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Growth of Tumour using Mathematical Modelling with the Help of Discrete Fractional Calculus

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ABSTRACT----: ***In this paper we are trying to find the basics growth of mathematical modelling of the growth of cancerous cell or any type of growth cause by bacteria or any others by using discrete fractional calculus.***

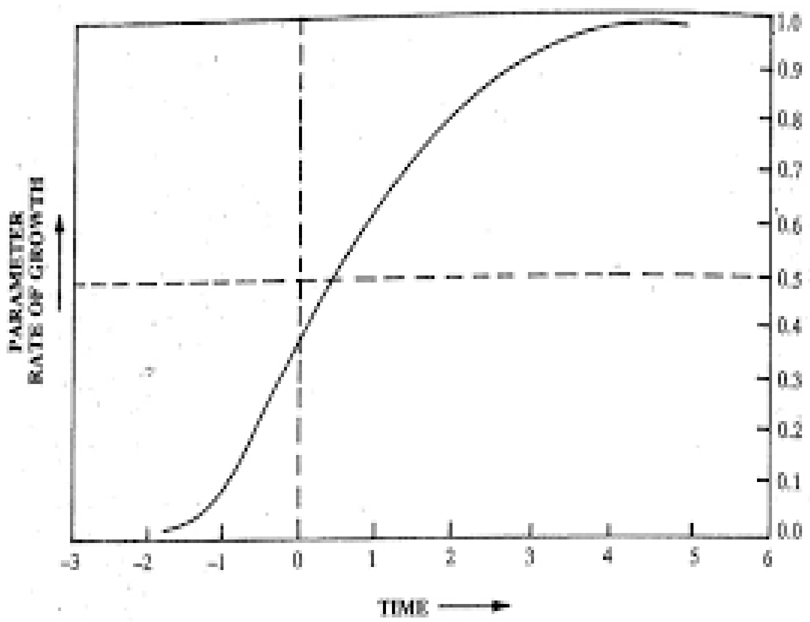
Keywords---- Discrete fractional calculus, Delta operator [Δ] , continuous partial derivatives, forward jump operator.

