*Asian Journal of Fuzzy and Applied Mathematics (ISSN: 2321 – 516X)*
*Volume 01– Issue 04, December 2013*

# On the Asymptotically Stability with Respect to Probability via Stochastic Matrix-valued Lyapunov Systems

[1]Ebiendele Peter, [2]Okodugha Edward

[1]Department of Mathematics and Applied Sciences, polytechnic
Auchi Edo State, South-west Nigeria.

[2]Department of Mathematics and Applied Sciences, polytechnic
Auchi Edo State, South-west Nigeria.

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**ABSTRACT –** ***In this paper, we study the stability to system in Kats-Krasovskii form, in terms of the stochastic Matrix-valued function.*** $\boldsymbol{\Pi(t,x,y)}$ ***constructed for system*** $\frac{dx}{dt} = f(t,x,y(t))\ x(t_0) = x_0, y(t_0) = y_0$ ***and use the Lyapunov matrix-valued function to established the necessary and sufficient conditions that guarantees the asymptotically stability of the control systems.***

**Keyords**: Stability, Asymptotically, Lyapunov matrix-valued function, Stochastic probability

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## 1. INTRODUCTION

The Stability of deterministic control system has been widely developed in the past few decades (see, for instance Sontag and wang [1] Grune [2], Karafyllis and Tsinias [3], Ning et.al[4] ). In this papers, we investigate The necessary and sufficient conditions for the asymptotically stability with respect to probability via stochastic Matrix –valued Lyapunov system. Mao [5] established the necessary and sufficient conditions for the existence of Exponential stability of stochastic differential equations. On the other hand, the asymptotic stability and input-to State stability in probability at the equilibrium state of stochastic control systems were established among others the following authors, Liu and Raffool [6], Lan and Dang[7], Abedi et.al[8], Khasminiskii[9] Tsinias.[ 10] Boulanger [11] derived sufficient conditions for asymptotic stability in probability and exponential stability in Mean square for a special case of composite stochastic control system. In his paper we study the stability to systems in Kats-Krasovskii form, in terms of the stochastic matrix-valued Function . $\Pi(t,x,y)$ constructed for system (1.1), and established necessary and sufficient conditions that Guarantees asymptotically stability of stochastic probability control system.

We consider in this paper a system modeled by equations of the form

$\frac{dx}{dt} = f(t,x,y(t)) \rightarrow 1.1$

with determined Initial conditions.

$x(t_0) = x_0 \rightarrow (1.2)$

$y(t_0) = y_0 \rightarrow (1.3)$

Here $x\, \epsilon R^n$, $t\epsilon T\ [or\ t\epsilon T_\tau = [\tau, +\infty), \tau \geq 0]$, $y(t)$ is a perturbation vector that can take the values from $y\, cR^n$ for every $t\epsilon T$. We assume that the vector function f is continuous with respect to every variable and Satisfies Lipchitz condition in variable $x$, that is $\|f(t,x',y) - f(t,x'',y)\| \leq L\|x' - x''\|$ in domain $B(T,\ell,y)$; $t\epsilon T, \|x\| < l, y\epsilon Y$ ($\ell = const.or\ \ell = +\infty$ uniformly in $t\epsilon T$ and $y\epsilon Y$, and is bounded for all $t, y\epsilon T \times y$ in every Bounded domain

$, \|x\| < \ell^* (\ell^* = const > 0)\ f(t,0,y(t)) = 0\ \forall (t,y) T \times Y \rightarrow (1.4)$

that is, the unperturbed motion of system (1.1) corresponds to the solution x(t)=0. In order to investigate the stability of a stochastic system of form (1.1) using matrix-valued Liapunov Function. Equation (1.1) is defined in relation of the form $\Pi(t,x,y(t)) = [V_{Kl}(t,x,y(t))]$. $K, l\, \epsilon [1,s] \rightarrow 1.5$

where $(t,x,y)\epsilon B$ and $V_{Kl}(t,0,y(t)) \equiv 0\ \ \forall\ t\epsilon T\ and\ y\epsilon Y$ and

