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Ranking Agro-Ecological & Socio-Economic Constraints in Agricultural using Fuzzy Ranking Method based on Radius of Gyration of Centroids

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ABSTRACT--- ***Ranking fuzzy numbers plays a very important role in linguistic decision making and some other fuzzy application systems such as data analysis, artificial intelligence and socio economic systems. Many methods have been proposed to deal with ranking fuzzy numbers. However, the methods were never been simplified and tested in real life application. In this paper a new method is proposed to rank fuzzy numbers based on radius of gyration point of centroids and its application in ranking technical constraints faced by the farmers in agriculture especially on paddy cultivation. This method is relatively simple and easier in computation to rank farmers' technical constraints in agriculture.***

Keywords – centroid points, fuzzy numbers, ranking, farmers' constraints in agriculture

1. INTRODUCTION

Ranking of fuzzy numbers is an important component of the decision process in many applications. In fuzzy multi-attribute decision making, most probably, the ratings and the decision maker's risk performance are evaluated on fuzzy numbers. When the information is expressed as fuzzy number, it is necessary to evaluate and compare fuzzy data before a decision is made. Therefore, ranking fuzzy numbers becomes an important problem in the decision process. By ranking fuzzy numbers, we can rank alternatives and find the best one from them.

Many researchers have proposed different raking methods from different angles in fuzzy ranking. First method for ranking fuzzy numbers was introduced by Jain **[6]** in the year 1976, then a large number of methods have been developed to rank fuzzy numbers. Some of them have been reviewed and compared by Bortolan & Degani **[1]**. Chen & Hwang **[3]**, and Wang & Kerre **[11]**. Among the existing ranking methods of fuzzy numbers, a number of them are focused on the centroid point and its distance from origin or the area measurement for the fuzzy numbers. Yagar**[14]** is a first researcher, who contributes the centroid concept in ranking method. Murakami **[9]** is the first researcher, who presented both horizontal X and Y coordinates of the centroid – point as the ranking index in 1983. Chu and Tsao have proposed a centroid index ranking method that calculates the distance between the centroid point of each fuzzy number and the original point to improve some ranking methods **[5]**. Cheng **[4]** proposed a distance index which is based on both horizontal and vertical coordinates of the centroid point. Wang et al **[12]** proposed a formula for finding horizontal and vertical coordinates of centroid point. Shief **[10]** proposed a correct formula for finding horizontal and vertical coordinates of centroid point. Wang & Lee **[13]** proposed the ranking index on horizontal or vertical coordinates of centroid. Yong & Liu **[15]** used radius of gyration with respect to horizontal axis and vertical axis as the coordinates of centroid point in area based ranking method. Recently, Phani Bushan Rao and others **[10, 11, 14, 13]** have proposed the formula for finding center of gravity point using circumcenter of centroids, orthocenter of centroids, incenter of centroids and centroid of centroids. Although the novelty of the centroid concept in ranking fuzzy numbers has been known for many years, its application in real case study is silent. Beside all these background, this paper proposes a new centroid formula of centroid by using radius of gyration of centroids for ranking fuzzy numbers and tests it to rank the technical constraints faced by the farmers in paddy cultivation. The application of fuzzy decision making in agriculture related research is not new which has been applied in many agricultural decision making problems **[7, 2]**. Specifically the aim of this paper is to propose new ranking methods based on radius of gyration of centroids and to rank the technical constraints faced by the farmers in agriculture by using that method.

The rest of this paper is structured as follows: Section 2 introduces basic concepts and definitions of fuzzy numbers. Section 3 proposes an approach to ranking generalized trapezoidal fuzzy numbers based on radius of gyration of centroid. Section 4 presents a case study of establishing a ranking of agro-ecological and socio-economic constraints faced by the farmers in agriculture, particularly on paddy cultivation. Finally this paper is concluded in Section 5.

2. PRELIMINARIES

In this section, we introduce the basic concepts of fuzzy numbers, centroid point of fuzzy numbers and the radius of gyration (ROG) point of fuzzy numbers, these are play an important role in our proposed method.

