q)=(u_h^*,p_h^*)$ in (16) and using (6),(11), we obtain that

$$\begin{aligned}\|\nabla u_h^*\|_0 &\le (\nu - N\|\nabla u_H\|_0)^{-1}(N\|\nabla u_H\|_0^2 + \|f\|_{-1})\\ &\le (\nu - N\nu^{-1}\|f\|_{-1})^{-1}(N\nu^{-2}\|f\|_{-1}^2 + \|f\|_{-1})\end{aligned}$$

Meanwhile,setting $(v,q)=(u^h,p^h)$ in (17) and applying (6), (11) we get that

$$\begin{aligned}\|\nabla u^h\|_0 &\le (\nu - N\|\nabla u_H\|_0)^{-1}(N\|\nabla u_H\|_0\|\nabla u_h^*\|_0 + N\|\nabla u_h^*\|_0^2 + \|f\|_{-1})\\ &\le (\nu - N\nu^{-1}\|f\|_{-1})^{-1}(\|f\|_{-1} + N\nu^{-1}\|f\|_{-1}\|\nabla u_h^*\|_0 + N\|\nabla u_h^*\|_0^2)\end{aligned}$$

which gives the desired result.

Theorem 4.2. Let (u,p) , (u_h^*,p_h^*) , (u^h,p^h) be the solutions of (7),(16),(17), respectively. Then, under the assumptions of Theorem (3.2), we get the estimates as follows:

$$\|\nabla(u-u_h^*)\|_0 \le (1 - N\nu^{-2}\|f\|_{-1})^{-1}(NH^4\nu^{-3}\|f\|_1^2 + \nu^{-1}h^2\|f\|_1)$$

and

$$\|\nabla(u-u^h)\|_0 + \|p-p^h\|_0 \le (\nu - N\nu^{-1}\|f\|_{-1})^{-1}(N\nu^{-1}H^2\|f\|_1\|\nabla(u-u_h^*)\|_0 + N\|\nabla(u-u_h^*)\|_0^2 + h^2\|f\|_1)$$

Proof. Setting $e_h = R_h(u,p) - u_h^*$, $\eta_h = Q_h(u,p) - p_h^*$,subtracting (7) from (16) and using (13) ,we have

$$B_h((e_h,\eta_h),(v,q)) + b(u_H;u-u_h^*,v) + b(u-u_h^*;u_H,v) - b(u-u_H;u_H-u,v) = 0 \quad (18)$$

Meanwhile,taking $(v,q)=(e_h,\eta_h)$ in (18) and using (6),(11) (12),we obtain

$$\begin{aligned}\nu\|\nabla e_h\|_0 &\le N\|\nabla u_H\|_0\|\nabla(u-u_h^*)\|_0 + N\|\nabla(u-u_H)\|_0^2\\ &\le N\nu^{-1}\|f\|_{-1}\|\nabla(u-u_h^*)\|_0 + N\nu^{-2}H^4\|f\|_1^2\end{aligned}$$

Applying (14),we obtain that

$$\|\nabla(u-u_h^*)\|_0 \le (1 - N\nu^{-2}\|f\|_{-1})^{-1}(NH^4\nu^{-3}\|f\|_1^2 + \nu^{-1}h^2\|f\|_1)$$

Next, setting $(\varphi_h,\phi_h) = (R_h(u,p) - u^h, Q_h(u,p) - p^h)$,subtracting (6) from (17) and using (13),we obtain

$$B_h((\varphi_h,\phi_h),(v,q)) + b(u_H;u-u^h,v) + b(u-u^h;u_H,v) - b(u-u_H;u_h^*-u,v) - b(u-u_h^*;u_H-u,v) - b(u_h^*-u;u_h^*-u,v) = 0 \quad (19)$$

Setting $(v,q)=(\varphi_h,\phi_h)$ in (19) and applying (6),(11),(12), we can get

$$\nu\|\nabla\varphi_h\|_0 \le N\nu^{-1}\|f\|_{-1}\|\nabla(u-u^h)\|_0 + N\nu^{-1}H^2\|f\|_1\|\nabla(u-u_h^*)\|_0 + N\|\nabla(u-u_h^*)\|_0^2$$

Using (14),we get

$$\|\nabla(u-u^h)\|_0 \le (\nu - N\nu^{-1}\|f\|_{-1})^{-1}(N\nu^{-1}H^2\|f\|_1\|\nabla(u-u_h^*)\|_0 + N\|\nabla(u-u_h^*)\|_0^2 + h^2\|f\|_1)$$

The estimate for the $\|p-p_h\|_0$ can be obtained from the inf-sup condition. So, we complete the proof of this theorem.