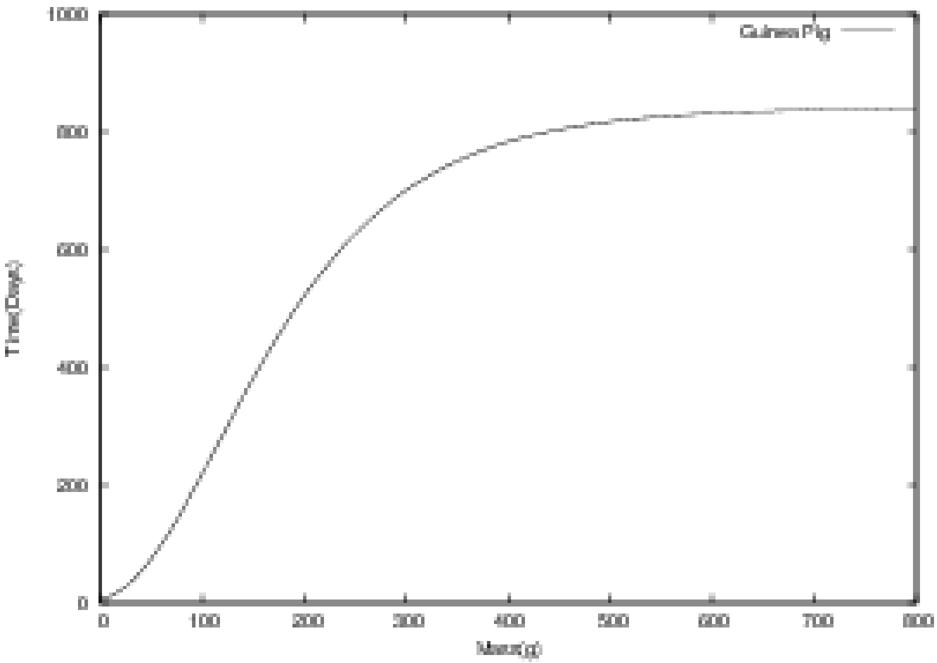
INTRODUCTION

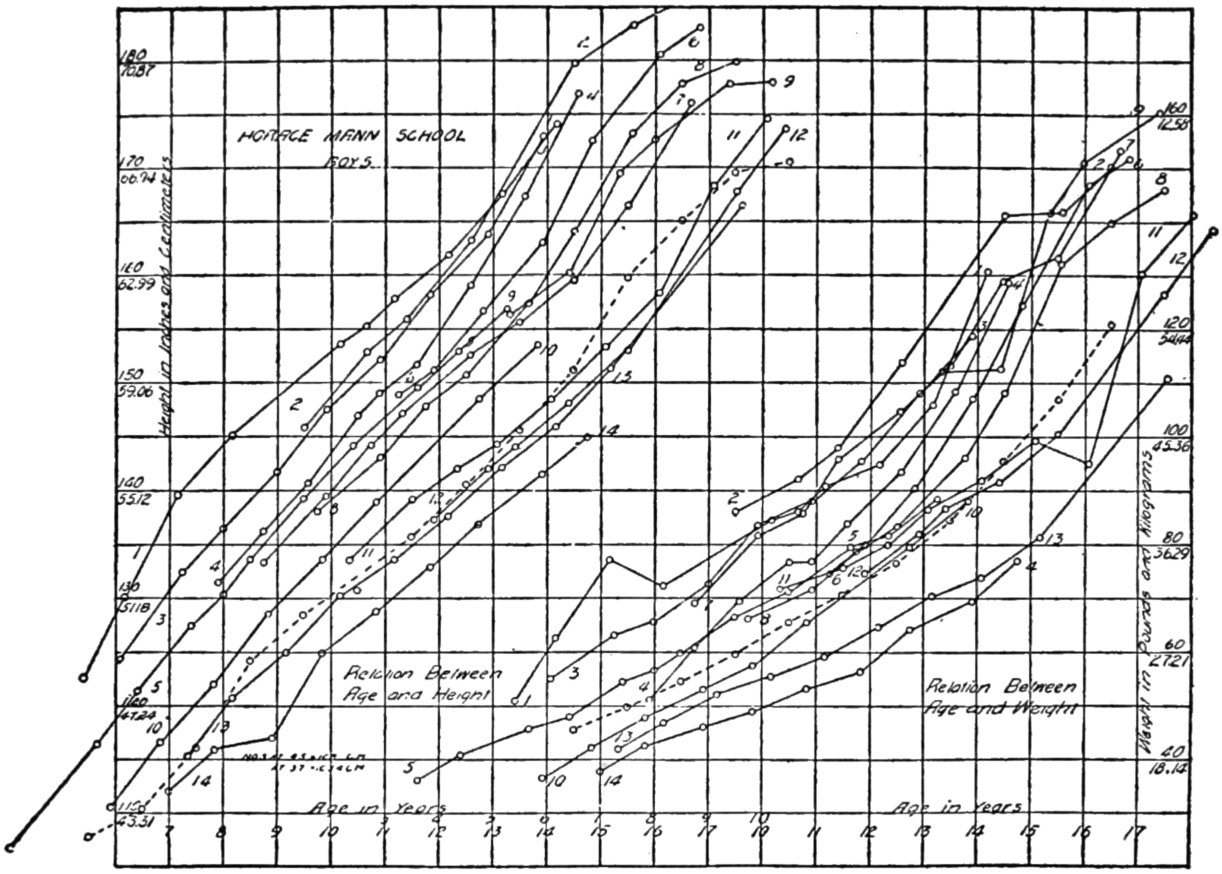
Cancer is a generic term for a large group of diseases that can affect any part of the body. It is a leading cause of death worldwide, about 8.2 million death due to cancer in 2012 worldwide and about 14.1 million new cancer cases diagnosed in 2012 worldwide, and also by 2030 13 million cancer death are predicted (ref. American cancer society).It is arises from one single cell. The transformation from a normal cell into a tumour cell is a multistage process. According to WHO, the risk factor for cancers are by using Tobacco, alcohol, unhealthy diet and physical inactivity.

