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A Trivial Note Concerning p And $n!$, Where p Is Prime $\geq n+1$ And n Is An Integer ≥ 1.

Ikorong A. Gilbert
Centre De Calcul;D'Enseignement Et De Recherche En Mathematiques Pures
Universite' de Paris VI; France
ikorong@ccr.jussieu.fr
Paul Archambault;
Research (Analytic Southern Europe)
Paul.Archambault@pb.com

Abstract. We show that if n is an integer ≥ 1 and if p is a prime $\geq n+1$, then for every integer k such that $1 \leq k \leq n$, p does not divide $k!$; where $k! = 1 \times .... \times k$; in particular, if p is a prime $\geq n+1$, then the greatest common divisor of p and $n!$ is 1 and therefore p does not divide $n!$.
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1. The proof of stated result.
We recall that if n is an integer ≥ 1, then $n!$ is defined as follow:

$$n! = \begin{cases} 1 & \text{if } n = 1, \\ 2 & \text{if } n = 2, \\ 1 \times 2 \times .... \times n & \text{if } n \geq 3. \end{cases}$$

Theorem 1.1. *Let n is an integer ≥ 1 and let p be a prime $\geq n+1$. Then for every integer k such that $1 \leq k \leq n$, p does not divide $k!$.*

Corollary 1.2. *Let n is an integer ≥ 1 and let p be a prime $\geq n+1$. Then the greatest common divisor of p and $n!$ is 1 (in particular, p does not divide $n!$).*
Proof. Immediate, and follows immediately by using Theorem 1.1. □

Now, to prove simply Theorem 1.1, we use the fundamental Theorem of Euclide.

Theorem 1.3 *(Euclide). Let* a*,* b *and* c*, be integers such that* $a \geq 1$*,* $b \geq 1$ *and* $c \geq 1$*. If* a *divides* bc *and if the greatest common divisor of* a *and* b *is* 1*, then* a *divides* c*.*□

Proof of Theorem 1.1. Otherwise [we reason by reduction to absurd], let k be a minimum counter-example to Theorem 1.1, clearly $k > 3$. It is immediate that the greatest common divisor of p and k is 1 [since p is prime $\geq n+1 \geq k+1$ (use the hypotheses) and since k is an integer > 3 (by the previous)]; now using the previous and Theorem 1.3, then we immediately deduce that p divides $(k-1)!$ [since $k > 3$ and p divides $k!$ and the greatest common divisor of p and k is 1]. This contradicts the minimality of k. □

Epilogue. Using Theorem 1.1, then it becomes natural and not surprising to conjecture (see [1]):

Conjecture.*Let* n *be an integer* ≥ 4*. If* $n+1$ *does not divide* $n!$ *and if* $n+3$ *does not divide* $n!$*, then* $n+5$ *divides* $(n+4)!$.

References .

[1] Ikorong Anouk Gilbert Nemron. *Then We Characterize Primes and Composite Numbers Via Divisibility.* International Journal of Advanced In Pure Mathematical Sciences; Volume 2, no. 1; 2014.

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