A Common Fixed Point Theorem for Three Self Maps in Cone Rectangular Metric Space

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ABSTRACT— *In this paper, we prove a common fixed point theorem for three self mappings in cone rectangular metric space. Our result generalizes and extends recent known results.*

Keywords— cone rectangular metric space, normal cone, common fixed point theorem, coincidence point.

1. INTRODUCTION

In 1906, the French mathematician M. Frenchet [4] introduced the concept of metric spaces. In this sequel, Branciari [3] introduced a class of generalized metric spaces by replacing triangular inequality of metric spaces by similar one which involves four or more points instead of three points and improved Banach Contraction Principle in such spaces. In 2007, Huang and Zhang [5] introduced the notion of cone metric spaces. They have replaced real number system by an ordered Banach space and established some fixed point theorems for contractive type mappings in a normal cone metric space. The study of fixed point theorems in such spaces is followed by some other mathematicians; see [1], [5], [6], [9], [13], [14], [16], [18].

Recently, A. Azam, M. Arshad and I. Beg [2] extended the notion of cone metric spaces by replacing the triangular inequality by a rectangular inequality and they proved Banach contraction Principle in a complete normal cone rectangular metric space. Several authors proved some fixed point theorems in such spaces see; [7], [10], [11], [12], [15], [17]. In 2012, R. A. Rashwan and S. M. Saleh [15] extended Banach Contraction Principle in cone rectangular metric space to two self mappings. In this paper we generalize and extended theorem 3 of Azam et al. [2] for three self mappings.

2. PRILIMINARIES

First, we recall some standard definitions and other results that will be needed in the sequel.

2.1. Definition [5]: A subset P of a real Banach space E is called a '*cone*' if it has following properties:

(1) P is nonempty, closed and $P \neq \{\theta\}$;

(2) $a, b \in R,\ a,\ b \geq 0,\ x,\ y \in P$ implies $ax + by \in P$;

(3) $P \cap (-P) = \{\theta\}$.

For a given cone $P \subseteq E$, we can define a partial ordering $\leq$ on E with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x < y$ if $x \leq y$ and $x \neq y$, while $x << y$ will stands for $y - x \in \text{int}\ (P)$, where $\text{int}(P)$ denotes the interior of P.

2.2. Definition [5]: The cone P is called '*normal*' if there is a number $k > 1$ such that for all x, y $\in$ E, $0 \leq x \leq y$ implies $\|x\| \leq k\|y\|$.

The least positive number k satisfying the above condition is called the '*normal constant*' of P.

2.3. Proposition [15]: Let (X, d) be a cone metric space with cone P is not necessary to be normal. Then for *a, c, u, v, w* ∈ *E,* we have

(1) If $a \le \lambda a$ and $\lambda \in [0,1)$, then $a = \theta$.

(2) If $\theta \le u \ll c$, for each $\theta \ll c$, then $u = \theta$.

(3) If $u \le v$ and $v \ll w$, then $u \ll w$.

2.4. Definition [5]: Let X is a nonempty set, E is a real Banach space and P is a cone in E with int P ≠ Φ and ≤ is a partial ordering with respect to P. Suppose that the mapping $d : X \times X \to E$ satisfies:

(1) $\theta \le d(x, y)$, for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$, for all $x, y \in X$;

(3) $d(x, y) \le d(x, z) + d(z, y)$, for all $x, y, z \in X$.

Then *d* is called a *'cone metric'* on X and (X, d) is called a *'cone metric space'*.

2.5. Definition [2]: Let X is a nonempty set, E is a real Banach space and P is a cone in E with int P ≠ Φ and ≤ is a partial ordering with respect to P. Suppose that the mapping $d : X \times X \to E$ satisfies:

(1) $\theta \le d(x, y)$, for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$, for all $x, y \in X$;

(3) $d(x, y) \le d(x, w) + d(w, z) + d(z, y)$, for all $x, y \in X$ and for all distinct points $w, z \in X - \{x, y\}$

[rectangular inequality].

