Fuzzy Nonparametric Predictive Inference for the Reliability of Series Systems

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ABSTRACT—*This paper presents fuzzy lower and upper probabilities for the reliability of series systems. Attention is restricted to series systems with exchangeable components. In this paper, we consider the problem of the evaluation of system reliability based on the nonparametric predictive inferential (NPI) approach, in which the defining the parameters of reliability function as crisp values is not possible and parameters of reliability function are described using a triangular fuzzy number. The formula of a fuzzy reliability function and its α-cut set are present. The fuzzy reliability of structures defined based on a fuzzy number. Furthermore, the fuzzy reliability functions of series systems discussed. Finally, some numerical examples are present to illustrate how to calculate the fuzzy reliability function and its α-cut set. In other words, the aim of this paper is present a new method titled fuzzy nonparametric predictive inference for the reliability of series systems.*

Keywords— Series Systems, Lower and Upper Probabilities, Nonparametric Predictive Inference, Fuzzy Number

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1. INTRODUCTION

Study on the reliability of the engineering design process is an important part of a system in which future performance will be evaluated. Since the future cannot be predicted with certainty be normal in the calculation of reliability, methods are used that allow the modeling of uncertainty[11] . This paper provides a new method for statistical inference on system reliability on the basis of limited information resulting from component testing. This method is called Fuzzy Nonparametric Predictive Inference (FNPI).

We present FNPI for system reliability, in particular, FNPI for series systems.The theory of imprecise probabilities [18], Possibility Theory [13], the theory of interval probability [19, 20] and fuzzy reliability theory [4] have been used as a general and promising tool for reliability analysis[11]. Coolen [6] provided an insight into imprecise reliability, discussing a variety of issues and reviewing suggested applications of imprecise probabilities in reliability, see [6, 8, 9, 10, 11] for a detailed overview of imprecise reliability and many references. A nonparametric predictive approach is a statistical approach based on few assumptions about probability distributions, with inferences based on data [7]. This method assumes exchangeability of random quantities, both related to observed data and future observations, and uncertainty is quantified using lower and upper probabilities that derived from Coolen [7]. The nonparametric predictive approach that proposed by [7] has proved to be efficient for measuring the probability of outcomes that cannot be done using precise probabilities. Nonparametric predictive inference (NPI) is a statistical framework which uses few modeling assumptions, with inferences explicitly in terms of future observations. NPI is close in nature to predictive inference for the low structure stochastic case as briefly outlined by Geisser [14], which is in line with many earlier nonparametric test methods where the interpretation of the inferences is in terms of confidence intervals. NPI provides exactly calibrated frequentist inferences [7], and it has strong consistency properties in theory of interval probability [1]. NPI is always in line with inferences based on empirical distributions, which is an attractive property when aiming at objectivity [7].

In recent years, many theoretical aspects and a variety of applications of inference based on Hill's assumption A(n) for prediction of probabilities, for one (or more) future values, on the basis of n prior observations, have been presented, referring to these as 'nonparametric predictive inference' (NPI), see e.g. [1, 5, 7, 8, 11].

This paper aims at studying the reliability of series systems base on nonparametric predictive inference in a fuzzy environment. In some cases, it may not be possible to define reliability of series systems parameters as crisp values. In these cases, these parameters can be expressed by linguistic variables. The fuzzy set theory can be used successfully to cope the vagueness in these linguistic expressions for the reliability of series systems base on nonparametric predictive inference. In this paper, a new method is presented for system reliability. This approach is called Fuzzy Nonparametric Predictive Inference (FNPI). It provides a new method for statistical inference on system reliability on the basis of limited information resulting from component testing. Formula of a fuzzy reliability function and its α-cut set are presented. The fuzzy reliability of structures is defined on the basis of fuzzy number. Furthermore, the fuzzy reliability functions of the series system discussed. Finally, some numerical examples are presented to illustrate how to calculate the fuzzy reliability function and its α-cut set. In other words, the aim of this paper is to propose a new method titled fuzzy nonparametric predictive inference for the reliability of series systems.

In Section 2 we review briefly the main idea of NPI and Nonparametric Predictive Inference for the reliability of series systems. The Fuzzy Nonparametric Predictive Inference for the reliability of series systems is presented in Section 3, and finally, in section four conclusions and discussion are presented.

