Evaluating Environmental Factors through a BL-Algebra

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ABSTRACT--- ***A method for environmental evaluation based on type-2 fuzzy sets and aBL-algebra defined therein is illustrated. Every environmental indicator (security, facilities, environment, social impact, etc.) is a type-2 fuzzy set. Thisapproach has been utilized both to rank the quality of life in Italian cities and then the environmental quality of four sites for a domestic airport near Reykjavik. As regards the quality of lifethis approach produces results similar to those obtained by statistical methods but one gets a linguistic classification of cities and not a numerical one. Also as regards the sites for the airport, the results are similar but the linguistic expressivity is greatly enhanced.***

Keywords – BL-Algebras, Type-2 fuzzy sets, Quality of life, Environmental evaluation.

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1. INTRODUCTION

The concept of fuzzy set was introduced by LotfiZadeh in 1975 [20, 21] and since then a lot of theoretical and practical results have been obtained by researchers and developers. A clear introduction to fuzzy theory is currently available in many textbooks, e.g. [15].

In the following the attention is focused on type-2 fuzzy sets, whose memberships grades are fuzzy sets themselves. These fuzzy sets are used to rank the quality of life in Italian cities and then the environmental quality of four sites for a domestic airport near Reykjavik. In both cases each environmental feature is viewed as a type-2 fuzzy set and these sets are composed by means of the operations of a suitable algebra. The results obtained by applyingthis method are similar to those obtained by traditional approaches, but it is worth emphasizing some meaningful differences.

The paper is organized as follows. In Section 2 the features of the BL-algebraare briefly sketched. Section 3 and 4 apply the methodology to main Italian Italian cities and to Reykjavik airport, respectively.

2. A BL-ALGEBRA ON TYPE-2 FUZZY SETS

In [6] the formal features of a specific BL-algebra are illustrated in some details. This algebraic structure has shown its usefulness in dealing with several applicative areas [4, 7, 8]. In order to make the paper self-consistent the basic features of the BL-algebra are now briefly recalled.

A *commutative partially ordered monoid* is a structure (L, *, e, ≤) such that (L, *, e) is a *commutative monoid,* where the element e is the unit, ≤ is a *partial order* on L and for all a, b, c, d ∈ L, if a ≤ b and c≤d then a*c ≤ b*d.

An algebra (L,∩, ∪) is a *lattice* if the following identities are true in L:

Idempotency) x∩x = x, x∪x = x

Commutativity) x∩y = y∩x, x∪y = y∩x

Associativity) x∩(y∩z)=(x∩y)∩z, x∪(y∪z)=(x∪y)∪z

Absorption) x∩(x∪y) = x∪(x∩y) = x.

A*residuated lattice* (L,∩, ∩, *, ⇒, e, 0) is a structure such that:

i)(L,∩, ∪, *, ⇒, e, 0) is a lattice with the greatest element e and the least element 0 (with respect to the ordering ≤);

ii) (L,*, e) is a commutative monoid with the unit element e;

iii) * and ⇒ form an adjoint pair, i.e., for all a, b ∈ L, c*a≤b iff c≤a ⇒ b (Galois relation). The binary operation ⇒ on L is called *residuum*.

A residuated lattice$(L, \cap, \cup, *, \Rightarrow, e, 0)$ is a *BL-algebra on L* [9, 16] iff the following identities hold for any $x, y \in L$:

i) $x \cap y = x * (x \Rightarrow y)$;

ii) $(x \Rightarrow y) \cup (y \Rightarrow x) = e$.

Let A be a non empty classical set. A *fuzzy sets* on A is a function s: A --> [0, 1]. If a $\in$A then s(a) is said the *membership degree* of a to A.

A *triangular fuzzy number* x=[a, b, c] on [0, 1] is a fuzzy set whose membership function is a triangle whose vertices are the points (a, 0), (b, 1) and (c, 0). In the sequel the following extended operations are used on the class of the [0,1]-triangular fuzzy numbers: i) $\alpha*[a,b,c]=[\alpha*a, \alpha*b, \alpha*c]$ (*product of a real number*); ii) [a, b, c] + [d, e, f] = [a+d, b+e, c+f] (*sum*).