Then *d* is called a *'cone rectangular metric'* on X and (X, d) is called a *'cone rectangular metric space'*.

2.6. Example [2]: Let X = N, $E = R^2$ and $P = \{(x, y): x, y \ge 0\}$ is a normal cone in E. Define $d : X \times X \to E$ as follows:

$$d(x, y) = \begin{cases} (0, 0) & \text{if } x = y \\ (3, 9) & \text{if x and y are in } \{1, 2\}, x \ne y \\ (1, 3) & \text{otherwise} \end{cases}$$

Now, (X, d) is a cone rectangular metric space, but (X, d) is not a cone metric space because it lacks the triangular property:

$$(3, 9) = d(1, 2) > d(1, 3) + d(3, 2) = (1, 3) + (1, 3) = (2, 6),$$

as (3, 9) - (2, 6) = (1, 3) ∈ P.

2.7. Definition [2]: Let (X, d) be a cone rectangular metric space. Let $\{x_n\}$ be a sequence in (X, d) and $x \in X$. If for every $c \in E$, with $\theta \ll c$ there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n, x) \ll c$, then $\{x_n\}$ is said to be convergent, $\{x_n\}$ converges to x and x is the limit of $\{x_n\}$. We denote this by $\lim_{n \to \infty} x_n = x$ or $x_n \to x$, as $n \to \infty$.

2.8. Definition [2]: Let (X, d) be a cone rectangular metric space. Let $\{x_n\}$ be a sequence in (X, d). If for every $c \in E$, with $\theta \ll c$ there is $n_0 \in N$ such that for all $m, n > n_0$, $d(x_n, x_m) \ll c$, then $\{x_n\}$ is called a Cauchy sequence in (X, d).

2.9. Definition [2]:   Let (X, d) be a cone rectangular metric space. If every Cauchy sequence is convergent in (X, d), then (X, d) is called a complete cone rectangular metric space.

2.10. Lemma [2]:   Let (X, d) be a cone rectangular metric space and P be a normal cone with a normal constant k, let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $\|d(x_n, x)\| \to 0,$ as $n \to \infty$.

2.11. Lemma [2]:   Let (X, d) be a cone rectangular metric space, P be a normal cone with a normal constant k. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $\|d(x_n, x_m)\| \to 0,$ as $n, m \to \infty.$

2.12. Definition [8]:   Let T and S are self maps of a nonempty set X. If $w = Tx = Sx,$ for some $x \in$ X, then x is called a *'coincidence point'* of T and S and w is called a *'point of coincidence'* of T and S.

2.13. Definition [8]:   Two self mappings T and S are said to be *'weakly compatible'* if they commute at their coincidence points, that is, $Tx = Sx$ implies that $TSx = STx$.

3.   MAIN RESULT

In this section we establish a common fixed point theorem for three self mappings in cone rectangular metric space. The following theorem extends and improves Theorem 3 of [2].

3.1. Theorem: Let (X, d) be a cone rectangular metric space and P be a normal cone with a normal constant k. Let S, T, and U: X →X be three self mappings of X satisfying the following conditions:

$$(1). \ d(Sx, Ty) \le \lambda d(Ux, Uy),$$

$$(2). \ d(Sx, Sy) \le \lambda d(Ux, Uy),$$

$$\text{and } (3). \ d(Tx, Ty) \le \lambda d(Ux, Uy),$$

for all x, y $\in$ X, where $\lambda \in [0,1)$. If $S(X) \cup T(X) \subseteq U(X)$ and $S(X) \cup T(X)$ or U(X) is a complete subspace of X, then the mappings S, T and U have a unique coincidence point in X. Moreover, if (S, U) and (T, U) are weakly compatible pairs then S, T and U have a unique common fixed point in X.