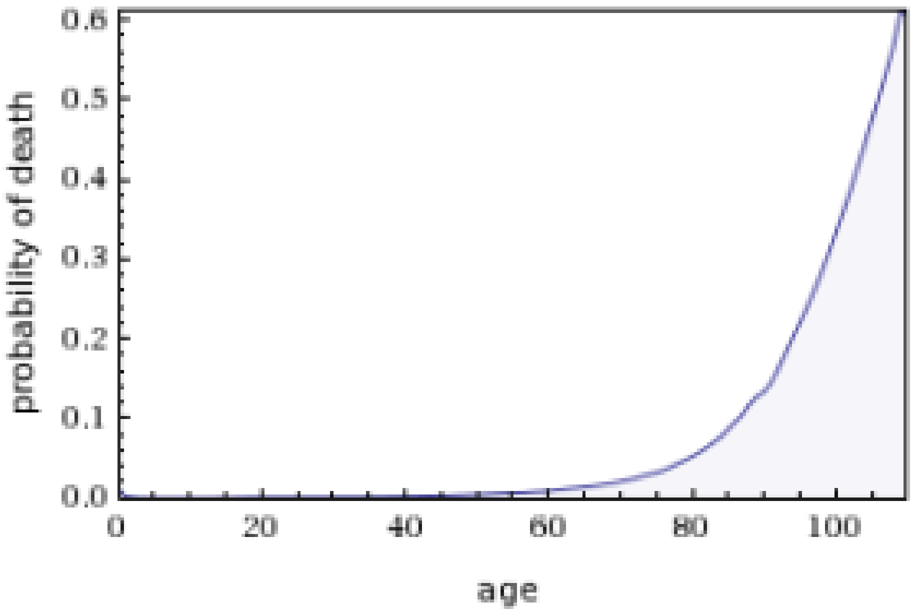
Mathematical modelling of tumours started in 1955, where Tomlinson and Gray (1955) proposed a mathematical model of the diffusion and consumption of oxygen to supplement. In 1825 **Benjamin Gompertz** (5 March 1779 – 14 July 1865) was a British self-educated mathematician and actuary, who became a Fellow of the Royal Society. Gompertz is now best known for his Gompers law of mortality, a demographic model published in 1825, he apply to study the growth in biological and economic contexts in 1932 by Winson(1932) and in 1965 laird showed that the Gompertzian equation could describe the normal growth of an organism such as the guinea pig over an incredible 10,000 fold range of growth [1].

Gompertz law: 1. The below figures show the growth of different object




Guinea Pig
Time(Days)
Mass(g)
0
200
400
600
800
1000
0
100
200
300
400
500
600
700
800


HORACE MANN SCHOOL
BOYS
Relation Between Age and Height
Relation Between Age and Weight
Age in Years
Age in Years
Height in Inches and Centimeters
Weight in Pounds and Kilograms



(based on death rates in 2005)

Gompertz fractional difference equations: Growth as we know that it is related to time i.e. time to time it is increases or decreases ,same for tumour growth also since growth estimation is very critical in clinical practice .In a model growth there are three terms growth behaviour in biology ;**Exponential**, **logistic**, and sigmoidal, and in a tumour growth however, is best described by sigmoidal function. In 1825, Benjamin Gompertz introduced the Gompertz function,a sigmoid function, which is found to be applicable to various growth phenomena. In particular tumour growth [2], the Gompertz difference equation describes the growth models and these models can be studied on the basis of the parameters.

