Algorithm to Solve Spherical Fuzzy Optimization Problems with Applications

Sudhir kumar[1], Anita kumari[2]

[1]Department of Mathematics, Dr. Shyama Prasad Mukherjee University
Ranchi, Jharkhand-834001, India
Email: sudhir.kr.8826@gmail.com

[2]Department of Mathematics, Dr. Shyama Prasad Mukherjee University
Ranchi, Jharkhand-834001, India
Email: anitamehta@gmail.com

ABSTRACT---- In operation research, a specific area being analysed in great depth is the optimization problem. The primary objective of this issue is to determine the most economical costs for commodities in order to satisfy customer requirements at various destinations, while taking into account the resources available at their points of origin or deals with assigning tasks to locations. In this paper, the spherical fuzzy optimization problem (SFOP) determines the optimal cost of carrying items from origin to destination (or job to machine). While reliable data is frequently employed, these variables are actually ambiguous and inaccurate. Many generalizations and expansions of fuzzy sets have been proposed and investigated in the literature. The spherical fuzzy set (SFS) is one of the most recent developments in fuzzy sets. It is capable of identifying neutral degrees in addition to membership and non-membership degrees. In this study, a proposed approach is developed to find the solution for each of all three form of the SFOP. For a better understanding, the suggested article includes six solved situations together with screen grabs of the output summaries from the software used for the calculations. Additionally, the unique approach's advantages over the current work are mentioned. Finally, conclusion and future scope direction are also given.

Keywords: Spherical fuzzy set, Optimal solution, TORA, SFAP, SFTP

1. INTRODUCTION

There are numerous real-world applications involving problems with transportation. Transportation issues are a particular type of linear programming problem (LPP) where the goal is to determine the most economical and efficient way to distribute a good from a group of sources to a set of destinations [1]. The system of transportation issues is depicted in Figure 1. The cost of transportation and a product's price are strongly associated; that is, changes in transportation costs will cause a product's price to vary as well. For the same reason, a suitable method of transporting the product from various sources to various destinations must be found [2]. It has long been held that precise amounts should be used to indicate the transversal expenses related to supply and demand.

These values are typically ambiguous or imprecise, though. Zadeh [3] developed the FS theory, which may be utilized to more efficiently handle data uncertainty in decision-making scenarios by reflecting uncertain data by its membership grade. Atanassov [4] originally defined the intuitionistic fuzzy set (IFS), which is made up of a membership value and a non-membership value. In the fuzzy environment, specialists have so tried to resolve a number of transportation-related problems. Chanas et al.[5] were the first to present fuzzy transportation challenges. Numerous scholars have since investigated transportation issues under various fuzzy conditions, including interval valued analysis, fuzzy arithmetic data envelopment analysis, fuzzy assignment, triangular intuitionistic fuzzy, completely intuitionistic multi-objective fractional [6], and fuzzy evaluation. Fuzzy (intuitive) [7] fuzzy delphi [8] and so on. IFSs can be used in a wide range of domains, however they are not able to supply all the information [9]. There may be a situation when the sum of membership and non-membership exceeds one. Pythagorean fuzzy sets (PyFS), which were proposed by Yager [10] as a useful expansion of IFS, are another evolution of fuzzy ideas. Additional features of PFS [11] include the membership level and the non-membership level, both of which have sums of squares that are less than or equal to one. Other academics later used PFS to address multi-criteria decision making and linear programming problems. Kumar and associates [12]. Real-world scenarios frequently suggest neutral ratings in addition to membership and non-membership grades. This kind of ambiguous data cannot be handled by fuzzy sets or IFS. Cuong [13] first proposed the novel concept of the picture fuzzy set (PFS) as a solution to this issue. The positive, neutral, and negative grade of a decision-maker's response to a remark are 0.5, 0.3, and 0.1, respectively. The development process for picture FS now includes a neutral function, which offers a better way to handle challenging circumstances. The concept of PFSs has been used to model a variety of real-world decision-making problems [14] using methods including similarity and proximity, among others. Sometimes PFS is unable to handle problems in real life, like when$\alpha_F + \beta_F + \Upsilon_F \leq 1$. SFS are a direct generalization of PFS and PyFS. There was an interesting instance when the situation was outside the scope of both PyFS and PFS. When opinions include some rejection or abstinence in addition to yes/no answers, spherical fuzzy sets might be helpful. In decision-making processes, such as those where four decision-makers assess applicants based on four different categories, a representative example of a spherical fuzzy set is commonly found. The voting procedure, in which there are four types of voters—those who vote for, against, do not vote, or abstain from voting—could serve as another example. Therefore, in order to handle this situation, the spherical fuzzy set is needed [15]. Ashraf et al. [16] introduce the idea of SFSs based on these circumstances as an extension of PFS. The situation in SFS is becoming better with membership degrees under the condition that $0 < \alpha_F^2 + {\beta_F}^2 + \Upsilon_F^2 < 1$. Donyatalab et al.[17] extended the conventional linear assignment strategy to a spherical fuzzy linear AM in order to handle a variety of factors group decision making challenges. Wei et al.[18] developed similarity metrics of spherical uncertain ideas, which are used in pattern identify and diagnostics in medicine by use of the cosine function. A fuzzy spherical TP and AP was examined using three models by Kumar et al.[42]. As was previously said, the SFTP and SFAP has not received much attention. Additionally, the researchers used Vogel's approximation technique (VAM) and software to solve the SFTP and SFAP. The literature research indicates that there are no special techniques for SFTP problem resolution. This encourages the authors to develop a new technique that does not require any mathematical tools to determine its optimal value of SFTP. Reducing overall optimal costs is the main goal of this study. The paper's main contribution is as follows:

(1) An innovative method for figuring out the optimization problem of kinds I, II, and III is presented.

(2) To validate the proposed approach, random values of the three types were applied.