Proof: Let x_0 be any arbitrary point of X. Since, $S(X) \cup T(X) \subseteq U(X)$, starting with x_0 we define the sequence $\{y_n\}$ such that,

$$y_{2n} = Sx_{2n} = Ux_{2n+1},$$

$$\text{and } \ y_{2n+1} = Tx_{2n+1} = Ux_{2n+2}, \text{ for all } n \ge 0.$$

We shall prove that $\{y_n\}$ is a Cauchy sequence in X.

Suppose that $y_m = y_{m+1},$ for some $m \in N$.

If $m = 2k$, then $y_{2k} = y_{2k+1},$ for some $k \in N,$ then from (1) we get,

$$d(y_{2k+2}, y_{2k+1}) = d(Sx_{2k+2}, Tx_{2k+1})$$

$$\le \lambda d(Ux_{2k+2}, Ux_{2k+1})$$

$$= \lambda d(y_{2k+1}, y_{2k}) = \theta.$$

So that $y_{2k+2} = y_{2k+1}.$

Similarly, we can deduce that $y_{2k+2} = y_{2k+3} = y_{2k+4}....$

Hence, $y_n = y_m,$ for all $n \ge m.$

Thus $\{y_n\}$ is a Cauchy sequence in X in this case.

Assume that, $y_n \neq y_{n+1},$ for all $n \in$ N.

Then from (1) it follows that,

$$d(y_{2k}, y_{2k+1}) = d(Sx_{2k}, Tx_{2k+1})$$

$$\le \lambda d(Ux_{2k}, Ux_{2k+1})$$

$$\text{i.e., } \ d(y_{2k}, y_{2k+1}) \le \lambda d(y_{2k-1}, y_{2k}), \tag{4}$$

for all $k \geq 1$.

Also we have, $\quad d(y_{2k+1}, y_{2k+2}) = d(Tx_{2k+1}, Sx_{2k+2})$

$$= d(Sx_{2k+2}, Tx_{2k+1})$$

$$\leq \lambda d(Ux_{2k+2}, Ux_{2k+1})$$

i.e., $\quad d(y_{2k+1}, y_{2k+2}) \leq \lambda d(y_{2k}, y_{2k+1}),$

(5)

for all $k \geq 1$.

Using (4) and (5) we get,

$$d(y_{2k}, y_{2k+1}) \leq \lambda d(y_{2k-1}, y_{2k})$$

$$\leq \lambda^2 d(y_{2k-2}, y_{2k-1})$$

$$\vdots$$

$$\leq \lambda^{2k} d(y_0, y_1),$$

(6)

for all $k \geq 1$.

Also we have, $\quad d(y_{2k+1}, y_{2k+2}) \leq \lambda d(y_{2k}, y_{2k+1})$

$$\leq \lambda^2 d(y_{2k-1}, y_{2k})$$

$$\vdots$$

$$\leq \lambda^{2k+1} d(y_0, y_1),$$

(7)

for all $k \geq 1$.

From (6) and (7) we have,

$$d(y_n, y_{n+1}) \leq \lambda^n d(y_0, y_1),$$

(8)

for all $n \geq 1$.

Using $(2), (8),$ rectangular inequality and using the fact that $\lambda < 1,$ we get,

$$d(y_{2k}, y_{2k+2}) \leq \lambda d(Sx_{2k}, Sx_{2k+2})$$

$$\leq \lambda d(Ux_{2k}, Ux_{2k+2})$$

$$\leq \lambda d(y_{2k-1}, y_{2k+1})$$

$$\leq \lambda \left[d(y_{2k-1}, y_{2k}) + d(y_{2k}, y_{2k+2}) + d(y_{2k+2}, y_{2k+1}) \right]$$

which implies that,

$$d(y_{2k}, y_{2k+2}) \leq \frac{\lambda}{1-\lambda} \left[d(y_{2k-1}, y_{2k}) + d(y_{2k+1}, y_{2k+2}) \right]$$

$$\leq \frac{\lambda}{1-\lambda} \left[\lambda^{2k-1} d(y_0, y_1) + \lambda^{2k+1} d(y_0, y_1) \right]$$

$$\leq \frac{1}{1-\lambda} \left[\lambda^{2k} d(y_0, y_1) + \lambda^{2k+2} d(y_0, y_1) \right]$$

$$\leq \frac{\lambda^{2k}}{1-\lambda} (1+\lambda^2) d(y_0, y_1),$$

$$\leq \frac{\lambda^{2k}}{1-\lambda} (1+\lambda) d(y_0, y_1),$$

(9)

for all $k \geq 1$.