The Gompertz difference equation is $\ln G(t+1) = a + b\ln G(t)$ where ''a'' is growth rate and ''b'' is the exponential rate of growth deceleration, from simplest variational problem for discrete fractional calculus, we consider the following optimization problem $J[y] = \min \sum_{t=0}^{T-1} U\left(y(t+\alpha-1)\right)$ with the constraint $\Delta^{\alpha} y(t-\alpha+1) = (b-1)y(t) + a\ ,\ \ y(0) = c$

Or with the transformation $t = t + \alpha - 1$ $\Delta^{\alpha} y(t) = (b-1)\ y(t+\alpha-1) + a$ where $\Delta = \textit{delta operator},\ b = b > 1\ \textit{is an}$ integer α is a positive real number (0,1) or $0 < \alpha < 1$ and ''a'' be any real number where y(t) is the size of a tumour and U is a function with continuous partial derivatives

Introduction to discrete fractional calculus in brief: Recently in the discrete fractional calculus is being studied by the ATICI, ALOE, GOODRICH, MILLER AND ROSS etc.

1. **Repeated summation rule**: if $f : \mathrm{N}_a \to R$ be given, then
$$\int_a^t \int_a^{\tau} ......\int_a^{\tau_{n-1}} f(\tau_n)\Delta\tau_n ....\Delta\tau_2\Delta\tau_1 = \int_a^t h_{n-1}(t,\sigma(s))\ f(s)\Delta s \qquad [3]$$
Where $n \geq 1,\ N_a = \{a, a+1, a+2\}, a \in R$ or a set of the form $N_a^b := \{a, a+1, a+2, ...\mathrm{b}\}$ $a,b \in R\ \ and\ \ b-a\ \ is\ positive$ and σ is forward jump operator where $\sigma(t) = t+1$

2. **Fractional power sum:** $if\ \ \mu \geq 0\ \ and\ \ \nu > 0\ \ then\ \ \Delta_{a+\mu}^{-\nu}(t-a)^{\nu} = \dfrac{\Gamma(\mu+1)}{\Gamma(\mu+\nu+1)}(t-a)^{\mu+\nu}$

   For $t \in N_{a+\mu+\nu}$

3. Fractional difference power rule: if $if\ \mu > 0\ and\ \nu \geq 0, N-1 < \nu < N$

$$\Delta_{a+\mu}^{\nu}(t-a)^{\mu} = \frac{\Gamma(\mu+1)}{\Gamma(\mu-\nu+1)}(t-a)^{\mu-\nu} \quad for\ \ t \in N_{a+\mu+N-\nu}$$

Model of tumour growth : it is given by Ludwing Von Bertalanffy(1996) $\frac{dy}{dt} = ay^{\alpha} + by^{\beta}$

Where ''y'' is a measure of the size of the organism, ''a'' ''b '' $\alpha\ and\ \beta$ are constants. And the solution is given by $\frac{dv}{dt} = av^{\alpha} - bv^{\beta}$ the special case $\alpha = 1 and\ \beta = 2$ gives the logistic equation

$\frac{dv}{dt} = av - bv^2$ where $v(0) = v_0$ and the solution is given by

$$\frac{dv}{av - bv^2} = dt$$

$$\frac{dv}{v(a-bv)} = \frac{A}{v} + \frac{B}{(a-bv)}$$

$$1 = A(a-bv) + Bv$$

solving it we get $\frac{1}{av} + \frac{b}{a(a-bv)}$ integrating it

$\frac{1}{a}\ln v - \frac{1}{a}\ln(a-bv) = t + c$ where $c = \frac{1}{a}\ln\frac{v_0}{a-bv_0}$ putting this we get the result like

$V(t) = e^{\frac{a}{b}\left(\frac{a}{b}\ln v_0\right)e^{-bt}}$ this is the model of tumour growth

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