(3) This technique is used to validate both balanced and unbalanced concerns.

The assignment problem is a fundamental concept in optimization theory, where the objective is to assign resources to tasks in such a way that some criteria, typically the cost or efficiency, are optimized. Over the past decade, the integration of fuzzy logic into assignment problems has become increasingly prevalent, offering a more nuanced and flexible approach to real-world problems. Spherical fuzzy sets (SFS) represent a further evolution in this line of research, allowing for more complex, uncertain, and imprecise decision-making. Spherical fuzzy sets, unlike classical fuzzy sets, introduce a three-dimensional structure in decision-making, which includes not only the membership and non-membership functions but also a degree of hesitancy. This additional dimension provides more flexibility in modeling uncertainty. The concept of spherical fuzzy sets was introduced to extend traditional fuzzy sets by incorporating three components: the degree of membership (μ), the degree of non-membership (ν), and the degree of hesitancy (π). These three components must satisfy the condition $\mu+\nu+\pi\leq 1$, which ensures the consistency of the fuzzy set. This addition of the hesitancy degree allows for a more expressive representation of uncertainty, making it particularly useful in decision-making situations where information is incomplete or contradictory. In the last decade, much of the focus has been on applying spherical fuzzy sets to multi-criteria decision-making (MCDM) problems, where the goal is to evaluate and select among a set of alternatives based on multiple criteria. The spherical fuzzy framework has been particularly useful for handling situations with high uncertainty, such as when preferences are vague or inconsistent.

In 2015, Xu and Da presented a spherical fuzzy decision-making model that used the three-dimensional fuzzy set to model uncertain preferences in MCDM [19]. Their model was designed to handle cases where experts' opinions were not precise but could be captured more effectively using the spherical fuzzy set. They applied their approach to an assignment problem where multiple tasks were allocated to different agents under uncertain criteria. Recent trends in SFAP research suggest a growing interest in hybrid models that combine spherical fuzzy sets with other fuzzy techniques. For example, in 2024, a study by Kumar et al. combined spherical fuzzy sets with interval-valued fuzzy sets to address assignment problems with both uncertainty and vagueness [20]. This hybrid approach improved the decision-making process by offering a more comprehensive representation of uncertain parameters. Another interesting direction is the integration of spherical fuzzy models with metaheuristic algorithms such as genetic algorithms (GA), particle swarm optimization (PSO), and ant colony optimization (ACO). These hybrid approaches have been shown to improve the efficiency and robustness of the assignment problem solution, particularly when the problem size is large or the uncertainty is high.

One of the major challenges in spherical fuzzy assignment problems is the computational complexity of the solution algorithms. While algorithms such as the spherical fuzzy Hungarian method have shown promise in small to medium-sized problems, they may struggle to scale when dealing with large datasets. To address this, researchers are focusing on developing more efficient algorithms, such as those based on optimization heuristics and machine learning, to speed up the solution process. The following are the planned stages of this study: Section establishes the foundation for the mathematical structure of the SFTP and SFAP, while the subsequent section 2 presents the initial results and associated mathematical processes. A method for obtaining optimal Solution is presented in Section 3, and Section 4 includes real-world examples. The results and suggestions are presented in Section 6. Additionally, Section 7 discusses the conclusion and further research.

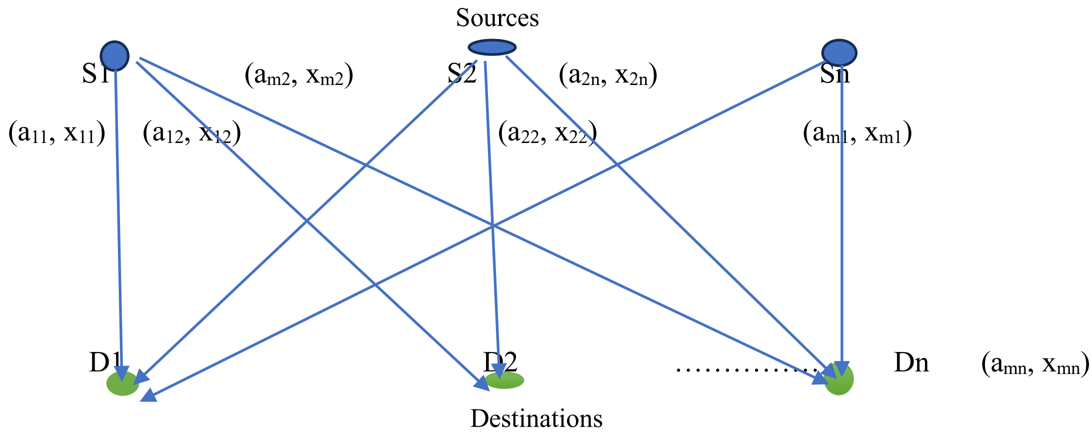


Fig. 1: Network Diagram Representing the Structure of a Fuzzy bi-objective TP

2. PRELIMINARIES

Let U be the universe throughout of this paper.

Definition 2.1 **(Atanossov,1983 – Yagar,2013)**

Let $\alpha_I: U \to [0,1]$ and $\beta_I: U \to [0,1]$ be two mappings

A set I = $\{\langle X, \alpha_I, \beta_I\rangle | X \in U\}$ is called

(1) Intuitionistic fuzzy set (IFS) if $0 \leq \alpha_I + \beta_I \leq 1 \; \forall \; X \in U$
(2) Pythagorean fuzzy set (PyFS) if $0 \leq \alpha_I^2 + \beta_I^2 \leq 1 \; \forall \; X \in U$

The values $\alpha_I, \beta_I \in [0,1]$ denote the degree of positive membership and negative membership of X to I, respectively.