Besides, $V_{Kl}(t;\cdot) = V_{lK}(t,\cdot)\ \forall (K \neq \ell)\epsilon [1,S], V_{Kl}\epsilon\ c(T \times R^n \times y, R[y,R])$

## 2. NOTATIONS AND DEFINITIONS

**Definition 2.1:** The stochastic matrix-valued function Π:R+X $B(\ell) \times y \rightarrow R[y, R^{s\times s}]$ IS Referred to as

(i). Positive (negative) definite, if and only if there exists a time-invariant connected neighborhood N of point $x = 0$ ($N \subseteq R^n$) and positive definite in the sense of Lyapunov function $w(x)$ such that
(a). $\Pi$ is continuous, that is $\Pi \epsilon C(R_+ \times N \times y, R[y, R^{s\times s}])$
(b). $\Pi(t, 0, y) = 0 \ \forall \ t\epsilon R_+ \ and \ y\epsilon y$;
(c).inf $V(t, x, y, z) = w(x) \ \forall (t, y, z)\epsilon \ R_+ \times y \times R^s$;
($\sup v(t, x, y, z) = -w(x) \ \forall (t, y, z)\epsilon \ R_+ \times y \times R^s$);
(ii). Positive (negative) definite on S, if and only if all conditions of Definition (2.1) (i) are satisfied for N=S;
(iii).Positive (negative) definite in the whole, if and only if all conditions of definition 2.1 (i) are satisfied for $N = R^n$.

**Remark 2.1.** If function $\Pi$ does not depend on $t\epsilon$ $R_+$ then in definition 2.1 the requirement of function w(x) Existence is omitted and conditions (a)-(c) are modified and condition (c) becomes

$$(C^{'})v(x, y, z) = Z^T\Pi(x, y)z > 0 \ \forall (x \neq 0, z \neq 0, y)\epsilon \ N \times R^S \times y, (v(x, y, z) < 0 \ \forall (x \neq 0, z \neq 0, y)\epsilon N \times R^S \times y.$$

**Definition 2.2**: The stochastic matrix-valued function $\Pi$: $R_+ \times B(\ell) \times y \to R[y, R^{s\times s}]$ is referred to as (i). Positive semi-definite, if and only if there exist a time-invariant connected neighborhood N of point $x = 0$ ($N \subseteq R^n$) such that
(a).$\Pi$ is continuous in $(t, x)\epsilon \ R_+ \times N$;
(b).$\Pi$ is non-negative on N: $Z^T\Pi(t, x, y)z \geq 0 \ \forall (t, x, y)\epsilon \ R_+ \times N \times y$.
(c).$\Pi$ vanishes at the origin $Z^T\Pi(t, x, y)z = 0 \ \forall (z \neq 0, y\epsilon y); R + \times N \times y$
(ii ) positive semi-definite on $R_+ \times S \times y$ if and only if (i) holds for N=S
(iii) positive semi-definite in the whole if and only if (1) holds for $N = R^n$
(iv). Negative semi-definite (in the whole) if and only if $(-\Pi)$ is positive semi-definite (in the whole) respectively.
**Definition 2.3.** The matrix-valued function $U: T_r \times B(1, l) \times D(l + 1, s) \to R^{s\times s} \ is$;
(1) Positive definite on $T_r, r\epsilon R$, With respect to variables $(x_1^T, \ldots, x_l^T)$ if and only if there exist time-invariant Connected neighborhoods $N^*, N^* C \ R^l$ of x=0. A vector $\varphi \epsilon R_+^s, \varphi > 0$ and a scalar positive definite in the Sense of Lypunov function $w$: $N^* \to R_+$ Such that
(a) $u(t, x) = 0$ for all $t\epsilon T_r$ and $(x_1^T, \ldots, x_l^T) = 0$
(b) $u(t, x) \ \epsilon \ C \ (T_r \times N^* \times D(l + 1, s)R^{s\times s})$;
(c).$\varphi^{TU}(t, x)\varphi \geq w(x_1 T, l)$ for all $(t, x \neq 0, \varphi \neq 0)\epsilon T_r \times N^* \times D(l + 1, s) \times R_+^s x_1^T, l = (x_1^T, \ldots, x_l^T)$;
(ii) Positive definite on $T_r \times G^*$ With Respect to variables $(x_1^T, \ldots, x_l^T)$ if conditions of definition 2.3 (i) holds for $B(1, l) = G^*$
(iii).Negative define (in the whole) on $T_r \times \ B(1, l) \ on \ T_r$ with respect to variables $(x_1^T, l)$, if and only if (-u) is Positive definite (in the whole) on $T_r \times \ B(1, l)( \ on \ T_r)$ with respect to variables $(x_1^T, l)$