2. NON-PARAMETRIC PREDICTIVE INFERENCE FOR A SERIES SYSTEM

Hill [15] proposed the assumption $A_{(n)}$ for the prediction about future observations. This assumption was proposed particularly for situations in which there is no strong prior information about the probability distribution for a random quantity of interest. $A_{(n)}$ does not assume anything else, and is a post-data assumption related to exchangeability [7]. Hill [16] discusses $A_{(n)}$ in detail. Inferences based on $A_{(n)}$ are predictive and nonparametric, and can be considered suitable if there is hardly any knowledge about the random quantity of interest, other than the *n* observations, or if one does not want to use such information, e.g. to study effects of additional assumptions underlying other statistical methods [7]. $A_{(n)}$ is not sufficient to derive precise probabilities for many events of interest, but it provides optimal bounds for probabilities for all events of interest involving X_{n+1}. These bounds are lower and upper probabilities in the theories of imprecise probability and interval probability, and as such, they have strong consistency properties. NPI is a framework of statistical theory and methods that use these $A_{(n)}$ based lower and upper probabilities, and also considers several variations of $A_{(n)}$ which are suitable for different inferences [7]. Augustin and Coolen [1] proved that the lower and upper probabilities obtained based only on the $A_{(n)}$ assumption has strong consistency properties in the theory of interval probability [7]. Coolen [5] used $A_{(n)}$ for NPI in the case of Bernoulli data, providing lower and upper probabilities for the number of successes in *m* future trials, based on the number of successes in *n* observed trials. This was possible by considering the same representation for such Bernoulli data as was used by Bayes [2], namely as balls on a table [7].

The class of *k*-out-of-*m* systems, also called 'voting systems', was introduced by Birnbaum [3]. These are systems that consist of *m* exchangeable [12] components (often the confusing term identical components is used), such that the system functions if and only if at least *k* of its components function. Since the value of *m* is usually larger than the value of *k*, redundancy is generally built into a *k*-out-of-*m* system. Both parallel and series systems are special cases of the *k*-out-of-*m* system. A series system is equivalent to an *m*-out-of-*m* system while a parallel system is equivalent to a *1*-out-of-*m* system [11].

Applications of *k*-out-of-*m* systems can e.g. be found in the areas of target detection, communication, safety monitoring systems, and, particularly, voting systems. The *k*-out-of-*m* systems are a very common type in fault-tolerant systems with redundancy. They have many applications in both industrial and military systems. Fault-tolerant systems include the multi-display system in a cockpit, the multiengine system in an airplane, and the multi-pump system in a hydraulic control system [11].

Definition 1 (The $A_{(n)}$ assumption of Hill)[1].

Assume that $X_1, X_2, \dots, X_n, X_{n+1}$ are continuous and exchangeable random quantities. Let the ordered observed values of $x_1, x_2, \dots, x_n$ be denoted by $x_{(1)} < x_{(1)}, \dots < x_{(n)} < \infty$, and let $x_{(0)} = -\infty$ and $x_{(n+1)} = \infty$ for ease of notation. Assume

that the possibility of the existence of a nod is zero, and observations specify the real line as $n+1$ intervals in the form of $I_j = (x_{(j-1)}, x_{(j)})$ for $j = 1,2,\ldots,n+1$.
For a future observation of X_{n+i} based on n observations, assumption $A_{(n)}$ is written as:

$$P(X_{n+i} \in I_j) = P\left(X_{n+i} \in (x_{(j-1)}, x_{(j)})\right) = \frac{1}{n+1} \quad for\ i \geq 1, \qquad j = 1,\ldots,n+1 \quad (1)$$

This assumption implies that the rank of X_{n+i} amongst the observed $x_{(1)} < x_{(2)} < \cdots < x_{(n)}$ has equal probability to be any value in $\{1,2,\ldots,n+1\}$.

Definition 2[1]. Assume that $\mathcal{B}$ is the Borel σ −field on $\mathbb{R}$. For each element $B \in \mathcal{B}$, function sets $\underline{P}(\cdot)$ and $\overline{P}(\cdot)$ for the event $X_{n+1} \in \mathcal{B}$ based on the intervals $I_1, I_2, \ldots, I_{n+1}$ and the assumption $A_{(n)}$ are defined as:

$$\underline{P}\,(X_{n+1} \in B) = \frac{1}{n+1}\,\left|\{j : I_j \subseteq B\}\right|, \qquad (2)$$

$$\overline{P}\,(X_{n+1} \in B) = \frac{1}{n+1}\,\left|\{j : I_j \cap B \neq \emptyset\}\right|. \qquad (3)$$

Theorem 1. Assume a $n+m$ number of Bernoulli's exchangeable experiments whose result can be success or failure. Assume:

$Y_{n+1}^{n+m} \rightarrow$ The random variable of number of successes of m Bernoulli's future ($n+1$to $n+m$) experiments.

$Y_1^n \rightarrow$ The random variable of the number of successes in n Bernoulli's previous (1 to n experiments.

For the sake of simplicity we define $\binom{s+r_0}{s} = 0$, therefore, the higher and lower probabilities of non-parametric predictive inference are

$$\overline{P}\left(Y_{n+1}^{n+m} \in R_t | Y_1^n = s\right) = \binom{n+m}{m}^{-1} \sum_{j=1}^{t}\left[\binom{s+r_j}{s} - \binom{s+r_{j-1}}{s}\right]\binom{n-s+m-r_j}{n-s} \qquad (1)$$

and

$$\underline{P}\left(Y_{n+1}^{n+m} \in R_t | Y_1^n = s\right) = 1 - \overline{P}\left(Y_{n+1}^{n+m} \in R_t^c | Y_1^n = s\right)$$

Where $R_t = \{r_1, \ldots, r_t\}$ with $\circ \leq r_1 < r_2 < \ldots < r_t \leq m$, $1 \leq t \leq m+1$ and $R_t^c = \{\circ, 1, \ldots, m\} \setminus R_t$.