The pair I = $\langle X, \alpha_I, \beta_I\rangle$ where $\alpha_I, \beta_I \in [0,1]$ and $\alpha_I + \beta_I \leq 1$ $(\alpha_I^2 + \beta_I^2 \leq 1)$, is called a intuitionistic fuzzy number(IFN)(Pythagorean fuzzy number(PyFN)).

Definition 2.2 **(Coung,2013-Ashrof et al.,2018)**

Let $\alpha_F: U \to [0,1]$: $\beta_F: U \to [0,1]$: $\Upsilon_F: U \to [0,1]$ be three mappings.

A set F = $\{\langle X, \alpha_F, \beta_F, \Upsilon_F\rangle | X \in U\}$ is called

(1) Picture fuzzy set (PFS) if $0 \leq \alpha_F + \beta_F + \Upsilon_F \leq 1$,$\forall \; X \in U$

(2) Spherical fuzzy set (SFS) if $0 \leq \alpha_F^2 + {\beta_F}^2 + \Upsilon_F^2 \leq 1, \forall \; X \in U$

The values $\alpha_F, \beta_F, \Upsilon_F \in [0,1]$ denote the degree of positive membership, neutral-membership and negative membership function of X to G, respectively.

The triplet S = $(\alpha_F, \beta_F, \Upsilon_F)$ where $\alpha_F, \beta_F, \Upsilon_F \in [0,1]$ and $\alpha_F^2 + {\beta_F}^2 + \Upsilon_F^2 \leq 1$ $(\alpha_F + \beta_F + \Upsilon_F \leq 1)$ is called SFN & PFN.

Remark:

(1) PFN interprets the total as linear and represents a plane in space since the sum of the three positive, neutral, and negative memberships is less than or equal to 1.
(2) However, for SFN, we take into account the non-linear shape of membership functions that depict a sphere in space.

Definition 2.3 **(Kutlu Gundogdu and Kahraman, 2019)**

Let F = $<\alpha_F, \beta_F, \Upsilon_F>$, F₁ = $<\alpha_{F_1}, \beta_{F_1}, \Upsilon_{F_1}>$, F₂ = $<\alpha_{F_2}, \beta_{F_2}, \Upsilon_{F_2}>$ be three SFNs, K≥ 0, and $\mu > 0$. Then the operations between SFNs are defined as follows:

(1) $F^C = <\Upsilon_F, \beta_F, \alpha_F>$
(2) $F_1 \leq F_2$ iff $\alpha_{F_1} \leq \alpha_{F_2}$, $\beta_{F_1} \leq \beta_{F_2}$ and $\Upsilon_{F_2} \geq \Upsilon_{F_1}$
(3) $F_1 = F_2$ iff $F_1 \leq F_2$ *and* $F_2 \leq F_1$
(4) $F_1 \oplus F_2$ iff $< \sqrt{\alpha_{F_1}^2 + \alpha_{F_2}^2 - \alpha_{F_1}^2\alpha_{F_2}^2}\ , \sqrt{(1-\alpha_{F_1}^2)\beta_{F_1}^2 + (1-\alpha_{F_2}^2)\beta_{F_1}^2 - \alpha_{F_1}^2\alpha_{F_2}^2}\ , \Upsilon_{F_1}\Upsilon_{F_2} >$
(5) $F_1 \otimes F_2$ iff $< \alpha_{F_1}\alpha_{F_2}\ , \sqrt{(1-\Upsilon_{F_1}^2)\beta_{F_1}^2 + (1-\Upsilon_{F_2}^2)\beta_{F_1}^2 - \beta_{F_1}^2\beta_{F_2}^2}\ , \sqrt{\Upsilon_{F_1}^2 + \Upsilon_{F_2}^2 - \Upsilon_{f_1}^2\Upsilon_{F_2}^2} >$
(6) $KG = < \sqrt{1-(1-\alpha_{F_1}^2)^k}\ , \sqrt{((1-\alpha_F^2)^k) - (1-\alpha_F^2-\beta_F^2)^k}\ , \Upsilon_F^k >$
(7) $G^k = < \alpha_F^K\ , \sqrt{(1-\Upsilon_F^2)^k - (1-\Upsilon_F^2-\beta_F^2)^k}\ , \sqrt{1-(1-\Upsilon_F^2)^k} >$

Definition 2.4 **(Ashraf at al., 2019)**

Let ϕ be the collection of all GSFNs and F ϵ ϕ where F = $<\alpha_F, \beta_F, \Upsilon_F>$.

(1) A score function SF: $\phi \to [0,1]$ is defined as

$$SF(F) = \frac{2+\alpha_F-\beta_F-\Upsilon_F}{3}$$

(2) An accuracy function AF: $\phi \to [-1,1]$ is defined as

$$AF(F) = \alpha_F - \Upsilon_F$$

Definition 2.5 **(Haque et al., 2020)**

Let $H_1 = <\phi_1, \phi_1, \phi_1>$ and $H_2 = <\phi_2, \phi_2, \phi_2>$ be two GSFNs. Then the ranking method (comparison technique) as follows:

(1) If SF(H_1) < SF(H_2), then $H_1 < H_2$
(2) If SF(H_1) > SF(H_2), then $H_1 > H_2$
(3) SF(H_1) = SF(H_2), then
 (i) AF(H_1) < AF(H_2), then $H_1 < H_2$
 (ii) AF(H_1) < AF(H_2), then $H_1 < H_2$
 (iii) AF(H_1) = AF(H_2), then $H_1 = H_2$

2.6 Score and Accuracy Functions

Let the SFS be represented by $S_F = <\alpha_F, \beta_F, \Upsilon_F>$. The accuracy and score functions have the following definitions:

$S(S_F) = (\alpha_F - \beta_F)^2 - (\Upsilon_F - \beta_F)^2$, $S_F \in [-1,1]$ (4)

Accuracy(S_F) = $\alpha_F^2 + {\beta_F}^2 + \Upsilon_F^2$, Accu.(S_F) $\in [0,1]$ (5)