Using (3) and (9) we get,

$$d(y_{2k+1}, y_{2k+3}) = d(Tx_{2k+1}, Tx_{2k+3})$$

$$\leq \lambda d(Ux_{2k+1}, Ux_{2k+3})$$

$$= \lambda d(y_{2k}, y_{2k+2})$$

$$\leq \frac{\lambda^{2k+1}}{1-\lambda}(1+\lambda)d(y_0, y_1), \tag{10}$$

for all $k \geq 1$.

Therefore from (9) and (10) we get,

$$d(y_n, y_{n+2}) \leq \frac{\lambda^n}{1-\lambda}(1+\lambda)d(y_0, y_1), \tag{11}$$

for all $n \geq 1$.

For the sequence $\{y_n\}$ we consider $d(y_n, y_{n+p})$ in two cases.

If p is odd say $2m + 1$, for $m \geq 1$, then by using rectangular inequality and (8) we get,

$$
\begin{aligned}
d(y_n, y_{n+2m+1}) &\leq d(y_{n+2m+1}, y_{n+2m}) + d(y_{n+2m}, y_{n+2m-1}) + d(y_{n+2m-1}, y_n) \\
&\leq d(y_{n+2m}, y_{n+2m+1}) + d(y_{n+2m-1}, y_{n+2m}) + d(y_{n+2m-1}, y_{n+2m-2}) \\
&\quad + d(y_{n+2m-2}, y_{n+2m-3}) + \ldots + d(y_{n+2}, y_{n+1}) + d(y_{n+1}, y_n) \\
&= d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \ldots + d(y_{n+2m-1}, y_{n+2m}) + d(y_{n+2m}, y_{n+2m+1}) \\
&\leq \lambda^n d(y_0, y_1) + \lambda^{n+1}d(y_0, y_1) + \lambda^{n+2}d(y_0, y_1) + \ldots + \lambda^{n+2m-1}d(y_0, y_1) + \lambda^{n+2m}d(y_0, y_1) \\
&= \left[1 + \lambda + \lambda^2 + \ldots + \lambda^{2m-1} + \lambda^{2m}\right]\lambda^n d(y_0, y_1) \\
&\leq \left[1 + \lambda + \lambda^2 + \lambda^3 + \ldots\right]\lambda^n d(y_0, y_1) \\
&\leq \frac{\lambda^n}{1-\lambda}d(y_0, y_1).
\end{aligned}
$$

Hence, $d(y_n, y_{n+2m+1}) \leq \dfrac{3\lambda^n}{1-\lambda}d(y_0, y_1),$ (12)

for all $n \geq 1, m \geq 1$.

If p is even say $2m$, for $m \geq 1$, then by using rectangular inequality, (8), (11), and using the fact that $\lambda < 1$, we get,