A *type-2 fuzzy set* s_2 [14, 17] on A is a function s_2: A --> $[0, 1]^{[0,1]}$.

Suppose that one has the following objects:

- **i)** **U**: a finite universe of discourse of cardinality p;
- **ii)** **Tr** = {[0, 0, 0], [1, 1, 1]} $\cup$ {[a, b, c]: {a, b, c} $\subset$ [0, 1]}: a set of totally ordered triangular fuzzy numbers. [a, b, c] $\leq$ [d, e, f] iff a$\leq$d, b$\leq$e, c$\leq$f. It is worth noting that the crisp numbers: [0, 0, 0] and [1, 1, 1] belong to **Tr**;
- ***iii)*** $\mathbf{F_2}$= {a: $\Sigma_{i:\ m...1,\ \text{with}\ m\leq p}\ x_i/u_i$}: class of the type-2 fuzzy sets **U** -->**Tr**, where $x_i \in$**Tr**, $x_i < x_{i+1}$, and $\{u_m, u_{m-1}, ..., u_1\}$ belongs to the class of crisp partitions ***P***(**U**) on **U**. In the sequel the elements $u_i$are called *crisp parts* and the elements x_i *fuzzy parts*
- **iv)** **S(U)**= {[**0**= [0, 0, 0]/U, (1, 0, 1)], [**1**=[1,1,1]/U, (0, 1, 1)]} $\cup$ {[a, t]: a $\in \mathbf{F_2}$, and t=(k, s, a_m, a_{m-1}, ..., a_1) is a suitable t-uple of positive integers, that satisfies the following constraints: *j)* if k= 1 then a_i=1 for any i:1...m; *jj)* if k>1 the t-uple (a_m, a_{m-1}, ..., a_1) is symmetric with respect to the central values }; *jjj)* s = 0 for **0**, instead s=1 for any A $\neq$**0** and **1**in**S(U)**. Moreover (k, s, a_m, ..., a_1) = (1, s, 1, 1, ..., 1) iff the related type-2 fuzzy set is not the product of other sets through the operation ◊ introduced in the sequel.

One can give the following intuitive meaning: the type-2 fuzzy set $\Sigma_{i:\ m...1,\ \text{with}\ m\leq p}\ x_i/u_i$ represents an *attribute* A in the sense that the elements $u_i \subseteq$**U** *satisfy* A with strength x_i. Moreover, one says that the elements of U are *classified* with respect to A by means of the linguistic terms represented by the type-1 fuzzy sets $x_i \in [0,1]^{[0,1]}$. With this interpretation the element **0** and **1** are read as "*No information*" and "*Not compatible*", respectively. The label standing for "*No information*" is utilized when there is no information available about the elements in U in order to assess the degree they satisfy the attribute A with, whereas "*Not compatible*" is used if the elements in U are not compatible with the property A.

Given

$\mathbf{A=[\Sigma_{i:\ n\leq p...1}\ x_i/u_i,\ (k_A,\ s_A,\ a_n,\ a_{n-1},\ ...,\ a_1)]}$ and

$\mathbf{B=[\Sigma_{i:\ m\leq p...1} y_i/v_i,\ (k_B,\ s_B,\ b_m,\ b_{m-1},\ ..,\ b_1)]} \in \mathbf{S(U)}$,

the binary operation ◊on **S(U)**x**S(U)** is defined as follows:

$$\mathbf{A \lozenge B = [\Sigma_{i:n+m-1...1} z_i/w_i,\ (k_{A+}k_B,\ 1,\ c_{n+m-1},\ ...,\ c_1)]}$$

where

$$\mathbf{w_i = \bigcup_{\substack{h=1...i \\ k=i...1 \\ h\leq n, k\leq m}} (u_h \cap v_k}$$

fuzzy parts

$$\mathbf{z_i = \frac{s_A s_B}{(k_A + k_B) c_i} \sum_{\substack{h=1...i \\ k=i...1 \\ h\leq n, k\leq m}} a_h b_k (k_A x_h + k_B y_k}$$

$$\mathbf{c_i = \sum_{\substack{h=1...i \\ k=i...1 \\ h\leq n, k\leq m}} a_h b_k}$$

crisp parts

It is worth noting that A ◊**0** = **0** and A ◊**1** = A.