Two SFSs, SF_1 and SF_2, have an order connection that is expressed as

$SF_1 < SF_2$ iff score (SF_1) < score (SF_2) or score (SF_1) = score (SF_2) or score (SF_1) > score (SF_2)

2.7 Mathematical Formulation of SFTP

Think about "m" suppliers and "n" locations. While the distribution network works to lower the cost of transporting goods from those suppliers to the locations, the demand and accessibility of goods are determined by a few assumptions and limitations. The mathematical expressions for a SFTP as follows in equation 6-9 and Table 1:

r – complete m source index

s – complete index of the destination for n

x_{rs} - is the quantity of products shipped in units from the starting point to the final destination

Minimize or Maximize $\tilde{z}^{SF} = \sum_{r=1}^{n} \sum_{s=1}^{m} \tilde{P}_{rs}^{SF} x_{rs}$ 6

Subject to conditions

$\sum_{s=1}^{n} x_{rs} = \tilde{\alpha}_r^{SF}$, r = 1 to m, (7)

$\sum_{r=1}^{n} x_{rs} = \tilde{\beta}_s^{SF}$, s = 1 to n, (8)

$x_{rs} \geq 0 \ \forall \, r, s$ (9)

Where, $\tilde{P}_{rs}^{SF}$ – The spherical fuzzy cost of shipping one unit from a certain good source i to recipient j.

$\tilde{\alpha}_r^{SF}$ – fuzzy spherical items of supply that must be transported between n locations.

$\tilde{\beta}_s^{SF}$ – The spherical fuzzy number of demand units required at the endpoints.

Table 1. Spherical Fuzzy Transportation Problem

Sources / Destinations	D_1	D_2	⋯	D_n	Supply
S_1	d_{11}^{SF}	d_{12}^{SF}	⋯	d_{1n}^{SF}	α_1^{SF}
S_2	d_{21}^{SF}	d_{22}^{SF}	⋯	d_{2n}^{SF}	α_2^{SF}
⋮	⋮	⋮	⋮	⋮	⋮
S_m	d_{m1}^{SF}	d_{m2}^{SF}	⋯	d_{mn}^{SF}	α_m^{SF}
Demand	β_1^{SF}	β_2^{SF}	⋯	β_n^{SF}	

2.8 Mathematical Formulation of SFAP

Spherical fuzzy sets are used in the Spherical Fuzzy Assignment issue (SFAP), a variation of the classic assignment issue, to address imprecision, vagueness, and uncertainty in decision-making. With spherical fuzzy sets, each element is represented by a triplet of membership values (represented by the letters μ, v, and π), which stand for the degrees of membership, non-membership, and hesitation, respectively. This approach to handling uncertainty is more thorough. When allocating agents to jobs, the goal is typically to minimize or maximize a cost or profit.

If $\tilde{P}_{rs}^{SF}$ represents the spherical fuzzy cost for assigning agent r to task s, then the objective function Z could be formulated as:

Minimize $or\ Maximize\ \tilde{z}^{SF} = \sum_{r=1}^{n} \sum_{s=1}^{n} \tilde{P}_{rs}^{SF} x_{rs}$ (10)

Subject to conditions

$\sum_{s=1}^{n} x_{rs} = 1$, r = 1 to n, (11)

$\sum_{r=1}^{n} x_{rs} = 1$, s = 1 to n, (12)

$x_{rs} \geq 0\ \forall\ r, s$ (13)

Table 2. Spherical fuzzy assignment problem

Jobs / Machines	M_1	M_2	⋯	Mn	Supply
S_1	d_{11}^{SF}	d_{12}^{SF}	⋯	d_{1n}^{SF}	1
S_2	d_{21}^{SF}	d_{22}^{SF}	⋯	d_{1n}^{SF}	1
⋮	⋮	⋮	⋮	⋮	⋮
S_m	d_{m1}^{SF}	d_{m2}^{SF}	⋯	d_{mn}^{SF}	1
Demand	1	1	⋯	1	

3. METHODOLOGY

In this section, we develop a novel method with spherical fuzzy information based on

Step 1: Select the optimization problem (SFAP OR SFTP) in the spherical fuzzy environment**.**

Step 2: To transform spherical fuzzy values into crisp values, apply the suggested scoring function found in Equation 4.

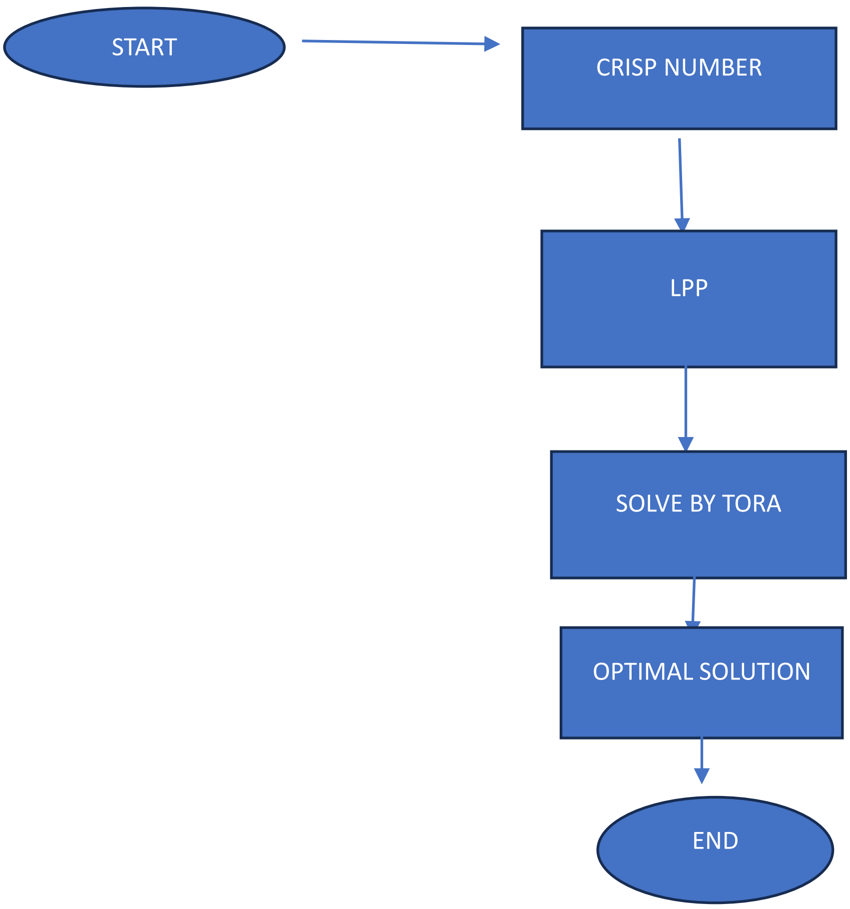
Step 3: Crisp numbers are into linear Programming problem.