Proof. See [5].

Corollary 1. Considering a *k*-out-of-*m* system, the event $Y_{n+1}^{n+m} \geq k$ is of interest as this corresponds to successful functioning of a *k*-out-of-*m* system, following *n* tests of components that are exchangeable with the *m* components in the system considered. Given data consisting of *s* successes from *n* components tested, the NPI lower and upper probabilities for the event that the *k*-out-of-*m* system functions successfully are also denoted by $\overline{P}\left(S(m:k)|(n,s)\right)$ and $\underline{P}\left(S(m:k)|(n,s)\right)$, respectively. For $k \in \{1, 2, \ldots, m\}$ and $0 < s < n$ [11]

$$\overline{P}\left(S(m:k)|(n,s)\right) = \overline{P}\left(Y_{n+1}^{n+m} \geq k | Y_1^n = s\right)$$

$$= \binom{n+m}{m}^{-1}\left[\binom{s+k}{s}\binom{n-s+m-k}{n-s} + \sum_{l=k+1}^{m}\binom{s+l-1}{s-1}\binom{n-s+m-l}{n-s}\right] \qquad (2)$$

and

$$\underline{P}\left(S(m:k)|(n,s)\right) = \underline{P}\left(Y_{n+1}^{n+m} \geq k | Y_1^n = s\right) = 1 - \overline{P}\left(Y_{n+1}^{n+m} \leq k-1 | Y_1^n = s\right)$$

$$= 1 - \binom{n+m}{m}^{-1}\left[\sum_{l=0}^{k-1}\binom{s+l-1}{s-1}\binom{n-s+m-l}{n-s}\right] \qquad (3)$$

Corollary 2. For the series systems, with *k=1*, NPI upper and lower probabilities can be substantially simplified to give the expressions below, which actually provide insight into the NPI approach for such systems. Representing corresponding lower and upper probabilities for an event A by $(\underline{P},\overline{P})(A)$, the general results above are, for series system [11]

$$(\underline{P},\overline{P})(m:1|n,s)=\left(\prod_{j=1}^{m}\frac{s-1+j}{n+j},\ \prod_{j=1}^{m}\frac{s+j}{n+j}\right) for\, 0<s<n \qquad (4)$$

3. FUZZY NON-PARAMETRIC PREDICTIVE INFERENCE FOR THE RELIABILITY OF SERIES SYSTEMS

In this Section, we consider the problem of the evaluation of system reliability based on the nonparametric predictive inferential (NPI) approach, in which the defining the parameters of reliability function in definite quantities is not possible and parameters of reliability function are described using a triangular fuzzy number.

3.1 Fuzzy Set Theory

The theory of sets and fuzzy logic was first proposed by Zadeh [21]. This theory has found wide applications in many fields such as computer, system analysis, electronic and recently in social sciences, economics, and industry. Fuzzy logic is a theory for uncertain conditions. This theory can form many of concepts, variables, and systems which are imprecise and vague in a mathematical form and provide the way for reasoning, control and decision-making in uncertain conditions. In popular speech, if a variable can take a number of terms from the natural language as amounts; we call it a linguistic variable. For the formulation of terms in mathematical expressions, we use fuzzy sets to designate terms. In other words, “if a variable can take terms from the natural language as its amounts, then it is called a linguistic variable in which terms are specified by fuzzy sets domains in which variables have been defined”. we recall same concepts of fuzzy set theory used in this article derived from [21, 22].

Definition 3. The set $\tilde{A}$ of R is called a fuzzy number if it satisfies in the following conditions:

1. $\tilde{A}$ is normal i.e. $\exists x_0 \in R\ ;\tilde{A}(x_0)=1$.
2. $\tilde{A}$ is convex i.e. for each $x_1,\ x_2 \in \mathrm{R}$ and each $\lambda \in [\circ\ ,1]$ we have

$$\tilde{A}(\lambda x_1+(1-\lambda)x_2)\geq \min(\tilde{A}(x_1),\ \tilde{A}(x_2))$$

3. $\tilde{A}$ is the upper semi continuous.

Definition 4 (α- cut of fuzzy set).

The α - cut, A_α, consists of elements whose membership degree in $\tilde{A}$ is not lower than α, i. e.

$$A_\alpha=\left\{x \in X\ |\tilde{A}(x)\geq\alpha\right\},\quad \circ<\alpha\leq 1$$

The α - cut set of a fuzzy number is a closed interval which is shown as $A_\alpha=\lfloor A_\alpha^-, A_\alpha^+\rfloor$ in which

$$A_\alpha^-=inf\ \left\{x \in R\ ;\tilde{A}(x)\geq\alpha\right\}$$

$$A_\alpha^+=sup\left\{x \in R\ ;\tilde{A}(x)\geq\alpha\right\}$$

The most used fuzzy numbers are the trapezoidal and triangular fuzzy numbers. Triangular fuzzy numbers, due to their simple computations, are used more.

situation which should be taken into account is to define the number of tested components by linguistic variables.