## Examples of Definitions (2.1, 2.2 and 2.3)

i. The concept of positive definiteness of a matrix-valued function (1.5) should be compatible with the well-known concept of positive definiteness of a matrix;
ii. The concept of positive definiteness of a matrix function (1.5) should be compatible with Lyapunov's original concept of positive of a scalar function. The principal idea of Lyapunov method is contained in the following resounding 'if the rate of change $\frac{dE}{dt}$ of a given energy $E(x)$ of an isolated physical system is negative or every possible state x except for as single equilibrium state $X_e$, then the energy will continually decrease until it finally assumes its minimum value $E(X_e)$. In mathematical form, the energy function of $E(x)$ is replaced by some scalar function $V(x)$. If for a given system, one is able to find a unction $v(x)$ such that it is always positive except at $x = 0$
$x = 0$ where it is zero , then we say the system returns to the origin if it is disturbed. The function $V(x)$is called the Lyapunov function.
A function is called positive semi-definite (negative – semi - definite) if $V(o) = 0$ and if around the origin $V(x) \geq 0(\leq 0)$ for $x \neq 0$.
Example
For n = 3
$V(x) = x_1^2 + x_2^2 + x_3^2$ is positive definite
ii. for n = 3

$V(x) = x_1^2 + (x_2 + x_3)^2$ is positive semi – definite since on $x_2 + x_3 = 0$ and $x_1 = 0, V(x) = 0$
iii. for n = 2
$V(x) = x_1^2 + x_2^2 - (x_1^4 + x_2^4)$ is positive definite near the origin. In general, it is sign in definite.
More precisely, it is positive definite inside the square definite by $|x_1| < 1, |x_2| < 1$.

## 3. PRELIMINARY RESULTS

In this section, we introduce and states propositions, that we enable us to establish the main results

**Proposition 3.1:** The matrix-valued function $U: T_r \times R^n \to R^{s\times s}$ IS Positive definite on $T_r$ with repect to $(x_1^T, \dots, x_l^T)$, if and only if it can be represented in the form $\varphi^{Tu}(t,x)\varphi = \varphi^{Tu} + (t,x)\varphi + w(x_1^T, \dots, x_l^T) \to (3.1)$ $where\ u + (t,x)$ is positive semi-definite with Respect to all variables $(x_1^T, \dots, x_s^T)$ $and\ w$ is a function explicity independent of $t\epsilon T_r$ and positive definite with Respect to variables $(x_1^T, \dots, x_l^T), l < s$.

**Proof**: Let the matrix-valued function $u(t,x)$ $be$ $(x_1^T, \dots, x_l^T)$ positive definite on $T_r$. Then, by definition 2.3, There exists a positive definite in the sense o f Lyapunov function $W(x_1^T, \dots, x_l^T)$ such that on the domain $T_r \times B(1,l) \times D(l+1,s) \times R_+^s$ condition (1) of definition 2.3 is satisfied. We introduce the function $\varphi^{Tu} + (t,x)\varphi = \varphi^{Tu}(t,x)\varphi - w(x_1^T, l)$

Which is non-negative by condition 2.3(c).Hence the function $\varphi^{Tu}(t,x)\varphi$ can be presented in the form (3.1).

Let the equality (3.1) be satisfied, where $\varphi^{Tu} + (t,x)\varphi \geq 0$ and $w(x_1^T, l)$ is a positive definite function with Respect to the variables $(x_1^T, \dots, x_l^T)$. Then equality (3.1) implies

$\varphi^{Tu}(t,x)\varphi - w(x_1^T, l) = \varphi^{Tu} + (t,x)\varphi \geq 0$

Hence condition 2.3(c) for the function $\varphi^{Tu}(t,x)\varphi$ holds. This proves the proposition 3.1.