Step 4: Applying TORA software, resolve the crisp optimization problem. This stage produces the best objective value and optimal solution for clear optimization issues.

Step 5: The fuzzy optimization problem's lowest and maximum objective values can be ascertained using equations (6) and (10) based on the solution that was achieved in step 3.

Step 6: The optimal solution and the optimal allocation for the assignment problem is achieved.

3.1 Flow Chart of Algorithm



4. NUMERICAL EXAMPLES

The proposed approach is demonstrated in this section with six issues that correspond to three distinct models.

Example 1

A company is selecting 4 suppliers (S1, S2, S3, S4) for 4 project tasks (T1, T2, T3 ,T4) based on their performance ratings given in spherical fuzzy numbers. Find the optimal assignment that minimizes and maximizes efficiency.

Table 3. SFAP

Suppliers/Tasks	T1	T2	T3	T4
S1	(0.8,0.3,0.2)	(0.6,0.4,0.3)	(0.7,0.5,0.2)	(0.5,0.4,0.5)
S2	(0.6,0.5,0.3)	(0.7,0.4,0.2)	(0.5,0.3,0.5)	(0.8,0.2,0.3)
S3	(0.7,0.3,0.4)	(0.5,0.5,0.3)	(0.6,0.4,0.5)	(0.7,0.3,0.2)
S4	(0.5,0.4,0.4)	(0.6,0.3,0.4)	(0.7,0.5,0.3)	(0.8,0.2,0.4)

Solution:

Steps-I Applying the suggested ranking function shown in Eq. 4 will transform spherical fuzzy values into crisp values.

Table 4. CAP

Suppliers/Tasks	T1	T2	T3	T4
S1	0.24	0.03	-0.05	0
S2	−0.03	0.05	0	0.35
S3	0.15	−0.04	0.03	0.15
S4	0.01	0.08	0	0.32

Steps-II Crisp numbers are converted into Lpp.

Min $\tilde{z}^{SF}$ = 0.24Z11 + 0.03Z12 +(- 0.05) Z13 + 0 Z14 +(-0.03) Z21 + 0.05Z22 + 0 Z23 + 0.35 Z24 + 0.15Z31 + (-0.04) Z32 + 0.03 Z33 + 0.15Z34 + 0.01Z41 + 0.08Z42 + 0Z43 + 0.32Z44

Subject to constraints

0.24Z11 + 0.03Z12 +(- 0.05) Z13 + 0 Z14 = 1 (Row 1 restriction)

(-0.03) Z_{21} + 0.05Z22 + 0 Z23 + 0.35 Z24 = 1 (Row 2 restriction)

0.15Z31 + (-0.04) Z32 + 0.03 Z33 + 0.15Z34 =1 (Row 3 restriction)

0.01Z41 + 0.08Z42 + 0Z43 + 0.32Z44 =1 (Row 4 restriction)

0.24Z11 + (-0.03) Z_{21} + 015Z31 + 0.01Z41 =1 (column 1 restriction)

0.03Z12 + 0.05Z22 + (-0.04) Z32 + 0.08Z42 =1 (column 2 restriction)

(- 0.05) Z13 + 0 Z23 + 0.03 Z33 + 0Z43 =1 (column 3 restriction)

0 Z14 + 0.35 Z24 +0.15Z34 + 0.32Z44 =1 (column 4 restriction)

Z11, Z12, Z13, Z14, Z21, Z22, Z23, Z24, Z31, Z32, Z33, Z34, Z41, Z42, Z43, Z44 ≥ 0(Non negative restrictions)

Step III Using TORA software to solve this problem, and the best result is shown in screenshots 1 (i.e., fig. 2).

The result image displays the crisp's optimal objective value and optimal solution.