3.2 Fuzzy Number of Success in Tested Components(s)

The number of success in tested components can be defined by linguistic variables. One of the situations which should be taken into account is to define the number of functioning items by linguistic variables. Fuzzy numbers can be used for showing functioning items. Assume that the number of functioning items is defined by triangular fuzzy number:

$\tilde{s}=TFN(s_1,s_2,s_3)$ and $s(\alpha)=(s_1+(s_2-s_1)\alpha\ ,s_3+(s_2-s_3)\alpha)$

Therefore fuzzy lower non-parametric predictive probability,

$$\underline{P}\left(S\left(m:m\right)|\left(n,s\right)\right)=\underline{P}\left(Y_{n+1}^{n+m}\geq m|Y_1^n=s\right)=\prod_{j=1}^{m}\frac{\tilde{s}-1+j}{n+j}$$

As a result

$$\underline{P}\left(\alpha\right)=\left\{\prod_{j=1}^{m}\frac{s-1+j}{n+j}|s\in s\left(\alpha\right)\right\}\qquad \circ\leq\alpha\leq 1$$

$$\underline{P}\left(\alpha\right)=\left[\underline{P}_l\left(\alpha\right),\underline{P}_r\left(\alpha\right)\right]$$

In a way that,

$$\underline{P}_l\left(\alpha\right)=min\left\{\prod_{j=1}^{m}\frac{s-1+j}{n+j}|s\in s\left(\alpha\right)\right\}$$

$$\underline{P}_r\left(\alpha\right)=max\left\{\prod_{j=1}^{m}\frac{s-1+j}{n+j}|s\in s\left(\alpha\right)\right\}$$

If s̃ be the triangular fuzzy number then

$$\underline{P}_l\left(\alpha\right)=\prod_{j=1}^{m}\frac{(s_1+\left(s_2-s_1\right)\alpha\ )-1+j}{n+j}$$

$$\underline{P}_r\left(\alpha\right)=\prod_{j=1}^{m}\frac{(s_3+\left(s_2-s_3\right)\alpha\ )-1+j}{n+j}$$

Too fuzzy upper non-parametric predictive probability,

$$\overline{P}\left(S\left(m:m\right)|\left(n,s\right)\right)=\overline{P}\left(Y_{n+1}^{n+m}\geq m|Y_1^n=s\right)=\prod_{j=1}^{m}\frac{\tilde{s}+j}{n+j}$$

As a result

$$\overline{P}\left(\alpha\right)=\left\{\prod_{j=1}^{m}\frac{s+j}{n+j}|s\in s\left(\alpha\right)\right\}\quad 0\leq\alpha\leq 1$$

or

$$\overline{P}\left(\alpha\right)=\left[\overline{P}_l\left(\alpha\right),\overline{P}_r\left(\alpha\right)\right]$$

$$\overline{P}_l\left(\alpha\right)=min\left\{\prod_{j=1}^{m}\frac{s+j}{n+j}|s\in s\left(\alpha\right)\right\}$$

$$\overline{P}_r\left(\alpha\right)=max\left\{\prod_{j=1}^{m}\frac{s+j}{n+j}|s\in s\left(\alpha\right)\right\}$$

If s̃ be the triangular fuzzy number then

$$\overline{P}_l\left(\alpha\right)=\prod_{j=1}^{m}\frac{(s_1+\left(s_2-s_1\right)\alpha\ )+j}{n+j}$$

$$\overline{P}_r\left(\alpha\right)=\prod_{j=1}^{m}\frac{(s_3+\left(s_2-s_3\right)\alpha\ )+j}{n+j}$$

3.2. Fuzzy Numbers of Tested Components(n)

Another situation which should be taken into account is to define the number of tested components by linguistic variables. Fuzzy numbers can be used to represent this definition successfully. Assume that n numbers of tested components are defined as the following triangular numbers:

$\tilde{n}=\mathrm{TFN}(\mathrm{n}_1,\ \mathrm{n}_2,\ \mathrm{n}_3)$ and $n\left(\alpha\right)=(n_1+\left(n_2-n_1\right)\alpha\ ,n_3+\left(n_2-n_3\right)\alpha)$

So fuzzy lower non-parametric predictive probability

$$\underline{P}\left(S\left(m:m\right)|\left(n,s\right)\right)=\underline{P}\left(Y_{n+1}^{n+m}\geq m\,|\,Y_1^n=s\right)=\prod_{j=1}^{m}\frac{\tilde{s}-1+j}{\tilde{n}+j}$$