The indices a_h e b_k represent the number of sets that have generated the i-th class of A and B, respectively. The indices k_A e k_B represent, in turn, the number of sets that have generated the classes of A and B, respectively. The

quantities $\mathbf{s_A}$ and $\mathbf{s_B}$ assume the values 1 for any attribute $\neq\mathbf{0}$ and **1** in **S(U)**. The operation for z_i represents essentially a mean among the type-2 fuzzy sets, where each fuzzy set takes a weight in some way related to the changes induced by the composition. Essentially these indices include the computational history of the type-2 fuzzy sets. The operation ◊is well defined: *i*) $(w_{n+m-1}, w_{n+m-2} ..., w_1) \in \boldsymbol{P}(\mathbf{U})$; *ii*) the t-uple $(c_{m+n-1}, ..., c_1)$ is strictly increasing and symmetric with respect to the central values; *iii)* A◊B$\in$**S**(U); *iv*) the elements z_i are triangular fuzzy numbers on [0, 1].
The algebraic properties of the structure have been widely investigated and the reader is referred to [6] where a comprehensive example is illustrated in details.

3. RANKING QUALITY OF LIFE IN MAIN ITALIAN CITIES

The leading daily Italian financialnewspaper "Il Sole 24 Ore" published on December 20th2015 [5] a dossier ranking the quality of life in 110 Italian cities. Bologna ranks first, Messina, in turn, last.
The quality of life indicators taken into account are the following:

Standard of living: household expenditure, income, costs, etc.
Economic development: business and job opportunities, employment, economic growth, retail sales, etc.
Environmental management: waste management and recycling, air quality, traffic, public transport, etc.
Safety: crime levels, child safety, road casualties, perceptions of safety, etc.
People: population growth, age, ethnicity, number of marriages and divorces, etc.
Time off: electronic communication, restaurants, cinemas, bookshops, gyms, etc.

For each of the six indicators, a score is assigned and an overall score is obtained by computing the arithmetical mean of the values. It is worth noting that the same weight is assigned to the six indicators, consequently, for example, a city with high scores in Economic development and low in Safety and People can rank high, whereas Safety and People would deserve more attention. For example, Milan ranks second although performs badly for Safety and People. Another example is given by Potenza which scores high for safety and people but remains at position 66.

Valuable fuzzy-based approaches to classify the quality of life have been developed [1, 2, 11, 12]. For the sake of simplicity, theBL-algebra-based method has been applied to the first 48 cities present in the report (in brackets it is reported the rank as regards the indicator "Standard of living"):

1 Bolzano(4)	*13 Parma (6)*	*25 Forlì (42)*	*37 Piacenza (17)*
2 Milano (1)	*14 Modena (10)*	*26 Reggio Emilia (21)*	*38 Verbano (14)*
3 Trento (16)	*15 Como (2)*	*27 Livorno (30)*	*39 Cagliari(80)*
4 Firenze (47)	*16 Roma (41)*	*28 Brescia (23)*	*40 Massa Carrara (62)*
5 Sondrio (28)	*17 Belluno (19)*	*29 Mantova(38)*	*41 Genova (43)*
6 Olbia (73)	*18 Udine (24)*	*30 Grosseto (61)*	*42 Prato (26)*
7 Cuneo (22)	*19 Gorizia (5)*	*31 Verona (18)*	*43 Nuoro (85)*
8 Aosta (7)	*20 Monza (15)*	*32 Pesaro (64)*	*44 Ancona (54)*
9 Siena (52)	*21 Ascoli Piceno(56)*	*33 Savona(44)*	*45 Lecco (11)*
10 Ravenna (39)	*22 Pisa (50)*	*34 Trieste (12)*	*46 Varese (3)*
11 Macerata (48)	*23 Rimini (57)*	*35 Vicenza (35)*	*47 Fermo (58)*
12 Bologna (9)	*24 Bergamo (31)*	*36 Arezzo (55)*	*48 Venezia (51)*

The basic linguistic terms used are poor, satisfactory, good, excellent. It is clear that four linguistic terms are not sufficient to express the quality of life level of a city. In fact, additional linguistic modifiers, such as very, more than, less than, almost, and so on can be used.