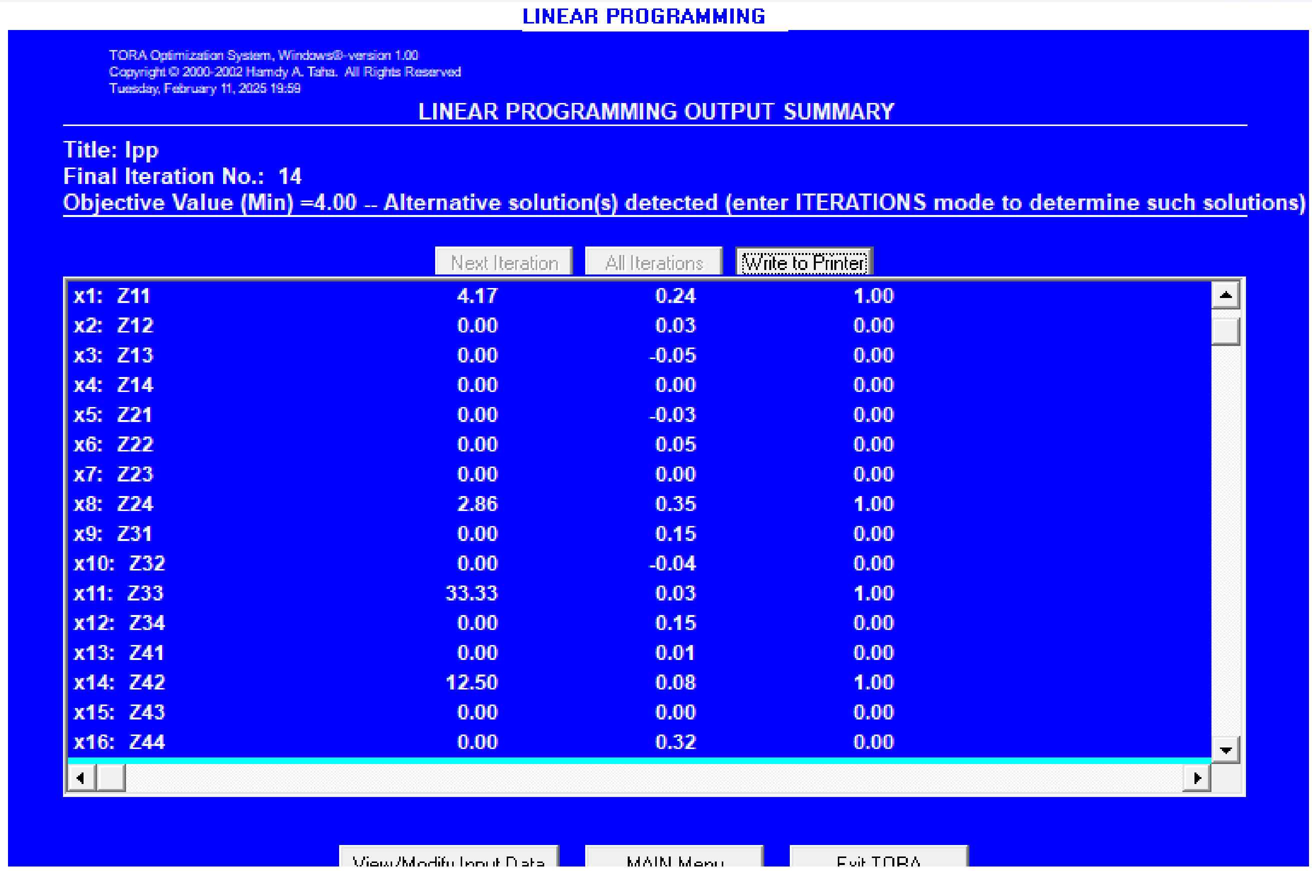


Fig. 2: LPP (Iteration No. 14)

Therefore, the optimal assignment to the given real-life problem is

Machine S1 job T1

Machine S2 job T4

Machine S3 job T3

Machine S4 job T2

Both the best solution to the given FAP and its corresponding CAP are identical. Because each problem's choice variables have distinct numerical values.

Min $\tilde{z}^{SF}$ = (0.24) × 4.17+(2.86) × 0.35+(33.33) × (-0.04) + (12.50) × 0.8 = 4.00

From step 4, we can write the optimal objective value for the given FAP is as follows.

Min $\tilde{z}^{SF}$ = (1,2,2,4,5;1)×1+(1,4,5,9,16;1)×1+(3,6,9,12,15;1)×1+(2,4,6,8,10;10)×1 = (7,16,23,33,46;1)

Example 2

A hospital needs to assign 4 doctors (D1, D2, D3, D4) to 4 patients (P1, P2, P3, P4) based on their expertise, availability uncertainty, and unsuitability levels. Find the best doctor-patient assignment.

Table 5. SFAP

Doctors(D)/Patients(P)	S1	S2	S3	S4
D1	(0.9,0.2,0.1)	(0.7,0.3,0.2)	(0.6,0.4,0.3)	(0.5,0.3,0.4)
D2	(0.6,0.3,0.4)	(0.8,0.2,0.3)	(0.7,.5,0.2)	(0.6,0.4,0.3)
D3	(0.7,0.4,0.2)	(0.5,0.3,0.5)	(0.8,0.2,0.3)	(0.6,0.4,0.2)
D4	(0.5,0.3,0.4)	(0.6,0.5,0.3)	(0.7,0.4,0.5)	(0.9,0.2,0.1)

Solution: Obtained optimal solution by using the suggested method, and 1.90 is the optimal value.

Example 3

A university is assigning 4 professors (P1, P2, P3, P4) to 4 courses (C1, C2, C3, C4) based on their qualifications, uncertainty in availability, and unsuitability. Determine the optimal assignment that improves teaching efficiency.

Table 6. SFAP

Professors(P)/Courses(C)	C1	C2	C3	C4
P1	(0.8,0.3,0.2)	(0.6,0.4,0.3)	(0.7,0.5,0.2)	(0.5,0.4,0.5)
P2	(0.6,0.5,0.3)	(0.7,0.4,0.2)	(0.5,0.3,0.5)	(0.8,0.2,0.3)
P3	(0.7,0.3,0.4)	(0.5,0.5,0.3)	(0.6,0.4,0.5)	(0.7,0.3,0.2)
P4	(0.5,0.4,0.5)	(0.6,	(0.7,0.5,0.3)	(0.8,0.2,0.4)

Solution: Obtained optimal solution by using the suggested method, and 1.84 is the optimal value.

Example 4

In order to calculate the lowest possible transportation cost, the supply and demand are expressed in crisp, and the costs in Table 6 of SFTP for model-I are expressed in spherical fuzzy.

