Vector Fitting-Based Models of Vertical and Horizontal Electrodes under Lightning Strikes to Interface with EMTP

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ABSTRACT—*In this paper, wide band circuit models based on vector fitting method are proposed for the grounded vertical and horizontal electrodes. At first, the input impedance of the electrodes in the frequency domain is computed by accurate methods such as the method of moments (MoM). The circuit models are then achieved through converting input impedance of the electrodes to rational functions via vector fitting method. These functions are formed in such a way that at first a set of starting poles in the frequency range of interest are chosen, and then the locations of poles are reformed via an iteration process. Finally, these rational functions are converted to equivalent circuits in time domain and then imported into EMTP software for modeling the ground systems so that transient voltage is efficiently evaluated.*

Keywords— Grounding system, EMTP, Vector fitting.

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1. INTRODUCTION

Grounding systems such as vertical and horizontal electrodes are often used in power systems to discharge lightning current into earth without any damage to people and installations [1-6]. Figure 1 shows schematic diagram of such grounding systems under lightning stroke.

Transient voltage of grounding system (defined electrical potential of the grounding electrodes with respect to a reference point at infinite) is of great practical importance, because firstly it is able to reveal the maximum voltage level that is submitted to the ground, secondly it is evaluates the time that the ground is subjected to certain levels of transient voltage. Safety criteria are based upon minimizing this parameter. It should be thus correctly computed. This parameter is usually computed via EMTP (Electromagnetic Transient Program) software through importing grounding system as equivalent circuit into EMTP.

There are a number of equivalent circuits for the grounded electrodes [1] as shown in figure 2. The values of R, L, and C in figure 2 are listed in table 2. Unfortunately these models are based on quasi static approximation and valid when the length of electrode is less than one-tenth of the wave length in the earth as shown in figure 3.

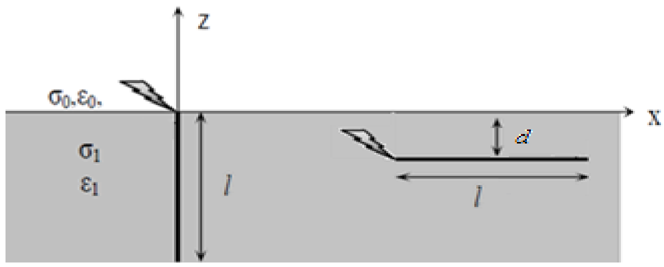
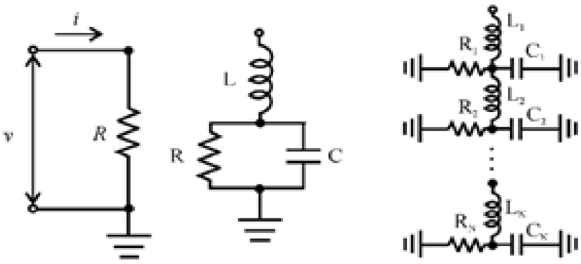


Fig 1. Schematic diagrams of conventional grounding systems.



(a) (b) (c)

Fig 2. Models of the grounding systems, (a) resistivity model, (b) RLC model, and (c) distributed RLC model.

There are two current waveforms for lightning, that is, first and subsequent strokes [7] which are expressed as following:

$$i(t) = \frac{I_0}{\eta} \frac{(t/\tau_1)^n}{1+(t/\tau_1)^n} \exp(-t/\tau_2) \tag{1}$$

and

$$\eta = \exp\left[-(\tau_1/\tau_2)(n\tau_2/\tau_1))^{1/n}\right] \tag{2}$$

Where I_0 is the amplitude of current, τ_1 is the front time constant, τ_2 is decay time constant, n is an exponent having value between 2 to 10, and η is the amplitude of the correction factor. These parameters are given in the table 1.

Table 1. Heidler parameters for lightning current of first and subsequent stroke.