**Proposition 3.2**: The Stochastic matrix-valued function $\Pi$: $R_+ \times B(\ell) \times y \to R[y, R^{s\times s}]$ is positive definite, if and only if there exists a vector $Z\epsilon R^s$ and a positive definite in the sense of Lyapunov function $a\epsilon K$ Such that.

$Z^T \Pi(t,x,y)Z = Z^T \Pi + (t,x,y)z + a(x) \to 3.2$

Where $\Pi + (t,x,y)$ is a stochastic positive semi-definite matrix-valued function.

**Proposition 3.3:** The stochastic matrix-valued function $\Pi: R_+ \times B(\ell) \times y \to R[y, R^{s\times s}]$ is decreasing if and only if there exist a vector $z\epsilon R^s$ and a positive definite in the sense of Lyapunov function $c\epsilon k$ such that

$Z^T \Pi(t,x,y)Z = Z^T Q - (t,x,y)Z + C(x)$. Where $Q - (t,x,y)$ is a stochastic negative semi-definite matrix-valued function.

**Proposition 3.4:**If all conditions of assumption (1.1) are satisfied, then for the function

$v(t,x,y,\eta) = \eta^T \Pi(t,x,y)\eta \to 3.3$

With a constant positive vector $\eta\epsilon R_+^s$ The bilateral estimate

$u^T H^T A_1 H_U \leq v(t,x,y,\eta) \leq W^T H^T A_2 H_W \to 3.4$

Take place for all $(t,x,y)\epsilon R_+ \times N_0 \times y$, where

$u^T = (\varphi_1(\|P\|)), \Psi_1(\|q\|), \Phi_1(\|r\|),$

$w^T = (\varphi_2(\|P\|), \Phi_2(\|q\|), \Phi_2(\|r\|), and\ A_1 = [\underline{\propto}\ Kl], A_2 = [\overline{\propto}\ \ kl,]$

$H = diag(\eta_1 \eta_2 \eta_2)$

## 4. THE MAIN RESULTS

Theorem 4.1: Let the equation of perturbed motion (1.1) are such that:

(1).There exists a matrix-valued function $\Pi: R_+ \times B(P) \times y \to R[y, R^{s\times s}]$ in the time-invariant neighborhood $N \leq R^n$ of equilibrium state $x = 0$;

(2). There exists a vector $\eta\epsilon R^s$ $(\eta\epsilon R_+^s)$;

(3).Stochastic scalar function (3.3) is positive definite;

Then the equilibrium state $x = 0$ of system (1.1) is stable with respect to probability.

**Proof:** Let arbitrary numbers $\varepsilon\ \epsilon(0,\ell), \ell\epsilon(0,1)$ $and$ $t_0\epsilon R_+$ be given under the conditions (1)-(2) of theorem 4.1

We have the function $v(t,x,y,\eta) = \eta^T \Pi(t,x,y)\boldsymbol{\eta}, (\eta\epsilon R_+^s)$that is positive definite by condition (3) of theorem 4.1

Therefore, a number $\varepsilon_1 > 0$ is found, such that $\inf v(t,x,y,\eta) = \varepsilon_1\ for\ t\epsilon R_+,\ \ y\epsilon y, \eta\epsilon R^s\ (\eta\epsilon R_+^s)$

We designate $B(\varepsilon) = \{(x,y)\epsilon R^n \times y: \|x\| < \varepsilon, y\epsilon y\}$.let $\tau\varepsilon$ be the time of trajectory $(x(t), y(t))$ first leaving the Domain $B(\varepsilon)\ and\ let\ \tau\varepsilon(\tau) = min(\tau, \tau_\varepsilon)$. we have by averaged derivative of (1.1)