Corollary 4.1. If we choose H such that $h = O(H^2)$ for the Algorithm 4.1, then, the method we study provides an approximate solution (u^h,p^h) with the convergence rate of same order as the one level method solution (u_h,p_h) ,which involves solving one large Navier-Stokes problem on a fine mesh with mesh size h. So,our method will be more effective and convenient .

5. NUMERICAL EXPERIMENTS

In this sections, we present numerical experiments to validate the theory developed in the above sections and illustrate the efficiency of our two-level method based on Newton iteration ,This method is characterized by using quadratic polynomial functions for both the velocity and pressure field. Now, the stabilized term is defined by local Gauss integration[6] as follow:

$$G_\mu(p_\mu,q)=\sum_{(k_\mu)_j\in K_\mu}\left\{\int_{(K_\mu)_{j,2}}\nabla p_\mu\cdot\nabla q dx-\int_{(K_\mu)_{j,1}}\nabla p_\mu\cdot\nabla q dx\right\},\forall p_\mu,q\in M_\mu$$

where $\int_{(K_\mu)_{j,i}}\nabla p_\mu\cdot\nabla q dx$ indicates a local Gauss integral over $(K_\mu)_j$ that is exact for polynomials of degree i (i=1,2), and $\nabla p_\mu\cdot\nabla q$ is a polynomial of degree not greater than 2.Thus,the trial function $\nabla p_\mu\in P_1$ must be projected to piecewise constant space, when i=1 for any $\nabla q\in P_1$.

Firstly,we consider an exact solution problem,Let Ω be the unit square in R^2.The exact solution for the velocity and pressure is given as follows:

$$u_1(x,y)=10x^2(x-1)^2y(y-1)(2y-1)$$
$$u_2(x,y)=-10x(x-1)(2x-1)y^2(y-1)^2$$
$$p(x,y)=10(2x-1)(2y-1)$$

and the right-hand side $f=(f_1(x,y),f_2(x,y))$ is determined by original problem (1).

Our goal is to show the prominent properties of our method as compared with the one-level method.So we choose the same fixed value of h for the one-level method as the mesh spacing h in the finest grid process for the two-level method. In two-level computations,h is given,so we choose H such that $h=O(H^2)$ in Algorithm 4.1. At last ,we assume that the viscosity $\nu=1$ in the equations of (1)-(4), and pick five value of h, i.e., 1/5, 1/10, 1/15, 1/20 and 1/25.

Table1:Comparisons of the one-level with the two-level-method

Method	H	h	CPU-time	$\frac{\|\nabla(\mathbf{u}-\mathbf{u}_h)\|_0}{\|\nabla\mathbf{u}\|_0}$	$\frac{\|p-p_h\|_0}{\|p\|_0}$
One-level	~	1/5	0.121	1.10028E-01	2.80282E-02
Algorithm4.1	1/2	1/5	0.032	1.09764E-01	3.10451E-02
One-level	~	1/10	0.302	2.77092E-02	6.56642E-02
Algorithm4.1	1/3	1/10	0.093	2.88795E-02	7.76887E-03
One-level	~	1/15	0.708	1.29342E-02	3.00332E-03
Algorithm4.1	1/4	1/15	0.291	1.29981E-02	3.46510E-03
One-level	~	1/20	1.252	7.27666E-03	1.03542E-03
Algorithm4.1	1/4	1/20	0.391	7.34774E-03	1.97850E-03
One-level	~	1/25	2.013	4.69352E-03	7.26892E-04
Algorithm4.1	1/5	1/25	0.625	4.70351E-03	1.28911E-03

From table 1,we can find that the numerical results of one-level and two-level method just like the theoretical analysis in previous section. As expected, the CPU time of the two-level method is less than the one-level method under nearly the same relative error. Other example have been tested, and similar performance has been observed.