The membership degree of the cities to the six indicators is represented by four linguistic labels (low, sufficient, fair, high) which get associated with a fuzzy partition including the following fuzzy triangular numbers: [0, 0, 1/3], [0, 1/3, 2/3], [1/3, 2/3, 1], [2/3, 1, 1]. The attribute strings, namely the type-2 fuzzy sets, that denote the membership degree of the cities to the six indicators are then constructed. For example, the string for "Standard of living" is:

High */ { Milano, Como, Varese, Bolzano, Gorizia, Parma, Aosta, Bologna, Modena, Lecco, Trieste, Verbano} +*

+ Fair / *{Monza, Trento,Piacenza, Belluno, Verona, Reggio Emilia, Cuneo, Brescia, Udine,Prato,Sondrio,Livorno}* +

+ Sufficient / *{Bergamo,Vicenza,Olbia,Mantova,Ravenna,Roma,Forlì,Savona,Firenze,Macerata,Genova,Pisa}* +

+ Low / *{Siena,AscoliPiceno,Rimini, Grosseto,Pesaro,Arezzo,Cagliari,MassaCarrara,Nuoro,Ancona,Venezia,Fermo}.*

By applying the operator ◊ one gets the fuzzy set "Quality of life":

Quality of life = Standard of living ◊ Economic development ◊ Environmental management ◊ Safety ◊ People ◊ Time off =

verygood/ *{Aosta, Firenze, Siena, Trento}* + more thangood/ *{ Sondrio, Bolzano, Milano, Cuneo }* + good/ *{ Olbia, Como, Macerata, Parma}* + almostgood/ *{Ravenna, Bologna, Udine, Gorizia, Belluno, Modena, Monza, Roma}* + fair/ *{Verona, Mantova, Brescia, Reggio Emilia, Bergamo, Pisa}* + more thansufficient/ *{Livorno, Forlì, Ascoli Piceno, Pisa}* + sufficient */ { Cagliari, Piacenza, Vicenza, Trieste, Genova, Lecco, Varese, Venezia, Fermo}* + almostsufficient / *{Pesaro, Grosseto,Savona, Arezzo, Verbano}* + insufficient/*{Ancona, Nuoro, Prato, Massa Carrara}*

Now it is worth briefly discussing the results obtained. No city ranks "excellent" and this is quite obvious as no city ranks first in all six strings. In particular, Bolzano and Milano (first and second in the newspaper's ranking) are now viewed as "more than good" as a consequence of the values of "Safety" and "People". Best positions are obtained by Trento, Firenze, Siena, Aosta which have good grades in all six indicators. However, arithmetical mean is not adequate to model satisfactorily the cities. In fact, the top performance of Bolzano and Milano stems from the high values of "Standard of living" and "Economic development", but it is correct that the low values of "Safety" and "People" affect the final ranking. Also, in the last positions of the ranking, a similar effect is present: in fact, the city of Fermo, last but one for "Il Sole", gets a satisfactory position thanks to the indicator "People".

The following table summarizes the new tabulated data using the BL-algebra approach:

Ranking	***Cities***
1 : very good	*Aosta, Firenze, Siena, Trento*
2: more than good	*Sondrio, Bolzano, Milano, Cuneo*
3: good	*Olbia, Como, Macerata, Parma*
4:almost good	*Ravenna, Bologna, Udine, Gorizia, Belluno, Modena, Monza, Roma*
5: fair	*Verona, Mantova, Brescia, Reggio Emilia, Bergamo, Rimini*
6:more than sufficient	*Livorno, Forlì, Ascoli Piceno, Pisa*
7: sufficient	*Cagliari, Piacenza, Vicenza, Trieste, Genova, Lecco, Varese, Venezia, Fermo*
8:almost sufficient	*Pesaro, Grosseto,Savona, Arezzo, Verbano*
9: insufficient	*Ancona, Nuoro, Prato, Massa Carrara*

Now there are only nine linguistic grades that take into account the six indicators. Instead of assigning a numeric position in the ranking, that cannot exactly reflect the specific situation of the city, a more comprehensive and general linguistic label is attached to a group of cities that overall deserve that grade.