$E[v(\tau\varepsilon(\tau), x(\tau\varepsilon(\tau)], y(\tau\varepsilon(\tau)|\eta)\ |xt_0 = x_0, y(t_0) = y_0] \leq v(t_0, x_0, y_0, \eta) \to 4.1$

now we take $\delta > 0$ so that

$\sup v(t_0, x, y_0) < p_{\varepsilon_1} \to 4.2\ whenever\ \|x\| \leq \delta$. The estimate (4.1) and (4.2) imply

$$p_{\varepsilon_1} > v(t_0, x_0, y_0, \eta) \geq E[v(\tau\varepsilon(\tau), x(\tau\varepsilon(\tau)], y(\tau\varepsilon\ (\tau)), \eta)\ |x_0 y_0] \geq \varepsilon_1 p \begin{Bmatrix} \sup\|x(t)\| \geq \varepsilon|x_0, y_0 \\ t_0 \leq t \leq \tau \end{Bmatrix}$$

Hence we get for $\tau \to +\infty$. $p\begin{Bmatrix} \sup\|x(t)\| \geq \varepsilon|x_0, y_0 \\ t_0 \leq t \end{Bmatrix} < p.$

This proves the theorem.

**Theorem 4.2:** Let the equations of perturbed motion (1.1) are such that:

(1).Hypotheses (1) and (2) of Theorem 4.1 are satisfied;

(2).The stochastic matrix-valued function $\Pi(t,x,y)$ is positive definite and decreasing.

(3).The averaged derivative $\frac{d\epsilon[v]}{dt}$ is negative definite. Then the equilibrium state $x = 0$ of the system (1.1) is

Asymptotically stable with probability P(H); That is if $\|x_0\| \leq H_0$ *and* $y_0 \epsilon y, t_0 \geq 0$ then
$p\left\{\begin{matrix}\sup\|x(t)\| \geq H|x_0, y_0 \\ t \geq t_0\end{matrix}\right\} \geq 1 - P(H), H_0 < H$

**Proof:** Let a number $P(H) \angle 1$ be given. Theorem 4.1 implies that under the conditions of Theorem 4.2 the Equilibrium state $x = 0$ of system (1.1) is stable with respect to probability. Therefore, for any $\varepsilon\ \epsilon(0, \ell)$ and $t_0 \geq 0\ \ \delta = \delta(t_0, \varepsilon) > 0$ can be found such that
$p = \left\{\begin{matrix}\sup\|x(t)\| \geq \varepsilon|x_0, y_0 \\ t \geq t_0\end{matrix}\right\} \geq 1 - P(H) \rightarrow 4.3$ Whenever $\|x_0\| \leq \delta$ *and* $y_0 \epsilon y$.
Let us show that the number $H_0$ mentioned in conditions of theorem 4.2 can be taken as $H_0 = \delta$. to this end, we Define for arbitrary numbers $K\epsilon[0,\varepsilon)$ *and* $0 < q < +\infty$ the number $k_1 > 0$ from the inequality.
$sup[v(t, x, y, \eta)\ for\ t\epsilon R_+, \|x\| < k_1, y\epsilon y, \eta\epsilon R_+^s] < \frac{q}{2} inf[v(t, x, y, \eta) for\ t\epsilon R_+\ k_1 \leq \|x\| \leq \varepsilon, y\epsilon y\ and\ \eta\epsilon R_+^s\ ] \rightarrow 4.4$
The arguments similar to those used in the proof of theorem 4.1 yield
$p\left\{\begin{matrix}sup\|x(\tau)\| < k|x(t), y(t) \\ \tau > t\end{matrix}\right\} > 1 - \frac{1}{2}q$, Whenever $\|x(t)\| \leq k_1$ *and* $y(t)\epsilon y$. We claim that ther exist a $\tau > t_0$
Such that $p\{\|x(t_0 + \tau)\| < k_1|x_0, y_0\} > 1 - \frac{1}{2}q - P(H) \rightarrow 4.6$
If this is not true, then for trajectory $\{x(t), y(t)\}$ the inequality
$p\{k_1 \leq \|x(t)\| < \varepsilon, t \geq t_0|x_0, y_0\} > \frac{1}{2}q$.
Holds, that yields by condition (3) of theorem 4.2
$\lim_{t\to\infty} E\left[v\left(\tau_\alpha\ (t), x(\tau_\alpha(t)), y(\tau_\alpha(t)), \eta\right)|x_0, y_0\right] = -\infty \rightarrow 4.7$
Here $\tau_\alpha(t) = min(\tau^*, t)$, where $\tau^*$ is a time of
Trajectory $\left(x(t), y(t)\right)$ first leaving the set $B_1 = \{(x, y]: k_1 < \|x\| < \varepsilon, y\ \epsilon\ y\}$
Since the function $\Pi(t, x, y)$ is positive definite, the correlation (4.7) cannot be satisfied. This proves inequality (4.6) The estimate (4.3), (4.5) and (4.6) imply that for arbitrary $q > 0\ a\ \tau > 0$ is found so that
$p\left\{\begin{matrix}sup\|x(t)\| < k|x_0, y_0 \\ t \geq t_0 + \tau\end{matrix}\right\} > 1 - q - P(H)$ Whenever $\|x_o\| < H_0$ *and* $y_0 \epsilon Y$.
This proves the Theorem 4.2.