6. CONCLUSIONS

In this work we have proposed the two-level quadratic equal-order finite element method in solving the steady Navier-Stokes equations based on local Gauss integration. The main feature of our method is combine the quadratic equal-order stabilized method with two-level discretization.It includes three steps: first we solve one small, nonlinear system on a coarse mesh, then solve one linearized problem on fine mesh based on Newton's method and at last solve one linear correction problem on the coarse mesh. It is shown that the given method is stable .Moreover, we have derived the error estimates for the discrete stabilized finite element solution (u^h,p^h). By above analysis, if we choose H such that $h=O(H^2)$, then the two-level method we study is of the convergence rate of same order as the usual one-level stabilized finite element method. At last, numerical tests for solving the Navier-Stokes equations have shown the better performance of our method.

7. REFERENCES

[1] J.C.Xu,A novel two-grid method for semiliear elliptic equations, SIAM J.Sci.Comput., 15(1994),231-237
[2] J.C.Xu,Two-grid discretization techniques for linear and nonliear PDEs,SIAM J.Numer.Anal 33(1996),1759-1777
[3] Y.N.He,J.Li,Two-level methods based on three corrections for the 2D/3D steady Navier-Stokes equations.,2(2011),42-56

[4] J.Li,Z.Chen, A new local stabilized nonconforming finite element for the Stokes equations,Computing 82(2008):157-170

[5] P.Z.Huang,Y.N.He,X.L.Feng,Convergence and stability of two-level penalty mixed finite element method for stationary Navier-Stokes equations,Front.Math.China.,8(2013):837-854

[6] J.Li, Y.He, A stabilized finite element method based on two local Gauss integrations for the Stokes equations, J. Comput. Appl. Math. 214 (2008) :58-65

[7] X.X.Dai,X.L.Cheng ,A two-grid method based on Newton iteration for the Navier-Stokes equations,J.Comput.Appl.Math. 220(2008)566-573

[8] L.P.Zhu,J.Li,Z.X.Chen, A new local stabilized noncomforming finite element method for solving stationary Navier-Stokes equations, J.Comput.Appl.Math 235(2011)2821-2831

[9] A.Labovsky, W.J.Layton,C.C.Manica,M.Neda and L.G.Rebholz, The stabilized extrapolted trapezoidal finite element method for the Navier-Stokes equations, Comput.Methods Appl. Mech. Engrg.,198(2009):263-274

[10] P.Bochev,C.R.Dohrmann,M.D.Gunzburger,Stabilization of low-order mixed finite elements for the Stokes equations, SIAM J. Numer. Anal. 44(2006): 82-101

[11] Y.N.He,A.W.Wang,L.Q.Mei,Stabilized finite-element method for the stationary Navier-Stokes equations, J.Engrg.Math.51(2005):367-380

[12] J.Li,Y.N.He,and Z.X.Chen, Performance of several stabilized finite elements for Stokes equations based on the lowest equal-order pairs,Computing,86(2009):37-51

[13] P.Z.Huang,X.L.Feng,Y.N.He,Two-level defect-correction oseen iterative stabilized finite element methods for the stationary Navier-Stoeks equations,Appl.Math.Modelling.,37(2013):728-741

[14] P.Z.Huang, X.L.Feng, D.M.Liu, Two-level stabilized method based on three corrections for the stationary Naiver-Stokes equations, Appl. Numer.Math.,62(2012):988-1001

[15] J.Li,Y.N.He,Z.X.Chen, A new stabilized finite element method for the transient Navier-Stokes equations, Comput.Methods Appl. Mech. Engrg. 197(2007):22-35

[16] H.B.Zheng,L.Shan, Y.R. Hou, A quadratic equal-order stabilized method for the Stokes problem based on two local Gauss integrations, Numer.Methods Partial. Differential Equations 26(2010):1180-1190

[17] J.Li, Investigations on two kinds of two-level stabilized finite element methods for the stationary Navier-Stokes equations, Appl. Math. Comput.182(2006):1470-1481