As a result

$$\underline{P}\left(\alpha\right)=\left\{\prod_{j=1}^{m}\frac{s-1+j}{n+j}|s\in s\left(\alpha\right),n\in n\left(\alpha\right)\right\}\quad 0\leq\alpha\leq 1$$

or

$$\underline{P}\left(\alpha\right)=\left[\underline{P}_{l}\left(\alpha\right),\underline{P}_{r}\left(\alpha\right)\right]$$

$$\underline{P}_{l}\left(\alpha\right)=min\left\{\prod_{j=1}^{m}\frac{s-1+j}{n+j}|s\in s\left(\alpha\right),n\in n\left(\alpha\right)\right\}$$

$$\underline{P}_{r}\left(\alpha\right)=max\left\{\prod_{j=1}^{m}\frac{s-1+j}{n+j}|s\in s\left(\alpha\right),n\in n\left(\alpha\right)\right\}$$

If $\tilde{s}$ and $\tilde{n}$ be the triangular fuzzy numbers then

$$\underline{P}_{l}\left(\alpha\right)=\prod_{j=1}^{m}\frac{\left(s_1+\left(s_2-s_1\right)\alpha\right)-1+j}{\left(n_1+\left(n_2-n_1\right)\alpha\right)+j}$$

$$\underline{P}_{r}\left(\alpha\right)=\prod_{j=1}^{m}\frac{\left(s_3+\left(s_2-s_3\right)\alpha\right)-1+j}{\left(n_3+\left(n_2-n_3\right)\alpha\right)+j}$$

Too fuzzy upper non-parametric predictive probability,

$$\overline{P}\left(S\left(m:m\right)|\left(n,s\right)\right)=\overline{P}\left(Y_{n+1}^{n+m}\geq m|Y_1^n=s\right)=\prod_{j=1}^{m}\frac{\tilde{s}+j}{\tilde{n}+j}$$

$$\overline{P}\left(\alpha\right)=\left\{\prod_{j=1}^{m}\frac{s+j}{n+j}|s\in s\left(\alpha\right),n\in n\left(\alpha\right)\right\}\quad 0\leq\alpha\leq 1$$

or

$$\overline{P}\left(\alpha\right)=\left[\overline{P}_{l}\left(\alpha\right),\overline{P}_{r}\left(\alpha\right)\right]$$

$$\overline{P}_{l}\left(\alpha\right)=min\left\{\prod_{j=1}^{m}\frac{s+j}{n+j}|s\in s\left(\alpha\right),n\in n\left(\alpha\right)\right\}$$

$$\overline{P}_{r}\left(\alpha\right)=max\left\{\prod_{j=1}^{m}\frac{s+j}{n+j}|s\in s\left(\alpha\right),n\in n\left(\alpha\right)\right\}$$

If $\tilde{s}$ and $\tilde{n}$ be the triangular fuzzy numbers then

$$\overline{P}_{l}\left(\alpha\right)=\prod_{j=1}^{m}\frac{\left(s_1+\left(s_2-s_1\right)\alpha\right)+j}{\left(n_1+\left(n_2-n_1\right)\alpha\right)+j}$$

$$\overline{P}_{r}\left(\alpha\right)=\prod_{j=1}^{m}\frac{\left(s_3+\left(s_2-s_3\right)\alpha\right)+j}{\left(n_1+\left(n_2-n_1\right)\alpha\right)+j}$$

3.3 Numerical Examples

Example 1 Consider a series system with 5 exchangeable components (so *m*=5), and the only information available is the result of a test of 4 components, also exchangeable with the 5 to be used in the system. Assume that the numbers of successes in the tests are expressed as "Approximately 2". Triangular fuzzy numbers are more suitable to convert this definition into a fuzzy number. The number of successes in the tests to be converted to a triangular fuzzy number as $\tilde{s}=\text{TFN}\left(1,2,3\right)$. The FNPI lower and upper probabilities for successful functioning of the system are

$$\tilde{s} = \mathrm{TFN}(1,2,3)$$

$$s(\alpha) = (1+\alpha\ ,3-\alpha)$$

$$\underline{P}(\alpha) = \left\{\prod_{j=1}^{5} \frac{s-1+j}{4+j} | s \in s(\alpha)\right\} \qquad \circ \le \alpha \le 1$$

or

$$\underline{P}(\alpha) = \left[\underline{P}_{l}(\alpha), \underline{P}_{r}(\alpha)\right]$$

$$\underline{P}_{l}(\alpha) = min\left\{\prod_{j=1}^{5} \frac{s-1+j}{4+j} | s \in s(\alpha)\right\}$$

$$\underline{P}_{r}(\alpha) = max\left\{\prod_{j=1}^{5} \frac{s-1+j}{4+j} | s \in s(\alpha)\right\}$$

$$\overline{P}(\alpha) = \left\{\prod_{j=1}^{5} \frac{s+j}{4+j} | s \in s(\alpha)\right\} \quad 0 \le \alpha \le 1$$

or

$$\overline{P}(\alpha) = \left[\overline{P}_{l}(\alpha), \overline{P}_{r}(\alpha)\right]$$

$$\overline{P}_{l}(\alpha) = min\left\{\prod_{j=1}^{5} \frac{s+j}{4+j} | s \in s(\alpha)\right\}$$

$$\overline{P}_{r}(\alpha) = max\left\{1-\prod_{j=1}^{5} \frac{s+j}{4+j} | s \in s(\alpha)\right\}$$

