Study and Modeling of Waste Material Level in the Blood after Dialysis

Mojtaba Karimyar Jahromi

[1]Department of Mathematical Sciences
University of Jahrom, Iran

ABSTRACT— ***The main task of kidneys is removing waste materials like urea and toxins resulting from metabolism of the body. Other functions of the kidneys are regulating water balance and electrolytes, acid and alkali balance adjustment, the secretion of a hormone-like substances and so on.***

When the kidneys are impaired in their work and the toxic waste are accumulated in the blood, it can cause diabetes or increase blood pressure. A quarter of people with diabetes need kidney dialysis. Remove waste materials and toxins from the blood, dialysis makes the kidneys work better. In this paper, waste material level in the blood has been investigated after dialysis using modeling software Metmatyka and drawing chart of required time to reach waste material level to the normal level in blood.

Keywords— dialysis; dialysis solution; waste material; the differential equation.

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1. INTRODUCTION

In the process of hemodialysis, the patient's blood is sent into the dialysis machine 1 to 3 dl per minute. The base work of dialysis is transfering fluid and particles through a semi-permeable membrane through diffusion, osmosis and ultrafiltration. Using this membrane, separation of blood from the dialysis solution is performed. Dialysis solution is a buffered electrolyte in which it is close to the blood electrolyte composition. Incoming water to the dialysis machine is combined with this solution before exposure to the patient's blood and passing through the filter. The dialysis solution in is also called the cleaning solution. This solution moves in the opposite direction from 2 to 6 deciliter per minute and absorbs the waste materials from the blood and force out with itself.

The most important evaluation index in hemodialysis is that the patient blood is sent into the machine 1 to 3 DL per minute (see [2, 5]).

2. BASIC INFORMATION NEEDED FOR MODELING

The most important measuring index of kidney function is the glomerular or GFR in which it can be measured by urinary excretin which is completely filtered and reabsorbed by the tubules. The amount of such substances in the urine in the time unit is resulted from filteration of unit volume of plasma which contaims that material. This volume is called the clearance indirectly reflects the GFR. Practically, this Kratnyn is used to measure GFR. The urea is excreted through the kidneys and increase of it is the signs of reduced kidney function (see [4, 7, 8]).

3. MODELING

In a lot of problems in science, engineering and medicine, it is necessary to find a function that applies to the equation involves derivatives. This equation is called defferntial and the function is called the answer of differential equation. In this article, we attemted to investigate and formulate the level of waste materials in the blood using differential equation in the patient's blood after hemodialysis. For this reason, suppose that x is the every moment of dialysis, $u[x]$ and $v[x]$ are the volume of waste materials in the blood and the volume of waste in dialysis respectively and Q_D and Q_B are dialysis solution flow rate and blood circulation rate in the dialysis machine (see [1, 3]). Mathematical relationship between the above symbols are calculated as the follwing: (k is a fixed value)

$$Q_B u'[x] = -k(u[x] - v[x])$$
$$-Q_D v'[x] = k(u[x] - v[x])$$

If the L is the whole time of the dialysis and the initial concentration of wastes in the blood is $u[\circ] = u_\circ$ and the level of the waste materials in the washing solution is $v[L] = \circ$, in this case we have: $(u[x] - v[x])' = -(\frac{k}{Q_B} - \frac{k}{Q_D})(u[x] - v[x])$

Sunstituting $y[x] = u[x] - v[x]$ and $\alpha = \frac{k}{Q_B} - \frac{k}{Q_D}$, the above equation changes to (*) $y'[x] = -\alpha y[x]$. To solve this equation, software Metmatika is applied and functions $u[x]$ و $v[x]$ are the waste material volume in the blood and the waste material volume in the dialysis solution respectively and we can simply calculate x (see [1, 3, 6]).

First we do the differential equation (*)

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Clear[x, y]
step1 = DSolve[y'[x] == -α * y[x], y[x], x]
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$$\{\{y[x] \to e^{-x\alpha} C[1]\}\}$$

To ease the calculation we substitute c with c[1].

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y = step1[[1, 1, 2]] /. C[1] → c
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$$c\, e^{-x\alpha}$$

$$\texttt{step2 = DSolve}\left[\left\{u'[x] == -\frac{k}{Q_B} * c * e^{-x*\alpha},\ u[0] == u0\right\},\ u[x],\ x\right]$$

$$\left\{\left\{u[x] \to \frac{e^{-x\alpha}\left(c\,k - c\,e^{x\alpha}k + e^{x\alpha}u0\,\alpha\,Q_B\right)}{\alpha\,Q_B}\right\}\right\}$$

Now, we consider *capl* which is the total time of dialysis and reach c (see [1] (also [5, 7, 8] for the commutative case) .

$$\texttt{leftside} = \frac{e^{-x*\alpha}\left(c*k + e^{x*\alpha}\alpha\left(u0 - \frac{c*k}{\alpha*Q_B}\right)Q_B\right)}{\alpha*Q_B} - y\ /.\ x \to capl$$

$$-c\,e^{-capl\,\alpha} + \frac{e^{-capl\,\alpha}\left(c\,k + e^{capl\,\alpha}\alpha\left(u0 - \frac{c\,k}{\alpha\,Q_B}\right)Q_B\right)}{\alpha\,Q_B}$$

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cval = Solve[leftside == 0, c]
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$$\left\{\left\{c \to \frac{e^{capl\,\alpha}u0\,\alpha\,Q_B}{-k + e^{capl\,\alpha}k + \alpha\,Q_B}\right\}\right\}$$

And finally the functions $u[x]$ و $v[x]$ are reached as the followings:

$$u = \mathrm{Simplify}\left[\frac{e^{-x*\alpha}\left(c*k + e^{x*\alpha}*\alpha*\left(u0 - \frac{c*k}{\alpha*Q_B}\right)Q_B\right)}{\alpha*Q_B} \;/.\; \mathrm{cval}[[1]]\right]$$

$$\frac{u0\left(\left(-1 + e^{(c\varphi 1 - x)\alpha}\right)k + \alpha\, Q_b\right)}{\left(-1 + e^{c\varphi 1\,\alpha}\right)k + \alpha\, Q_b}$$

$$v = \mathrm{Simplify}[u - y \;/.\; \mathrm{cval}[[1]]]$$

$$\frac{e^{-x\,\alpha}\left(e^{c\varphi 1\,\alpha} - e^{x\,\alpha}\right)u0\,(k - \alpha\, Q_b)}{\left(-1 + e^{c\varphi 1\,\alpha}\right)k + \alpha\, Q_b}$$

4. REFERENCES

[1] Erwin Kreyszig, Introductory Functional Analysis With , Applications, New York. 1978:

[2] Peniscola, Jose Bonet, John Horvath, Klaus D. Bierstedt, Manuel Valdivia, Progress in Functional Analysis, Proceedings of the International Functional Analysis Meeting on the Occasion of the 60th, Birthday of Professor M. V.

[3] John B. Conway, A Course in Functional Analysis, Second Edition (Graduate Texts in Mathematics).

[4] Walter Rudin, Functional Analysis, 2nd Edition.

[5] Michael Reed, Methods of Modern Mathematical Physics, Vol. 1, Functional Analysis (Methods of Modern Mathematical Physics).

[6] Berberian, Introduction to Hilbert Space, New York, 1961.

[7] Halmos, Finite Dimensional Vector Spaces, New York, 1958.

[8] Klir and Yuan, Fuzzy Sets and Fuzzy Logic; Theory and Applications, New Dehli, 2003.