Parameters	I_{01} (KA)	τ_{11} (μs)	τ_{12} (μs)	n_1	I_{02} (KA)	τ_{21} (μs)	τ_{22} (μs)	n_2
First Stroke	28	1.8	92	2	---	---	---	---
Subsequent stroke	10.7	0.25	2.5	2	6.5	2.1	230	2

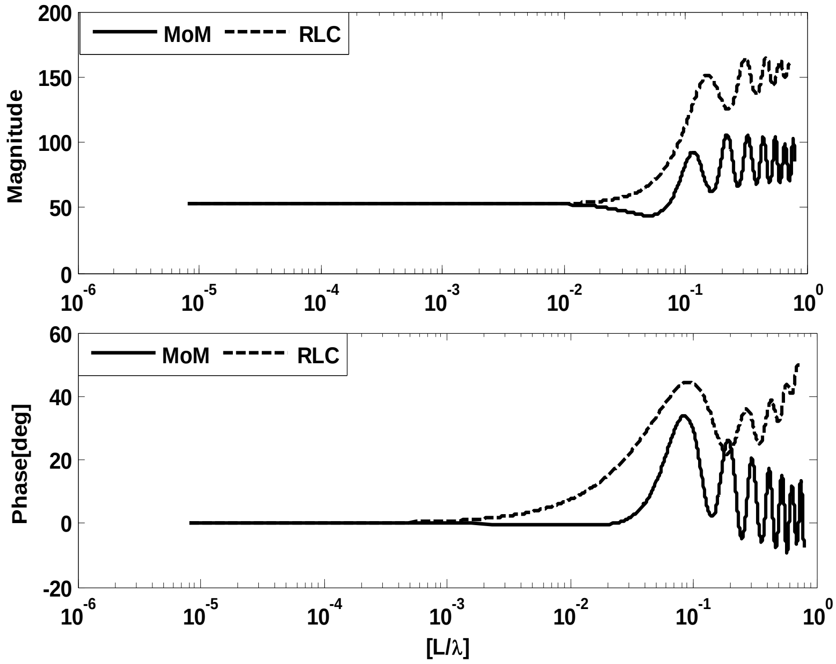


Fig 3. The input impedance of vertical electrode of length 24m which is computed by MoM and RLC.

The two lightning currents are shown in figure 4. According to this figure, the induced- lightning pulses induce currents of high frequencies up to 5 MHz or so. Therefore proposing equivalent circuits for grounding systems in wide band of frequency is still a challenging task.

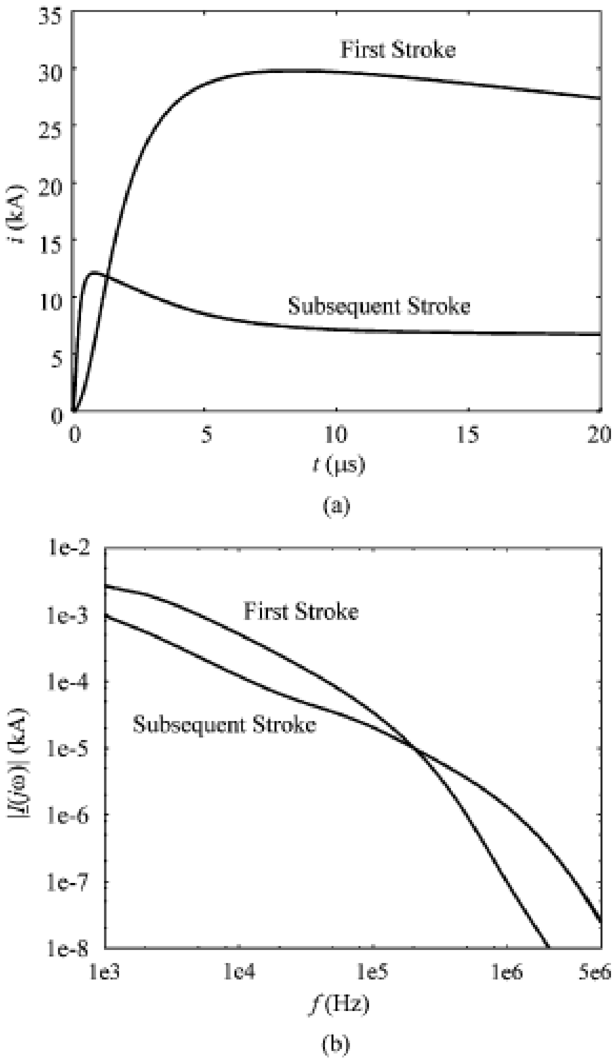


Fig 4. First and subsequent current pulses related to lightning strike, (a) in time domain, and (b) in frequency domain.