Table (1) and (2) shows α-cuts related to $\tilde{\underline{P}}$ fuzzy lower nonparametric predictive probability and $\tilde{\overline{P}}$ fuzzy upper nonparametric predictive probability and Figures (1) and (2) show diagrams corresponding membership function.

Table 1: α-cuts related to $\tilde{\underline{P}}$ fuzzy lower non-parametric predictive probability.

α	$\underline{P}_l(\alpha)$	$\underline{P}_r(\alpha)$	α	$\underline{P}_l(\alpha)$	$\underline{P}_r(\alpha)$
0	0.0079	0.1667	0.55	0.0234	0.0874
0.05	0.0089	0.1578	0.60	0.0255	0.0821
0.10	0.0099	0.1492	0.65	0.0277	0.0769
0.15	0.0110	0.1410	0.70	0.0301	0.0721
0.20	0.0122	0.1332	0.75	0.0326	0.0674
0.25	0.0135	0.1257	0.80	0.0353	0.0630
0.30	0.0149	0.1186	0.85	0.0381	0.0589
0.35	0.0164	0.1118	0.90	0.0411	0.0549
0.40	0.0180	0.1052	0.95	0.0443	0.0512
0.45	0.0197	0.0990	1	0.0476	0.0476
0.50	0.0215	0.0931			

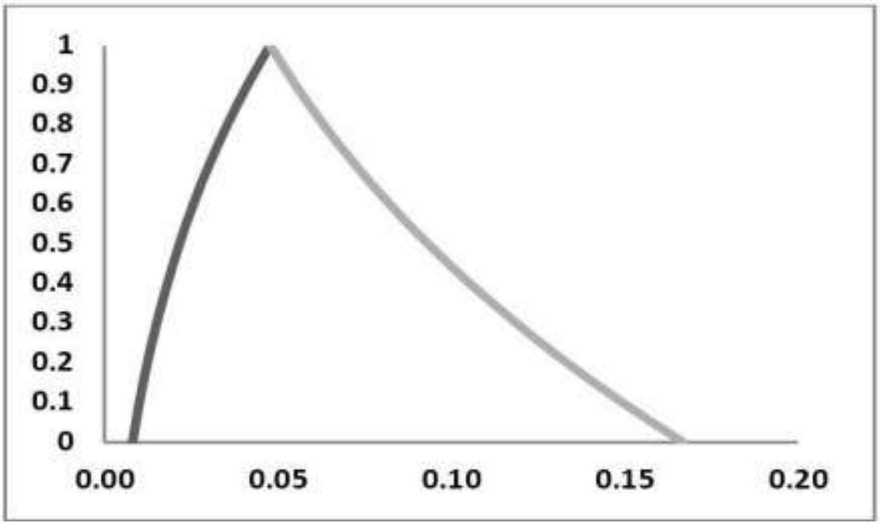

Figure 1: the diagram of membership function of lower non-parametric predictive probability.

Table 2: α-cuts related to $\overline{P}$ fuzzy upper non-parametric predictive probability.

α	$\overline{P}_l(\alpha)$	$\overline{P}_r(\alpha)$	α	$\overline{P}_l(\alpha)$	$\overline{P}_r(\alpha)$
0	0.0476	0.4444	0.55	0.0990	0.2659
0.05	0.0512	0.4251	0.60	0.1052	0.2530
0.10	0.0549	0.4065	0.65	0.1118	0.2407
0.15	0.0589	0.3885	0.70	0.1186	0.2288
0.20	0.0630	0.3711	0.75	0.1257	0.2173
0.25	0.0674	0.3544	0.80	0.1332	0.2063
0.30	0.0721	0.3382	0.85	0.1410	0.1958
0.35	0.0769	0.3226	0.90	0.1492	0.1857
0.40	0.0821	0.3076	0.95	0.1578	0.1760
0.45	0.0874	0.2932	1	0.1667	0.1667
0.50	0.9069	0.9785			