$$
\begin{aligned}
d(y_n, y_{n+2m}) &\leq d(y_{n+2m}, y_{n+2m-1}) + d(y_{n+2m-1}, y_{n+2m-2}) + d(y_{n+2m-2}, y_n) \\
&\leq d(y_{n+2m-1}, y_{n+2m}) + d(y_{n+2m-2}, y_{n+2m-1}) + \ldots + d(y_{n+4}, y_{n+3}) + d(y_{n+3}, y_{n+2}) + d(y_{n+2}, y_n) \\
&= d(y_n, y_{n+2}) + d(y_{n+2}, y_{n+3}) + \ldots + d(y_{n+2m-2}, y_{n+2m-1}) + d(y_{n+2m-1}, y_{n+2m}) \\
&\leq \frac{\lambda^n}{1-\lambda}(1+\lambda)d(y_0, y_1) + \left[\lambda^{n+2}d(y_0, y_1) + \lambda^{n+3}d(y_0, y_1) + \ldots + \lambda^{n+2m-2}d(y_0, y_1) + \lambda^{n+2m-1}d(y_0, y_1)\right] \\
&= \frac{\lambda^n}{1-\lambda}(1+\lambda)d(y_0, y_1) + \left[\lambda^2 + \lambda^3 + \ldots + \lambda^{2m-2} + \lambda^{2m-1}\right]\lambda^n d(y_0, y_1) \\
&\leq \frac{\lambda^n}{1-\lambda}(1+\lambda)d(y_0, y_1) + \left[1 + \lambda + \lambda^2 + \lambda^3 + \ldots\right]\lambda^n d(y_0, y_1) \\
&\leq \frac{1+\lambda}{1-\lambda}\lambda^n d(y_0, y_1) + \frac{\lambda^n}{1-\lambda}d(y_0, y_1) \\
&= \left[\frac{2+\lambda}{1-\lambda}\right]\lambda^n d(y_0, y_1)
\end{aligned}
$$

Hence, $d(y_n, y_{n+2m}) \leq \dfrac{3\lambda^n}{1-\lambda}d(y_0, y_1),$ (13)

for all $n \geq 1, m \geq 1$.

Therefore, from (12) and (13) we have,

$$d(y_n, y_{n+p}) \leq \frac{3\lambda^n}{1-\lambda}d(y_0, y_1), \tag{14}$$

for all $n \in N, p \in N$.

Since, P is a normal cone with a normal constant k and $0 \le \lambda < 1$, we have

$$\left\| d(y_n, y_{n+p}) \right\| \le \frac{3k\lambda^n}{1-\lambda} \left\| d(y_0, y_1) \right\| \to 0, \text{ as } n \to \infty,$$

$$\text{i.e., } \left\| d(y_n, y_{n+p}) \right\| \to 0, \text{ as } n \to \infty, \ \forall p \in N.$$

(15)

Hence, $\{y_n\}$ is a Cauchy sequence in X.

Suppose, U(X) is a complete subspace of X, there exists $z \in U(X)$ such that

$$\lim_{n \to \infty} y_n = z \tag{16}$$

Also, we can find $x \in X$ such that $z = Ux$ (17)

Now, we prove x is a coincidence point of pairs (S, U) and (T, U).

That is we prove, $Sx = Tx = Ux = z.$

First we prove $Sx = z.$

Using rectangular inequality, (1) and (17) we get,

$$d(Sx, z) \le d(Sx, y_{2n-1}) + d(y_{2n-1}, y_{2n}) + d(y_{2n}, z)$$

$$= d(Sx, Tx_{2n-1}) + d(y_{2n-1}, y_{2n}) + d(y_{2n}, z)$$

$$\le \lambda d(Ux, Ux_{2n-1}) + d(y_{2n-1}, y_{2n}) + d(y_{2n}, z)$$

$$= \lambda d(z, y_{2n-2}) + d(y_{2n-1}, y_{2n}) + d(y_{2n}, z), \text{ for all } n \ge 1.$$

Since, P is a normal cone with a normal constant k, using (15) and (16) we get,

$$\left\| d(Sx, z) \right\| \le k \left[\lambda \left\| d(z, y_{2n-2}) \right\| + \left\| d(y_{2n-1}, y_{2n}) \right\| + \left\| d(y_{2n}, z) \right\| \right] \to 0, \text{ as } n \to \infty,$$

$$\text{i.e., } \left\| d(Sx, z) \right\| = \theta$$

$$\text{i.e., } Sx = z.$$

Hence, $Sx = Ux = z$ (18)