2.1. Fuzzy Numbers

Let R be the set of all real numbers. We assume a fuzzy number A that can be expressed for all $x \in R$ in the form

$$A(x) = \begin{cases} A_L(x) & a \le x \le b \\ w & b \le x \le c \\ A_R(x) & c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

Where $0 \le w \le 1$ is a constant, a, b, c, d are real numbers, such that $a < b \le c < d$, $A_L(x):[a,b] \to [0,w]$, $A_R(x):[c,d] \to [0,w]$ are two strictly monotonic and continuous functions from R to the close interval $[0,w]$.

Since $A_L(x)$ is continuous and strictly increasing, the inverse function of $A_L(x)$ exists. Similarly $A_R(x)$ is continuous and strictly decreasing, the inverse function of $A_R(x)$ also exist. The inverse functions of $A_L(x)$ and $A_R(x)$ can be denoted by $A_L^{-1}(x)$ and $A_R^{-1}(x)$, respectively. $A_L^{-1}(x)$ and $A_R^{-1}(x)$ are continuous on [0,w] that means both $\int_0^w A_L^{-1}(x)$ and $\int_0^w A_R^{-1}(x)$ exist.

Definition 1:

A fuzzy number A = (a, b, c, d) is called a trapezoidal fuzzy number if its membership function A(x) has the following form:

$$A(x) = \begin{cases} \dfrac{w(x-a)}{b-a} & a \le x \le b \\ w & b \le x \le c \\ \dfrac{w(d-x)}{d-c} & c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

In this case, the inverse functions $A_L^{-1}(x)$ and $A_R^{-1}(x)$ can be expressed analytically as

$$A_L^{-1}(x) = a + (b - a)/w \qquad 0 \le y \le w$$
$$A_R^{-1}(x) = d - (d - c)/w \qquad 0 \le y \le w$$

Definition 2:

A fuzzy number A = (a, b, c**;** w) is called a triangular fuzzy number if its membership function A(x) has the following form:

$$A(x) = \begin{cases} \dfrac{w(x - a)}{b - a} & a \le x \le b \\ \dfrac{w(d - x)}{d - c} & b \le x \le c \\ 0 & \text{otherwise} \end{cases}$$

2.2. The centroid point of fuzzy numbers

The centroid point of a fuzzy number A, Cheng **[4]** provided the following centroid formulae:

$$\tilde{x}(A) = \frac{\int_a^b (xA_L(x))dx + \int_b^c (x)dx + \int_c^d (xA_R(x))dx}{\int_a^b (A_L(x))dx + \int_b^c dx + \int_c^d (A_R(x))dx}$$

$$\tilde{y}(A) = \frac{\int_0^w (yA_L^{-1}(y))dy + \int_0^w (yA_R^{-1}(y))dy}{\int_0^w (A_L^{-1}(y))dy + \int_0^w (A_R^{-1}(y))dy}$$

Where A_L and A_R are the left and right membership functions of the fuzzy number A. $A_L^{-1}(x)$ and $A_R^{-1}(x)$ are the inverse functions of A_L and A_R, respectively.

Normal fuzzy numbers can be obtained as special cases of non-normal fuzzy numbers with w=1.

2.3. The Radius of Gyration of Fuzzy Numbers:

Radius of gyration (ROG) is a concept in mechanics. The ROG point (r_x, r_y) for a fuzzy number $\tilde{A}$ is provided as **[15]**

$$r_x = \sqrt{\frac{I_x(\tilde{A})}{A(\tilde{A})}}$$

$$r_y = \sqrt{\frac{I_y(\tilde{A})}{A(\tilde{A})}}$$

Where $I_x(\tilde{A})$ and $I_y(\tilde{A})$ are moment of inertia of $\tilde{A}$ with respect to x and y axis respectively and $A(\tilde{A})$ is the area of $\tilde{A}$.