Table 2. The values of R, L, and C in the RLC model for vertical and horizontal electrodes.

Vertical electrode	**Horizontal electrode**
$R=\frac{\rho}{2\pi l}\left[\ln\left(\frac{4l}{a}\right)-1\right](\Omega)$ $C=2\pi\varepsilon l/\left[\ln(\frac{4l}{a})-1\right](F)$ $L=\frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{2l}{a}\right)-1\right](H)$	$R=\frac{\rho}{\pi l}\left[\ln\left(\frac{2l}{\sqrt{2ad}}\right)-1\right](\Omega)$ $C=\pi\varepsilon l/\left[\ln\left(\frac{2l}{\sqrt{2ad}}\right)-1\right](F)$ $L=\frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{2l}{a}\right)-1\right](H)$

In this study, equivalent circuits based upon vector fitting method (VF) [8-10] for horizontal and vertical electrodes in wide band of frequency are introduced. Vector fitting method is an efficiently approximated approach for fitting frequency response by rational functions. In this method, at first a set of starting poles in the frequency range of interest are chosen, and then the locations of poles are reformed via an iteration process. Finally, these rational functions are converted to an equivalent circuit of lumped elements so as to import into EMTP software for modeling the ground systems. Further information about this method is given in the next section.

2. VECTOR FITTING METHOD

Extracting equivalent circuit of electrical network is based on approximating frequency response with rational functions of the following form:

$$f(s) = \sum_{n=1}^{N} \frac{c_n}{s - a_n} + d + sh \tag{3}$$

Where residues c_n and poles a_n are either real quantities or come in complex conjugate pairs, while d and h are real. The aim is to approximate these coefficients using least square technique. Also note that the equation (3) is a nonlinear problem versus unknown coefficients. Vector fitting method solves this problem as a linear problem under assumption of known poles in an iteration process as follows.

At the first stage, a set of starting poles are assumed and multiply $f(s)$ by an unknown function $\sigma(s)$ Also a rational function for $\sigma(s)$ is introduced as following:

$$\begin{bmatrix} \sigma(s) f(s) \\ \sigma(s) \end{bmatrix} = \begin{bmatrix} \displaystyle\sum_{n=1}^{N} \frac{c_n}{s - \tilde{a}_n} + d + sh \\ \displaystyle\sum_{n=1}^{N} \frac{\tilde{c}_n}{s - \tilde{a}_n} + 1 \end{bmatrix} \tag{4}$$

Multiplying the second row of (4) by $f(s)$ gives:

$$\left(\sum_{n=1}^{N} \frac{c_n}{s - \tilde{a}_n} + d + sh \right) = \left(\sum_{n=1}^{N} \frac{\tilde{c}_n}{s - \tilde{a}_n} + 1 \right) f(s) \tag{5}$$

Rewriting the above equation gives:

$$\left(\sum_{n=1}^{N} \frac{c_n}{s - \tilde{a}_n} + d + sh \right) - \left(\sum_{n=1}^{N} \frac{\tilde{c}_n}{s - \tilde{a}_n} \right) f(s) = f(s) \tag{6}$$