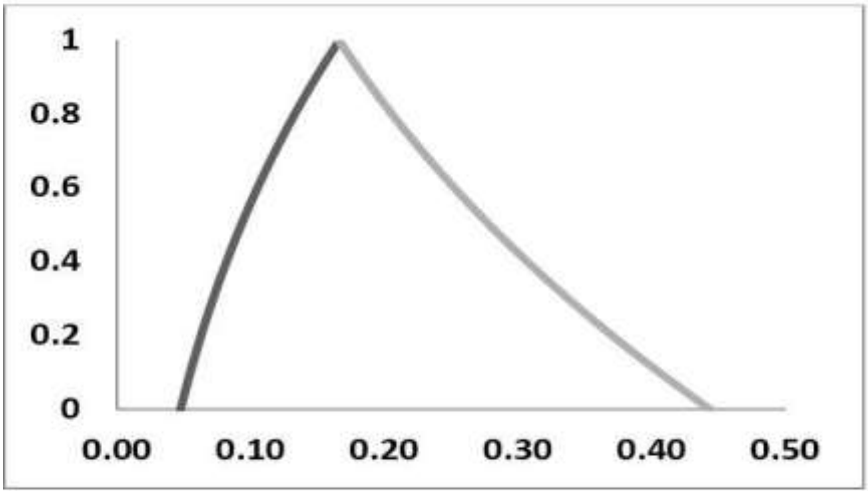

Figure 2: the diagram of membership function of upper non-parametric predictive probability

Example 2 Consider a series system with 5 exchangeable components (so m=5), and the only information available is the result of a test of "Approximately 4" components, also exchangeable with the 5 to be used in the system. Assume that the numbers of successes in the tests are expressed as "Approximately 2". Triangular fuzzy numbers are more suitable to convert this definition into a fuzzy number. The number of components to be converted to a triangular fuzzy number as

$\tilde{n} = \mathrm{TFN}(3,4,5)$ and The number of successes in the tests to be converted to a triangular fuzzy number as $\tilde{s} = \mathrm{TFN}(1,2,3)$. The FNPI lower and upper probabilities for successful functioning of the system are

$$\tilde{n} = \mathrm{TFN}(3,4,5)$$

$$\tilde{s} = \mathrm{TFN}(1,2,3)$$

$$n(\alpha) = (3+\alpha\ ,5-\alpha)$$

$$s(\alpha) = (1+\alpha\ ,3-\alpha)$$

$$\underline{P}(\alpha) = \left\{\prod_{j=1}^{5}\frac{s-1+j}{n+j} | s \in s(\alpha), n \in n(\alpha)\right\} \quad 0 \le \alpha \le 1$$

or

$$\underline{P}(\alpha) = \left[\underline{P}_l(\alpha), \underline{P}_r(\alpha)\right]$$

$$\underline{P}_l(\alpha) = min\left\{\prod_{j=1}^{5}\frac{s-1+j}{n+j} | s \in s(\alpha), n \in n(\alpha)\right\}$$

$$\underline{P}_r(\alpha) = max\left\{\prod_{j=1}^{5}\frac{s-1+j}{n+j} | s \in s(\alpha), n \in n(\alpha)\right\}$$

Too fuzzy upper non-parametric predictive probability,

$$\overline{P}(\alpha) = \left\{\prod_{j=1}^{5}\frac{s+j}{n+j} | s \in s(\alpha), n \in n(\alpha)\right\} \quad 0 \le \alpha \le 1$$

or

$$\overline{P}(\alpha) = \left[\overline{P}_l(\alpha), \overline{P}_r(\alpha)\right]$$

$$\overline{P}_l(\alpha) = min\left\{\prod_{j=1}^{5}\frac{s+j}{n+j} | s \in s(\alpha), n \in n(\alpha)\right\}$$

$$\overline{P}_r(\alpha) = max\left\{\prod_{j=1}^{5}\frac{s+j}{n+j} | s \in s(\alpha), n \in n(\alpha)\right\}$$

Table (3) and (4) shows α-cuts related to $\tilde{\underline{P}}$ fuzzy lower non-parametric predictive probability and $\overline{P}$ fuzzy upper non-parametric predictive probability and Figures (1) and (2) show diagrams corresponding membership function.

Table 3: α-cuts related to $\tilde{\underline{P}}$ fuzzy lower non-parametric predictive probability.

α	$\underline{P}_l(\alpha)$	$\underline{P}_r(\alpha)$	α	$\underline{P}_l(\alpha)$	$\underline{P}_r(\alpha)$
0	0.0179	0.0833	0.55	0.0332	0.0632
0.05	0.0191	0.0815	0.60	0.0347	0.0614
0.10	0.0204	0.0796	0.65	0.0363	0.0597
0.15	0.0217	0.0778	0.70	0.0378	0.0579
0.20	0.0231	0.0759	0.75	0.0394	0.0562
0.25	0.0245	0.0741	0.80	0.0410	0.0544
0.30	0.0258	0.0723	0.85	0.0427	0.0527
0.35	0.0273	0.0704	0.90	0.0443	0.0510
0.40	0.0287	0.0686	0.95	0.0459	0.0493
0.45	0.0302	0.0668	1	0.0476	0.0476
0.50	0.0317	0.0650			