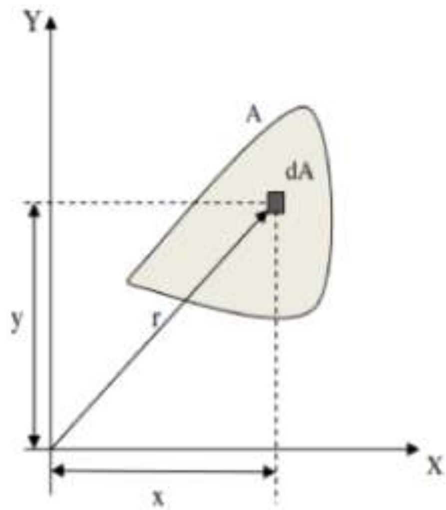

Figure 1: The moment of inertia of an area

3. THE PROPOSED APPROACH TO RANKING FUZZY NUMBERS

The centroid of a trapezoidal is considered as the balancing point of the trapezoid (Figure 2). Divide the trapezoid into three triangles which are AEP, EPF, and PFD. The radius of gyration of the centroids of these triangles is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy members. The reason for selecting this point as a point of reference is that each centroid point (G_1 of triangle EPF, G_2 of triangle AEP, and G_3 of triangle PFD) are balancing points of each individual triangle, and the ROG of these centroid points is much more balancing point for a generalized trapezoidal fuzzy numbers. Therefore, this point would be a better reference point than the centroid point of a trapezoid.

Consider a generalized trapezoidal fuzzy number A = (a, b, c, d; w) (Figure 2). The centroids of the three plane figures are $G_1 = \left(\frac{(a+d)+2(b+c)}{6}, \frac{2w}{3}\right)$, $G_2 = \left(\frac{(a+d)+2(a+b)}{6}, \frac{w}{3}\right)$ and $G_3 = \left(\frac{(a+d)+2(c+d)}{6}, \frac{w}{3}\right)$ respectively. Equation of the line $\overline{G_2G_3}$ is y = w / 3 and G_1 does not lie on the line $\overline{G_2G_3}$. Therefore, G_1, G_2 and G_3 are non – collinear and they form a triangle.

We define the radius of gyration point of a triangle with vertices G_1, G_2 and G_3 of the generalized trapezoidal fuzzy number $(a, b, c, d; w)$ as

$$\left(r_x^{\tilde{A}}, r_y^{\tilde{A}}\right) = \left(\sqrt{\frac{((c+d)-(a+b))^2}{216} + \frac{(5(a+d)+4(b+c))^2}{324}}, \sqrt{\frac{11}{54}}w\right)$$

As a special case, for triangular fuzzy number (a, b, c; w) that is, c = b the radius of gyration point of centroids is given by

$$\left(r_x^{\tilde{A}}, r_y^{\tilde{A}}\right) = \left(\sqrt{\frac{(d-a)^2}{216} + \frac{(5(a+d)+8b)^2}{324}}, \sqrt{\frac{11}{54}}w\right)$$

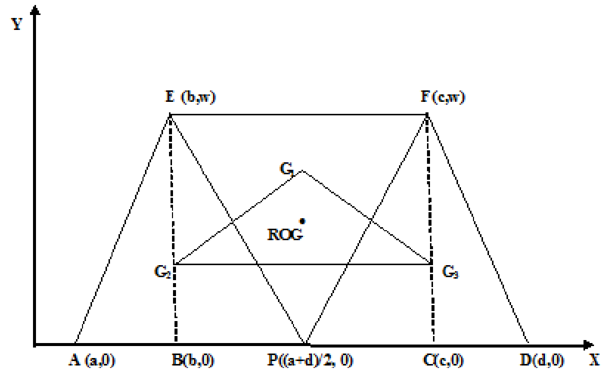

Figure 2: Radius of gyration of centroids

Now A = (a_1, b_1, c_1, d_1; w_1) and B = (a_2, b_2, c_2, d_2; w_2) to two generalized trapezoidal fuzzy numbers. The working procedure to compare A and B is as follows:

Find $r_y(A)$ and $r_y(B)$

Case (i): If $r_x(A) > r_x(B) \Rightarrow A>B$
Case (ii): If $r_x(A) < r_x(B) \Rightarrow A<B$
Case (iii): If $r_x(A) = r_x(B) \Rightarrow$

$$\begin{cases} r_y(A) > r_y(B) \Rightarrow A > B \\ r_y(A) < r_y(B) \Rightarrow A < B \\ r_y(A) = r_y(B) \Rightarrow A = B \end{cases}$$

4. NUMERICAL EXAMPLE

Let $\tilde{A}$ = (2, 2, 2 ; 1) and $\tilde{B}$ = (1, 2, 3; 1)
Then $G_{\tilde{A}}(\bar{x}_0, \bar{y}_0)$ = (2.0000, 0.4513) $G_{\tilde{B}}(\bar{x}_0, \bar{y}_0)$ = (2.0023, 0.4513)
Therefore, $r_x(\tilde{A})$ = 2.0000 and $r_x(\tilde{B})$ = 2.0023
since $r_x(\tilde{A}) < r_x(\tilde{B}) \Rightarrow \tilde{A} < \tilde{B}$

Therefore, from this example the proposed method ranks the crisp number which is the particular case of fuzzy number. But in some ranking functions like, cheng **[4]** Chu & Tsaos' **[5]** ranking functions cannot rank the crisp numbers. It can be seen from their formulae that the denominator in the first coordinate of their centroid formula is zero, and hence centroid of crisp number are undefined for their formulae.

5. CASE STUDY

Rice and wheat are major food crops in India. For the rice crop, that accounts for nearly 41% of the total area under cereals production in India and it is the staple food for more than half of the world population. Rice export in India has contributed to food security of many countries all over the world. The yield level of rice is comparatively low at present need to be increased substantially. To enhance the rice production and export, we need to know about the constraints faced in rice production and export at different levels to find out the suitable solution for overcoming the constraints.

There are three major categories of constraints faced by the farmers in agriculture, which affects the high production and export. These three categories of constraints are focused on agro-ecological constraints, technical constraints and socio-economic constraints. Out of which technical constraints have been studied and ranked according to their seriousness by

Jayakumar and Hari Ganesh **[8]**. By the pilot study conducted in various delta regions in India, five agro-ecological constraints and eight socio-economic constraints have been identified. Discussions were carried out with rice farmers, farms labours, agricultural officers and agricultural engineers. Five agro-ecological constraints and eight socio-economic constraints have been listed below:

Table 1: List of Socio – Economic and Agro – Ecological Constraints

Socio-Economic Constraints	Agro-Ecological Constraints
Inadequate inputs (SE_1)	Environmental pollution (AE_1)
Poor infrastructures (SE_2)	Dependence on monsoon(AE_2)
Lack of trainings (SE_3)	Lack of water (AE_3)
High cost of inputs (SE_4)	Small land (AE_4)
Lack of helpfulness from local governments (SE_5)	Land / soil problems (AE_5)
Poor extension services (SE_6)	
Lack of information (SE_7)	
Credit problems (SE_8)	

So these constraints have to be ranked according to the perception of their seriousness to identify the most sensitive constraints. Furthermore, this study will be more useful to the farmers to find out the suitable solutions for overcoming such constraints and promoting rice production and export in future.

The purpose of this study is to introduce an application of fuzzy ranking methods to rank the agro-ecological and socio-economic constraints faced by the farmers in paddy cultivation. To demonstrate the capability of the proposed ranking method, this study has been conducted in various delta regions of Tamil Nadu, India. An experiment was conducted to elicit linguistic judgment over the seriousness of constraints faced by the farmers in paddy cultivation. Eleven decision makers comprise a farm scientist, two progressive farmers, two educated farmers, two illiterate farmers, two agricultural officers, one farm worker and one agriculture engineer were formed a group of experts. Decision makers were asked to express their opinion about the seriousness of constraints faced by the farmers in paddy cultivation that affect the high rice production. Two different questionnaires for agro-ecological and socio-economic constraints have been used for the evaluation and decision makers need to make their decisions about the seriousness of constraints in agriculture. Decision makers need to respond in five linguistic scales very low seriousness, low seriousness, normal seriousness, high seriousness, and very high seriousness to indicate their views over experiences of farmers' constraints in paddy cultivation. Linguistic scales and their respective Triangular Fuzzy Numbers are given in Table 1.