Writing the equation (6) for a given frequency s_k , we obtain

$$A_k x = b_k \tag{7}$$

where

$$A_k = \begin{bmatrix} \frac{1}{s_k - \tilde{a}_1} & \cdots & \frac{1}{s_k - \tilde{a}_N} & 1 & s_k & -\frac{f(s_k)}{s_k - \tilde{a}_1} & \cdots & -\frac{f(s_k)}{s_k - \tilde{a}_N} \end{bmatrix} \tag{8}$$

$$x = [c_1 \quad \cdots \quad c_N \quad d \quad h \quad \tilde{c}_1 \quad \cdots \quad \tilde{c}_N]^T \tag{9}$$

$$b_k = f(s_k) \tag{10}$$

Equation (7) is linear in terms of its unknowns c_n, d , h, and $\tilde{c}_n$. If each sum of partial fractions in equation (6) is written as fraction:

$$(\sigma.f)(s) = h \frac{\prod_{n=1}^{N+1} (s - z_n)}{\prod_{n=1}^{N+1} (s - a_n)} \tag{11}$$

$$\sigma(s) = h\frac{\prod_{n=1}^{N+1}(s-\tilde{z}_n)}{\prod_{n=1}^{N+1}(s-a_n)} \tag{12}$$

From the two above equations, we get

$$f(s) = \frac{(\sigma.f)(s)}{\sigma(s)} = h\frac{\prod_{n=1}^{N+1}(s-z_n)}{\prod_{n=1}^{N+1}(s-\tilde{z}_n)} \tag{13}$$

Equation (14) shows that the poles of $f(s)$ is equal to zeros of $\sigma(s)$. Therefore, by computing the zeros of $\sigma(s)$, a set of starting poles can be chosen. According to [8-10], the zeros are computed through computing eigenvalues of the following matrix:

$$H = A - b\tilde{c}^T \tag{14}$$

Where A is a diagonal matrix including staring points, and b is column vector of ones. $\tilde{c}$ is a row vector including residues of $\sigma(s)$. To obtain more accurate result for unknowns, one should substitute these zeros in equation (6) as new poles, and this process is continued up to predefined error is achieved.

Finally, once the iteration process is finished, equivalent circuit is achieved as shown in figure 5. If there is no poles, R_0, C_0 are computed as $R_0 = 1/d$ and $C_0 = h$, else all lumped elements in the other branches are given in table 2.

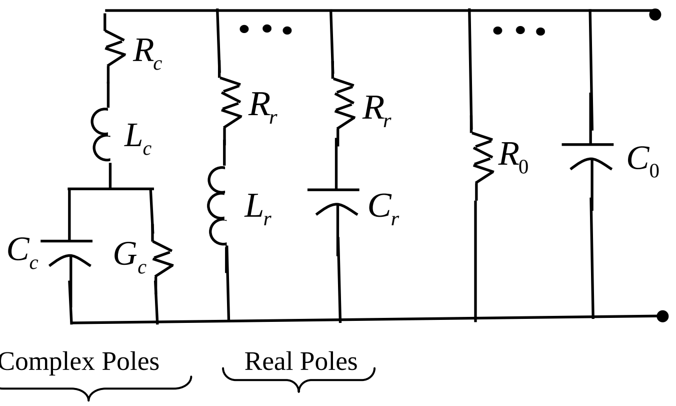


Fig 5. General equivalent circuit proposed by vector fitting method.

Table 3. Computing the elements of the parallel branches in figure 5.

Poles and Residues	Parallel branch	
a_n is real, and c_n is positive real	$R_r - L_r$	$R_r = -a_n / c_n \quad L_r = 1/c_n$
a_n is real, and c_n is negative real	$R_r - C_r$	$R_r = -\frac{a_n}{c_n} \qquad C_r = -\frac{c_n}{a_n^2}$
a_n is complex, and c_n is complex ($a_n = a_r + ja_i$, $c_n = c_r + jc_i$)	$R_c - L_c$ in series with $G_c - C_c$	$L_c = \frac{1}{2c_r}$ $R_c = [-2a_r + 2(a_r c_r + a_i c_i)L_c]L_c$ $C_c = \frac{1}{[a_r^2 + a_i^2 + 2(a_r c_r + a_i c_i)R_c L_c]}$ $G_c = -2(a_r c_r + a_i c_i)C_c L_c$

3. EQUIVALENT CIRCUITS OF VERTICAL AND HORIZONTAL ELECTRODES

In this section, the vector fitting explained in the previous section is used to extract equivalent circuit of different grounding systems.