4. RANKING FOUR SITES FOR AN AIRPORT

In [18] a report for the environmental and socio-economic evaluation of four sites for a domestic airport isillustrated. The four sites are classified according to two classes of indicators: environmental indicators and socio-economic ones. In particular, the environmental indicators are: safety, social impact, and environmental impact and are suitably weighted.

Then, for each site, the environmental quality index (EQI) is computed as the weighted sum of the scores of the indicators. Then the development cost of the airport is taken into account for the four sites and finally the best tradeoff environment/cost is singled out.

The four sites are denoted by the acronyms RVA, RLA, HHA, KIA. The following table shows the scores obtained by the four sites:

Indicators	*weights*	*RVA*	*RLA*	*HHA*	*KIA*
SAFETY					
Air Accidents	*0.4*	*8*	*8*	*9*	*8*
Road Accidents	*0.6*	*9*	*9*	*6*	*3*
Final score		*8.6*	*8.6*	*7.2*	*5.0*
SOCIAL IMPACTS					
Regional planning	*0.4*	*4.5*	*8*	*9*	*9*
Other utilisation	*0.2*	*2.5*	*4*	*7.5*	*7.5*
Level of service	*0.4*	*8.6*	*8.4*	*6.2*	*5.7*
Final score		*5.74*	*7.36*	*7.42*	*7.38*
ENVIRONMENTAL IMPACT					
Noise and pollution	*0.3*	*6*	*7*	*9*	*8*
Aviation Impact	*0.3*	*5*	*2*	*3*	*9*
Impact of structures	*0.3*	*9*	*1*	*4*	*9*
Visual Impact	*0.1*	*9*	*1*	*3*	*9*
Final score		*6.9*	*3.1*	*5.1*	*8.7*

The linguistic interpretation of the scores is as follows:

Score	*Interpretation*
1-2	*Unacceptable*
2-3	*Poor*
4-5	*Acceptable*
5-7	*Fair*
7-8	*Good*
8-9	*Excellent*

Finally, the following table shows the final scores obtained by the four sites, as regards the indicators safety, social impact and environment impact:

Environmental Indicators	*Safety*	*Social Impact*	*Environmental Impact*	*EQI=U(x)*
Weights	0.660	0.165	0.175	
R.V.A.	8.6	5.74	6.9	7.83
R.L.A.	8.6	7.36	3.1	7.43
H.H.A.	7.2	7.42	5.1	6.87
K.I.A.	5	7.38	8.7	6.04

The suitability of fuzzy logic for dealing with environmental evaluation problems has been widely investigated [3, 10, 13,19]. Now thegoal is to classify the four sites using the same indicators used by Solnes and Porgeirsson. The following table summarizes the linguistic terms and the corresponding fuzzy numbers:

Linguistic terms	*Fuzzy numbers*
Unsatisfactory	*(0;0;0.2)*
Poor	*(0;0.2;0.4)*
Acceptable	*(0.2;0.4;0.6)*
Fair	*(0.4;0.6;0.8)*
Good	*(0.6;0.8;1)*
Excellent	*(0.8;1;1)*

As regards the indicators safety, social impact and environmental impact, the corresponding type-2 fuzzy sets are obtained as follows:

Air_Accidents = excellent/{rva, rla, hha ,kia}

Road_Accidents =excellent/{rva, rla} + good/{hha} + poor /{ kia }

*Safety = 4*air_accidents ◊ 6*road_accidents = excellent/{ rva ,rla }+ fair/{ hha } + acceptable/{ kia }.*

This table shows the crisp values of the indicator safety and the values of the related fuzzy set:

Airport	*Score*	*Solnes&Porgeirsson*	*Fuzzy values*	*Linguistic approximation*
RVA	*8.6*	*Excellent*	*0.80, 1.00, 1.00*	*Excellent*
RLA	*8.6*	*Excellent*	*0.80,1.00 ,1.00*	*Excellent*
HHA	*7.2*	*Good*	*0.573 ,0.773, 0.919*	*Fair*
KIA	*5.0*	*Acceptable*	*0.415, 0.608,0.784*	*Acceptable*

In a similar way the fuzzy set for the indicator Social impact is obtained:

Regional_Planning = excellent/{rla,hha,kia} + poor/{rva}.
Other_Utilisation = good{hha,kia}+ poor/{rla}+ non unacceptable/{rva}.
Level_of_Service = excellent/{rva,rla}+acceptable/{hha,kia}.
*Social Impact=4*regional ◊ 2*other utili. ◊ 4*level= very good/{rla} + good/{hha,kia} + acceptable/{rva}.*

Airport	*Score*	*Solnes&Porgeirsson.*	*Fuzzy values*	*Linguistic approximation*
RVA	*5.74*	*Acceptable*	*0.405,0.592, 0.765*	*Acceptable*
RLA	*7.36*	*Good*	*0.679, 0.879, 0.978*	*Very good*
HHA	*7.42*	*Good*	*0.583, 0.780,0.914*	*Good*
KIA	*7.38*	*Good*	*0.583, 0.780, 0.914*	*Good*

For the indicator Environmental impact, one gets:

Noise and pollution = *excellent/{hha,kia} + good/{rla} + acceptable/{rva}.*

Aviation impact = *excellent/{kia}+acceptable/{rva}+poor/{hha}+ unacceptable/{rla}.*

Impact of structure= *excellent/{rva,kia}+ poor/{hha}+ unacceptable/{rla}.*

Visual impact= *excellent/{rva,kia} + poor/{hha} + unacceptable/{rla}.*

Environmental impact=*3*noise ◊ 3*aviation ◊ 3*impact<>1*visual = excellent/{kia} almost good/{rva} + almost acceptable/{hha} + very poor{/rla}*

Airport	*Score*	*Solnes&Porgeirsson*	*Fuzzy values*	*Linguistic approximation*
RVA	*6.9*	*Acceptable*	*0.551, 0.745,0.880*	*Almost good*
RLA	*3.1*	*Poor*	*0.150, 0.296,0.493*	*Very poor*
HHA	*5.1*	*Acceptable*	*0.338, 0.514,0.692*	*Almost acceptable*
KIA	*8.7*	*Excellent*	*0.800,1.00, 1.00*	*Excellent*

Finally, one gets the environmental quality index:

*Environmental quality index = 7*Safety ◊ 1*Social ◊ 2*Environmental= good/{rva}+ almost good/{rla} + very acceptable/{kia}+more than acceptable/{hha}.*

Airport	*Score*	*Solnes&Porgeirsson*	*Fuzzy values*	*Linguistic Approx.*
RVA	*7.83*	*Good*	*0.615,0.811,0.916*	*Good*
RLA	*7.43*	*Good*	*0.565, 0.758,0.879*	*Almostgood*
HHA	*6.87*	*Acceptable*	*0.470,0.659,0.814*	*More than acceptable*
KIA	*6.04*	*Acceptable*	*0.484,0.673,0.824*	*Very acceptable*

5. CONCLUDING REMARKS

Two different applications have been illustrated in this paper, both concerning aspects that deeply affect people's daily life. This algebraic approach to ranking the quality of life is completely different from the traditional statistical ones, yet it is encouraging that final results are quite similar to those previously achieved. The additional benefit stands in the fact that cities get classified linguistically and this makes them more readable. The validity of the method is confirmed by the second case study. Indeed, as regards the sites for the airport, also in this application the results are very similar to those obtained by Solnes and Porgeirsson, yet the linguistic expressivity is enhanced. More complex application areas are currently being investigated and, at the same time, the possibility of modifyingthis approach so that different weights can be assigned to the relevant indicators. In conclusion, we think that this fuzzy-based approach to classification can lead to results more detailed and expressive from the linguistic point of view.

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