Table 2: Linguistic Variables and Fuzzy Numbers for the seriousness of constraints

Linguistic Variables	TFN
Very Low Seriousness (VL)	(1,1,2)
Low Seriousness (L)	(1,2,3)
Normal Seriousness (N)	(2,3,4)
High Seriousness (H)	(3,4,5)
Very High Seriousness (VH)	(4,5,5)

Analogously, fuzziness of judgment among the group of DMs can be translated to TFNs. The flexibility of linguistic judgment and their TFNs for the indicators can be visualized in Figure 1.

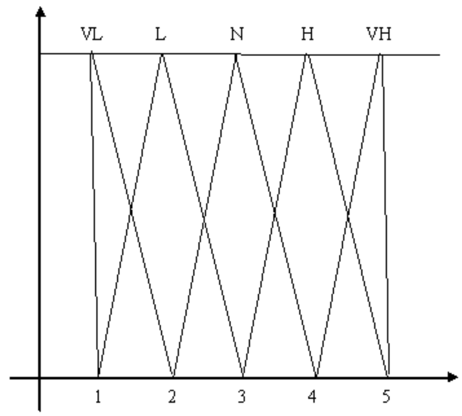

Figure 3: TFN to represent judgment score of the Farmers' constraints

The proposed ranking method in section 3 is employed in the following computations. For the sake of clarity, constrain SE_1 is used as an example.

Linguistic responses from the experts about the seriousness of constraints were compromised using arithmetic mean to obtain average weight score and calculated the average weight score for the constrain SE_1 is (1.45, 2.36, 3.36). The centroid – point of SE_1 was computed using a computer algebra system. Finally the controid -point for constrain SE_1 is obtained by using proposed ranking method as 2.3869. The centroid-points for remaining seven Socio - Economic Constraints and five agro – ecological constraints were computed with the same manner. The constraints are ultimately ranked according to the magnitude of the centroid-point. The centroid-point for the constraints and their respective ranking are presented in Table 2 and Table 3.

Table 3: Ranking of Socio – Economic Constraints

Socio-Economic Constraints	Average Weight Score	Centroid-Point	Rank
Inadequate inputs (SE_1)	(1.45, 2.36, 3.36)	2.3869	4
Poor infrastructures (SE_2)	(3.18, 4.18, 4.82)	4.0809	1
Lack of trainings (SE_3)	(1.27, 2.09, 3.09)	2.1420	5
High cost of inputs (SE_4)	(2.55, 3.55, 4.45)	3.5235	2
Lack of helpfulness from local governments (SE_5)	(1.09, 1.64, 2.64)	1.7670	8
Poor extension services (SE_6)	(1.18, 1.91, 2.91)	1.9870	6
Lack of information (SE_7)	(1.09, 1.73, 2.73)	1.8321	7
Credit problems (SE_8)	(1.73, 2.73, 3.73)	2.7317	3

Table 4: Ranking of Agro-Ecological Constraints

Agro-Ecological Constraints	Average Weight Score	Centroid-Point	Rank
Environmental pollution (AE_1)	(1.55, 2.45, 3.45)	2.4796	3
Dependence on monsoon(AE_2)	(2.82, 3.82, 4.55)	3.7461	1
Lack of water (AE_3)	(1.27, 1.91, 2.91)	2.0119	4
Small land (AE_4)	(1.18, 1.73, 2.73)	1.8569	5
Land / soil problems (AE_5)	(2, 2.91, 3.91)	2.9365	2

Based on the magnitudes, ranking of the constraints are obtained as $SE_2 \succ SE_4 \succ SE_8 \succ SE_1 \succ SE_3 \succ SE_6 \succ SE_7 \succ SE_5$ and $AE_2 \succ AE_5 \succ AE_1 \succ AE_3 \succ AE_4$ where the symbol $\succ$ represents "has less critical than".