In the first example, consider a vertical rod of length 24 m which is buried in the earth of electrical parameters of $\varepsilon_r = 10$ and $\sigma = 1mS/m$.

To achieve a comprehensive equivalent circuit, at first the input impedance of the vertical and horizontal electrodes is computed by method of moments [13] (MoM) in the range of 100Hz to 5MHz.

For the problem under consideration, four following poles are first selected, and then applying the above poles to vector fitting, and after two iterations, the input impedance in the frequency range of interest is approximated and shown in figures 6. As seen in this figure, in comparison with RLC model, excellent fitting is achieved, and accordingly equivalent circuit is extracted and shown in figure 7. Now, importing the obtained equivalent circuit into EMTP software, transient voltages of the vertical electrode subjected by first and subsequent stroke are easily extracted as shown in figures 8, and 9 respectively. As it is seen in these figures, the transient voltage is considerably affected by equivalent circuits, and accordingly protective systems.

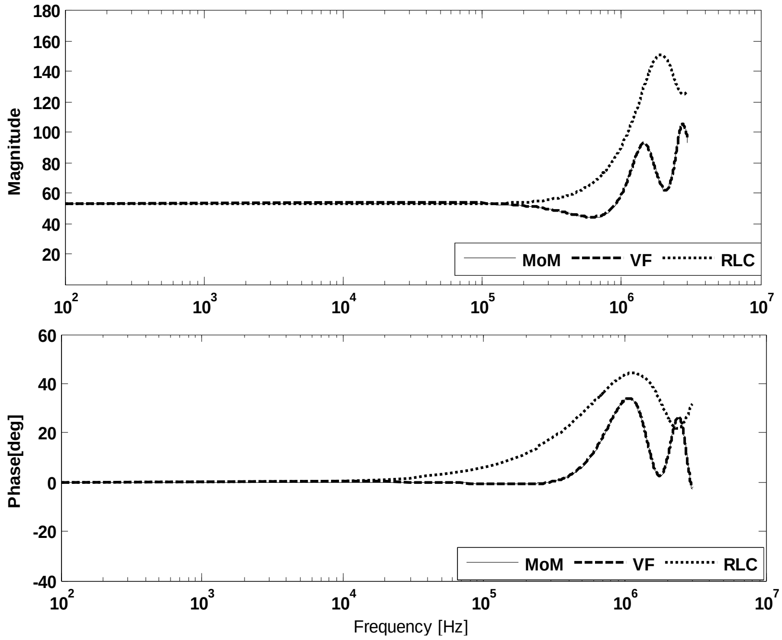


Fig 6. Equivalent circuit of vertical electrode predicted by MoM, VF, and RLC.

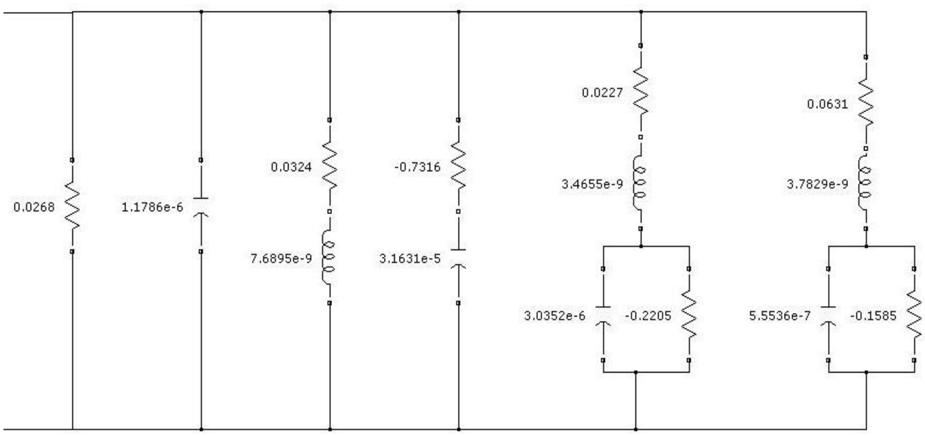


Fig 7. Equivalent Circuit of vertical electrode extracted by VF.