**Theorem 4.3:** Let the equations of perturbed motion (1.1) are such that
(1).hypotheses (1),(2) and (3) of the Theorem 4.1 are satisfied for $N = R^n$;
(2).The function $\Pi(t, x, y)$ is positive definite in the whole and radially unbounded;
(3).The averaged derivative $\frac{d\epsilon[v]}{dt}$ is negative definite in $B(T, \infty, y)$.

Then the equilibrium state x=0 of the system (1.1) is stable with respect to probability in the whole. Above theorem allowed us to find asymptotically stability with respect to probability in the whole on the basis of negative semi-definite, considering it's averaged derivative.
Let an open domain G Containing the origin be definite in space $R^n$ function $\varphi(t, x, y): T_0 \times G \times y \rightarrow R$ is referred to as positive definite on $G \times y$ if for any numbers $r > \varepsilon > 0$ there exists a number $\delta > 0$ such that $\varphi(t, x, y) \geq \delta$ holds for all $t \geq t_0, (x, y)\epsilon(N \cap \{\varepsilon \leq \|x\| \leq r\} \times y)$. Matrix-valued function $\Phi(t, x, y): T_0 \times G \times y \rightarrow R^{m\times m}$ satisfies hypotheses A if;
(a).The function $\Phi$ is bounded for all $t \geq t_0$ in any finite domain $\|x\| \leq \ell$, yϵy;
(b) Averaged derivative $\eta^T \frac{dM[\Phi]}{dt} \boldsymbol{\eta}$ Is bounded in any finite domain due to system (1.1), that is, there exists a Constant k such that $\left|\eta^T \frac{dM[\Phi]}{dt} \boldsymbol{\eta}\ \right| \leq k$;
(c) The function $\eta^T \frac{dM[\Phi]}{dt} \boldsymbol{\eta}$ is positive definite in domain $G \times y$. then the following statement is valid.
(1) Hypotheses (1) and (2) of Theorem 4.3 are satisfied;
(2) averaged derivative of $p\{\|x(t; t_0, x_0, y_0\| < \varepsilon|x(t_0) = x_0, y(t_0) = y_0\}$ >$1 - p$ under the condition $\|x_0\| < \delta$
And $y_0 \varepsilon y$. Satisfies the hypothesis $\eta^T \frac{dM[\Pi]}{dt} \boldsymbol{\eta} \leq$ H(x) ≤ 0, where H(x) is continuous in domain G;
(3). The set $D = \{x: x \neq 0, H(x) = 0\}$ is non-empty and does not possess mutual point with bound $\partial N$ domain N In the sense that $inf\|x_1 - x_2\| > k^2 > 0\ x_1\epsilon\partial G, x_2\epsilon D \cap \{\varepsilon \leq \|x\| \leq r\}$
(4) There exists a matrix-valued function $\Phi(t, x, y)$ satisfying hypotheses A. Then the equilibrium state x=0 of The System (1.1) is stable with respect to probability in the whole.

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