Table 7. SFTP of model-I

Source/Destination	D1	D2	D3	Supply
S1	(0.7,0.2,0.1)	(0.8,0.1,0.1)	(0.6,0.3,0.1)	100
S2	(0.6,0.2,0.1)	(0.9,0.05,0.05)	(0.5,0.4,0.1)	150
S3	(0.75,0.2,0.05)	(0.7,0.25,0.05)	(0.65,0.3,0.05)	200
Demand	120	180	150	Type equation here.

Solution:

Table 8. CAP

Source/Destination	D1	D2	D3	Supply
S1	0.24	0.49	0.05	100
S2	0.015	0.7225	-0.08	150
S3	0.28	0.1625	0.6	200
Demand	120	180	150	Type equation here.

Min $\tilde{z}^{SF}$ = 0.24Z11 + 0.49Z12 + 0.05Z13 +0.15 Z_{21} + 0.7225Z_{22} +(- 0 .8) Z23 + 0.28Z31 + 0.1625 Z32 + 0.6 Z33

Subject to constraints

0.24Z11 + 0.49Z12 + 0.05 Z13 = 100 (Row 1 restriction)

0.15 Z_{21} + 0.7225Z22 + (-0.08) Z23 = 150 (Row 1 restriction)

0.28Z31 + 0.1625 Z32 + 0.6Z33 =200 (Row 1 restriction)

$0.24Z11 + 0.15\ Z_{21} + 0.28Z31 = 120$ (Column 1 restriction)

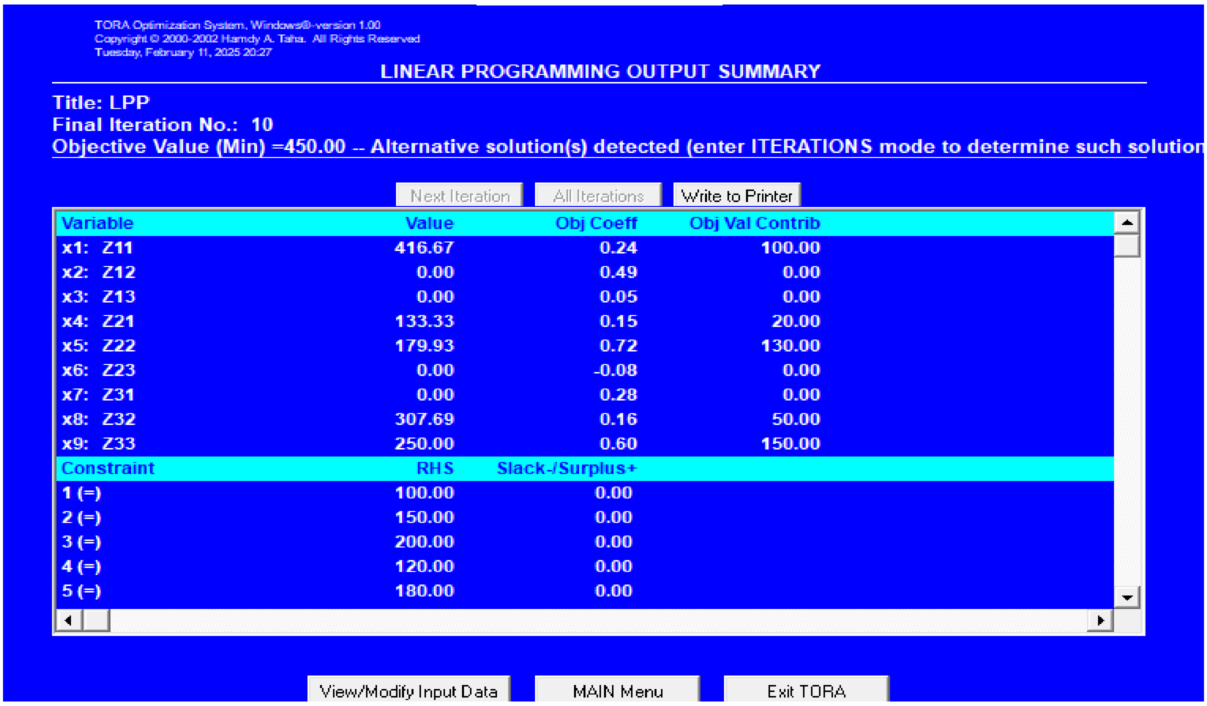
$0.49Z12 + 0.7225Z22 + 0.1625\ Z32 = 180$ (Column 1 restriction)

$0.05\ Z13 + 0\ .08Z23 + 0.6\ Z33 = 150$ (Column 1 restriction)

$Z11, Z12, Z13, Z21, Z22, Z23, Z31, Z32, Z33 \geq 0$(Non negative restrictions)

Step III Using TORA software to solve this problem, and the best result is shown in screenshots 1 (i.e., fig. 3).

The result image displays the crisp's optimal objective value and optimal solution.



Variable	Value	Obj Coeff	Obj Val Contrib
x1: Z11	416.67	0.24	100.00
x2: Z12	0.00	0.49	0.00
x3: Z13	0.00	0.05	0.00
x4: Z21	133.33	0.15	20.00
x5: Z22	179.93	0.72	130.00
x6: Z23	0.00	-0.08	0.00
x7: Z31	0.00	0.28	0.00
x8: Z32	307.69	0.16	50.00
x9: Z33	250.00	0.60	150.00
Constraint	**RHS**	**Slack-/Surplus+**	
1 (=)	100.00	0.00	
2 (=)	150.00	0.00	
3 (=)	200.00	0.00	
4 (=)	120.00	0.00	
5 (=)	180.00	0.00	

Fig. 3: LPP (Iteration No. 10)

Therefore, the optimal assignment to the given real-life problem is

Both the best solution to the given FTP and its corresponding CAP are identical. Because each problem's choice variables have distinct numerical values.