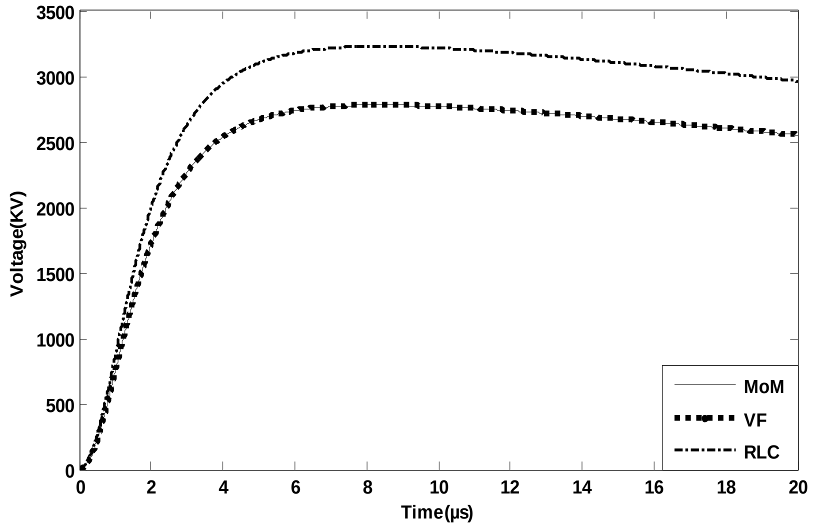


Fig 8. Transient voltage of vertical electrode under first stroke current in which the input impedance is computed by RLC, VF, and MoM.

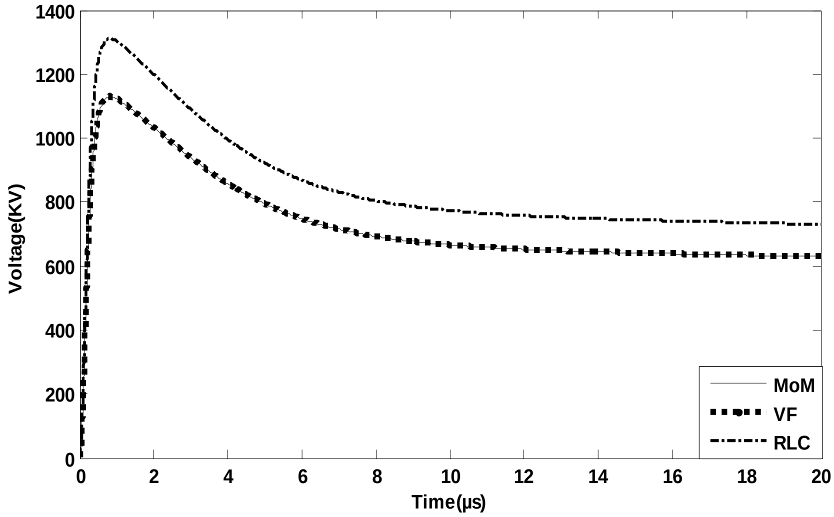


Fig 9. Transient voltage of vertical electrode under subsequent stroke current in which the input impedance is computed by RLC, VF, and MoM.

In the second example, a horizontal electrode of length 24m buried in depth of 1m is considered. In the same manner with vertical electrode, the input impedance is fitted and accordingly equivalent circuit is efficiently extracted easily as shown in figures 10, and 11 respectively. Finally using the obtained equivalent circuits, the transient voltage by EMTP software is evaluated.