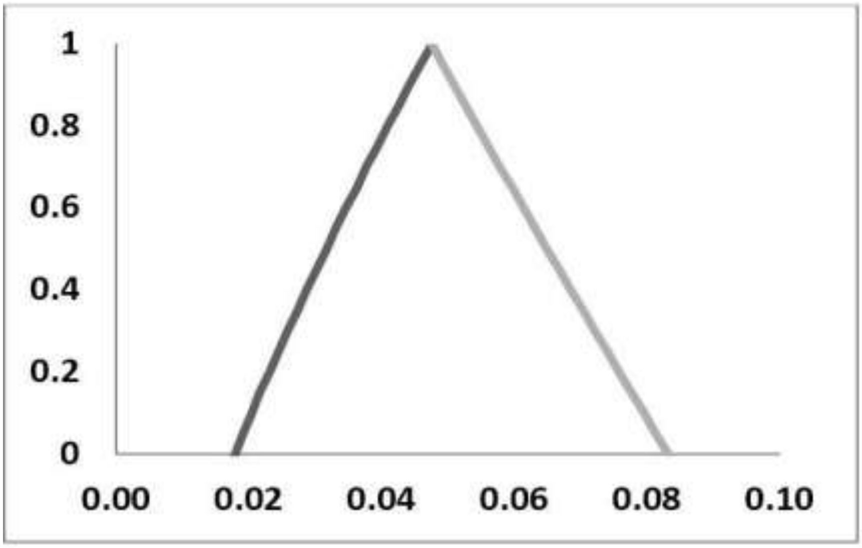

Figure 3: the diagram of membership function of lower non-parametric predictive probability

Table 4: α--cuts related to $\overline{P}$ fuzzy upper non-parametric predictive probability.

α	$\overline{P}_l(\alpha)$	$\overline{P}_r(\alpha)$	α	$\overline{P}_l(\alpha)$	$\overline{P}_r(\alpha)$
0	0.1071	0.2222	0.55	0.1402	0.1923
0.05	0.1102	0.2196	0.60	0.1432	0.1895
0.10	0.1132	0.2169	0.65	0.1462	0.1867
0.15	0.1162	0.2142	0.70	0.1491	0.1838
0.20	0.1192	0.2115	0.75	0.1521	0.1810
0.25	0.1223	0.2088	0.80	0.1550	0.1782
0.30	0.1253	0.2061	0.85	0.1579	0.1753
0.35	0.1283	0.2033	0.90	0.1609	0.1724
0.40	0.1313	0.2006	0.95	0.1638	0.1696
0.45	0.1343	0.1978	1	0.1667	0.1667
0.50	0.1373	0.1950			

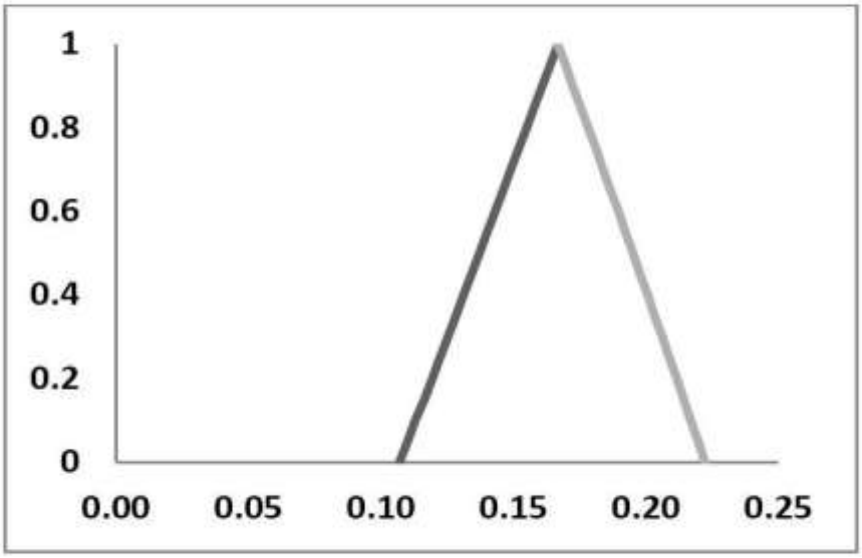

Figure 4: the diagram of membership function of upper non-parametric predictive probability

4. CONCLUSIONS

Despite the usefulness of reliability of series systems base of nonparametric predictive approach, it has the main difficulty in defining its parameters as crisp values. Sometimes it is easier to define these parameters by using linguistic variables. For these cases, the fuzzy set theory is the most suitable tool to analyze the reliability of series systems base of nonparametric predictive approach. The obtained results show that the fuzzy definitions of parameters provide more flexibility and more usability. In this article the nonparametric predictive probability has been analyzed for the reliability of series systems with fuzzy parameters. We have shown that when the definition of lower and upper predictive probability parameters is not possible as crisp values, and when defining the parameters of number of success in tested components and number of tested components as crisp values is not possible, these parameters can be expressed in linguistic terms, and the fuzzy set theory can be used successfully to overcome ambiguity in such expressions in the form of nonparametric predictive reliability of series systems. We also calculate the fuzzy reliability function and its α-cut set.

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