Now we prove, $Tx = z.$

Using rectangular inequality, (1) and (17) we get,

$$d(Tx, z) \le d(Tx, y_{2n}) + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z)$$

$$= d(Tx, Sx_{2n}) + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z)$$

$$= d(Sx_{2n}, Tx) + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z)$$

$$\le \lambda d(Ux_{2n}, Ux) + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z)$$

$$= \lambda d(y_{2n-1}, z) + d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, z), \text{ for all } n \ge 1.$$

Since, P is a normal cone with a normal constant k, using (15) and (16) we get,

$$\left\| d(Tx, z) \right\| \le k \left[\lambda \left\| d(y_{2n-1}, z) \right\| + \left\| d(y_{2n}, y_{2n+1}) \right\| + \left\| d(y_{2n+1}, z) \right\| \right] \to 0, \text{ as } n \to \infty,$$

$$\text{i.e., } \left\| d(Tx, z) \right\| = 0$$

$$\text{i.e., } Tx = z.$$

Hence, $Sx = Tx = Ux = z$ (19)

That is z is a point of coincidence of the pairs (S, U), and (T, U).

We shall prove that z is unique.

Suppose z^* is another point of coincidence of these pairs.

That is, $z^* = Ux^* = Tx^* = Sx^*$, for some $x^* \in X$ (20)

Using (1), (19), (20), and using the fact that $\lambda < 1$ we get

$$d(z, z^*) = d(Sx, Tx^*)$$

$$\le d(Ux, Ux^*)$$

$$\text{i.e., } d(z, z^*) < d(z, z^*).$$

Which is a contradiction.

Hence, $z = z^*.$

Thus, S, T and U have a unique point of coincidence in X.

Suppose, the pairs (S, U), and (T, U) are weakly compatible, then from (19) we get,

$$Sz = SUx = USx = Uz$$

and $\quad Tz = TUx = UTx = Uz.$

Therefore, $Sz = Tz = Uz = w$ (say)

Thus, *w* is another point of coincidence of S, T and U.

Therefore, by the uniqueness of point of coincidence, we must have *w* = *z*.

Hence, there exist unique point $z \in X$ such that $Sz = Tz = Uz = z.$

Thus, *z* is a unique common fixed point of self mappings S, T and U.

Similarly, if $S(X) \cup T(X)$ is a complete subspace of X, then the self mappings S, T and U have a unique common fixed point in X.

3.2. Example: Let X = {1/2, 1/3, 1/4, 1/5}, where E = $\mathbb{R}^2$ and P = $\{(x,\ y):\ x,\ y \geq 0\}$ is a normal cone in E.

Define $d : X \times X \to E$ as follows:

$$\begin{cases} d(x,\ x) = \theta, \quad \text{for all } x \in X, \\ d(1/2,\ 1/3) = d(1/3,\ 1/2) = (3,\ 6), \\ d(1/2,\ 1/4) = d(1/4,\ 1/2) = d(1/3,\ 1/4) = d(1/4,\ 1/3) = (1,\ 2), \\ d(1/2,\ 1/5) = d(1/5,\ 1/2) = d(1/3,\ 1/5) = d(1/5,\ 1/3) = d(1/4,\ 1/5) = d(1/5,\ 1/4) = (2,\ 4). \end{cases}$$

Then (X, d) is a complete cone rectangular metric space.

Now we define the mappings S, T and U: X →X as follows:

$$S(x) = 1/4, \quad \text{for all } x \in X$$

$$T(x) = \begin{cases} 1/4 & \text{if } x \neq 1/5 \\ 1/2 & \text{if } x = 1/5 \end{cases}$$

and $U(x) = x,$ for all $x \in X.$

It is clear that, $S(X) \cup T(X) \subseteq U(X)$ and (S, U) and (T, U) are weakly compatible. The inequalities (1), (2) and (3) of Theorem 3.1 hold for $\lambda = 1/2$ and 1/4 is a unique common fixed point of S, T and U.

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