Min $\tilde{z}^{SF} = 450$

Example 5

To find the lowest possible transportation cost, look at SFTP Table 11 for model II, where the costs are stated in clear form but the supply and demand are shown in spherical fuzzy form.

Table 9. SFTP of model-II

Source/Destination	D1	D2	D3	Supply
S1	10	12	8	(0.7,0.2,0.1)
S2	15	18	10	(0.6,0.3,0.1)
S3	20	22	12	(0.8,0.1,0.1)
Demand	(0.75,0.2,0.05)	(0.7,0.25,0.05)	(0.65,0.3,0.05)	Type equation here.

Solution: Obtained optimal solution by using the suggested method, and 9.99 is the optimal value.

Example 6

Examine SFTP Table 12 for model III, where supply, demand, and expenses are all spherically fuzzy, to find the lowest possible transportation cost.

Table 10. SFTP of model-III

Source/Destination	D1	D2	D3	Supply
S1	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.9,0.05,0.05)	(0.7,0.2,0.1)
S2	(0.6,0.3,0.1)	(0.8,0.15,0.05)	(0.7,0.2,0.1)	(0.6,0.3,0.1)
S3	(0.75,0.2,0.05)	(0.65,0.15,0.1)	(0.85,0.1,0.05)	(0.8,0.1,0.1)
Demand	(0.75,0.2,0.05)	(0.7,0.25,0.05)	(0.65,0.3,0.05)	Type equation here.

Solution: Obtained optimal solution by using the suggested method, and 0.716 is the optimal value.

5. RESULT AND DISCUSSION

In this paper, a new algorithm for figuring out SFTP's and SFAP`s optimal solution is presented. Table 16 and Fig. 2 make it clear that the recommended optimal solution approach produces better results. Together with the available data, random problems for each of the three categories are considered in order to confirm the accuracy of the suggested method. For both balanced and unbalanced issues, the suggested technique yields solutions that are equal to those of the standard approach. Vogel's approximation method and Hungarian method is regarded as conventional and efficient for cost optimization in a wide range of transportation and assignment applications. Based on a comparative analysis, the proposed method in this study outperforms VAM and, in some cases, yields results identical to VAM. When the proposed approach was shown on the mathematical examples of three different models, no issues were discovered in the desired inquiry. This work aims to reduce overall cost of transportation and assignment in a fuzzy spherical environment. The suggested algorithm thus achieved the goal of the transportation and assignment difficulties.

Table 16: Results compared to the current approach

Examples	HM	TORA	Proposed Method	Optimum
1	4.00	4.00	4.00	4.00
2	1.90	1.90	1.90	1.90
3	1.84	1.84	1.84	1.84

Table 17: Results compared to the current approach

Examples	Types	VAM	Proposed Method	TORA	Optimum
4	I	450	450	450	450
5	II	9.99	9.99	9.99	9.99
6	III	0.716	0.716	0.716	0.716

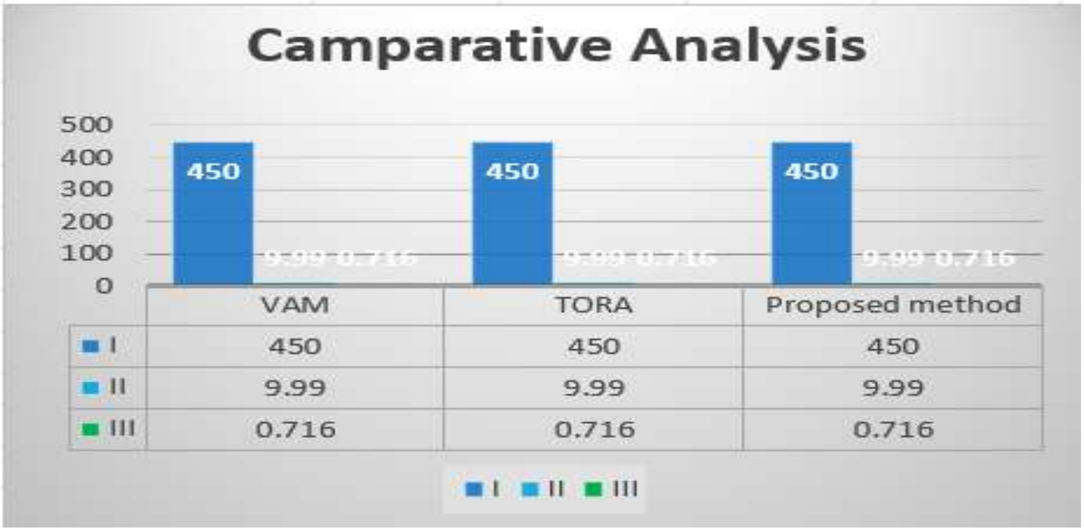


Fig. 4 : Comparative Analysis

6. CONCLUSION

There is a growing push on institutions to find more efficient ways to deliver goods to customers in the fiercely competitive market now. The transportation model provides a solid basis to solve the issue of how to distribute commodities to consumers in the most economical or timely way, which is the goal of many institutions. In this article, imprecise, incomplete, and ambiguous information is quantified using spherical fuzzy sets. This work has produced a new algorithm for SFTP's IBFS. Because SFSs are more crucial for describing information that is unknown. These models have made use of the score function to convert the unclear data into a definite optimization problem. Six numerical examples were used to illustrate the previously indicated approach, and the study's objectives were met while no flaws in the methodology were discovered. Examining the current approaches allowed for the determination of the suggested algorithms' efficacy. The suggested algorithms' efficacy was assessed by looking at the current techniques. This demonstrates the practicality and efficiency of the algorithm we suggested. The proposed algorithm presents a novel method for handling uncertainty in practical optimization problems. This suggested work is unable to tackle bi-objective optimization problems. This work will be expanded in the future to solve fuzzy bi-objective optimization problems.

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