6. CONCLUSION

In this paper, a new method for ranking fuzzy numbers, which is based on radius of gyration of centroids**,** is presented. An application to test this ranking method in identifying the seriousness of constraints faced by the farmers in paddy cultivation has been experimented. The centroid-point (x, y) of constraints determined the level of seriousness of farmers' technical constraints in paddy cultivation from experts' point of view. This proposed fuzzy ranking method has been successfully used to rank the constraints in paddy cultivation about their seriousness. This case study offers a new extension in application of ranking of fuzzy numbers. This study will be more useful to the farmers' community to know about the seriousness of constraints faced by the farmers in paddy cultivation and for improving rice production and export.

7. REFERENCES

[1] Bortolan, G., & Degani, R. A review of some methods for ranking fuzzy subsets, Fuzzy Sets and Systems, Vol. 15, No. 1, pp. 1 – 99, 1985.

[2] Burrough, P.A., Fuzzy Mathematical Methods for Soil Survey and land evaluation, Journal of Soil Science, Vol. 40, No. 8, pp. 477 – 492, 1989.

[3] Chen, S.J., & Hwang, C.L. Fuzzy Multiple Attribute Decision Making, Springer, Berlin, 1992.

[4] Cheng, C. H., A New approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Systems, Vol. 95, pp. 307 – 317, 1998.

[5] Chu, T., Tsao, C. Ranking fuzzy numbers with an area between the centroid point and original point, Computers and Mathematics with Applications, Vol. 43, pp. 111 – 117, 2002.

[6] Jain, R. Decision – making in the presence of fuzzy variable. IEEE Transactions on Systems Man, and Cybrenctics, Vol. 6, pp. 698 – 703, 1976.

[7] Jayakumar, S. & Hari Ganesh, A. Fuzzy Multi Criteria Group Decision Making (Mcgdm) Approach for variety selection in Rice Farming, Australian Journal of Basic and Applied Sciences, 6(12), 308 – 318, 2012.

[8] Jayakumar, S., Hari Ganesh, A., Ranking Farmers' Technical Constraints in Agricultural using Centroid point of fuzzy numbers: A Study, Proceedings of International Conference on Mathematical Methods and Computation, Jamal Academic Research Journal (JARJ), Special Issue, pp. 727 – 733, 2014.

[9] Murakami, S. et al., Fuzzy decision analysis on the development of centralized regional energy control system, in: IFAC symposium on Fuzzy Information Knowledge Representation and Decision Analysis pp.363 – 368, 1983.

[10] Shieh. B.S. An approach to centroids of fuzzy numbers, International Journal of Fuzzy Systems, Vol. 9, No.1, pp. 51 – 54, 2007.

[11] Wang, X., & Kerre, E.E. Reasonable properties for the ordering of fuzzy quantities (I), Fuzzy Sets and Systems, 118, 375 – 385, 2001.

[12] Wang, Y.M. et al. On the centroids of fuzzy numbers, Fuzzy Sets and Systems, 157, 919 – 926, 2006.

[13] Wang, Y.J., Lee, H.S. The revised method of ranking fuzzy numbers with an area between the centroid and original points, Computers and Mathematics with applications, Vol. 55, pp. 2033-2042, 2008.

[14] Yager, R.R. A procedure for ordering fuzzy subsets of the unit interval, Information Sciences, Vol. 24, pp. 143 – 161, 1981.

[15] Yong & Liu. Ranking fuzzy numbers with an area method using radius of gyration, Computers and Mathematics with applications. 51, 1127 – 1136, 2006.