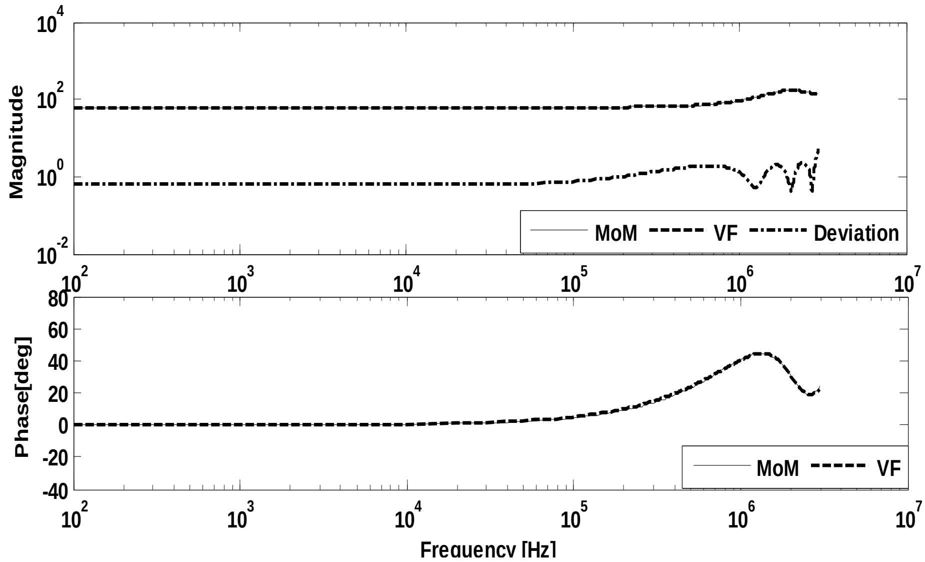


Fig 10. Equivalent circuit of horizontal electrode predicted by MoM, VF, and RLC.

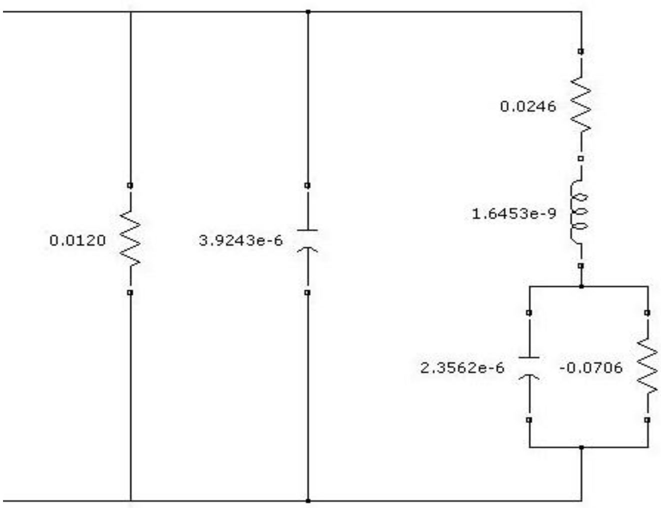


Fig 11. Equivalent Circuit of horizontal electrode extracted by VF.

3.1 Noise effect on the input impedance

In the previous sections, the exact input impedance was computed by MoM. It is well known that if this quantity is measured, some noise is added to it. One of ability of the VF is excellent curve fitting noisy data. As an example, assume that the input impedance of the vertical electrode is measured with $SNR = 2dB$. Figure12 shows that how well VF fits the measured data.

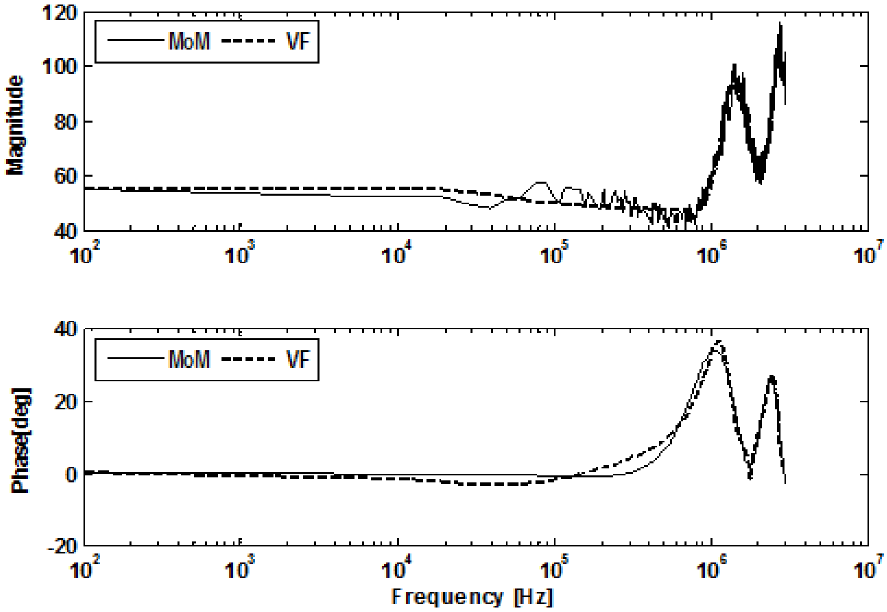


Fig. 13. Fitting the measured input impedance of the vertical electrode with $SNR = 2dB$.

4. IONIZATION EFFECT OF SOIL

In the previous sections, behavior of the grounding system was considered as linear. Researchers [14, 15], a nonlinear relation between the current injecting to grounding electrodes and resistance extracted, and demonstrated it as ionization effect of soil at high-current values of lightning strikes. This nonlinear resistance is represented as bellow:

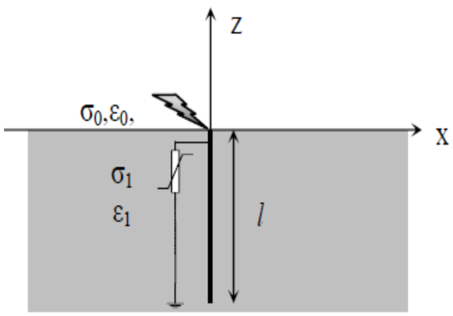
$$R(t) = \frac{R}{\sqrt{1 + i(t)/I_g}} \tag{15}$$

Where R is low-frequency low-current resistance of grounding electrode (in ohm), $i(t)$ is lightning current, and I_g is expressed as following:

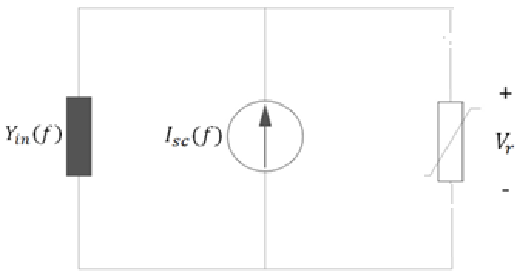
$$I_g = \frac{E_0 \rho}{2\pi R^2} \tag{16}$$

Where E_0 is the earth's electrical field intensity (in Kilo Volt/meter) varying between $300KV/m$ to $400KV/m$, ρ is resistivity of soil (in ohm. meter). Based upon the schematic diagram proposed by Sheshyekani et al [11, 12], the nonlinear behavior of grounding system at high-current values of lightning strikes is represented as figure 13(a). Figure 13(b) shows the nonlinear equivalent circuit of figure 13(a). In figure 13(b), $Y_{in}(f)$ is input admittance of the grounding system (inverse of input impedance discussed in previous sections), $I_{sc}(f)$ is lightning current, and the nonlinear load is $R(t)$.

In order to evaluate transient voltage of grounding system at high-current values of lightning strike, the equivalent circuit of figure 13(b) is solved by EMTP software in which $Y_{in}(f)$ is substituted by the linear equivalent circuits extracted by VF in figures 7, and 11.



(a)



(b)

Fig 13. (a) Schematic diagram of vertical electrode including ionization of soil, (b) circuit model of (a) based on [11, 12].

Figure 14 shows the nonlinear resistance of grounding system versus lightning current. To evaluate the transient voltage at high-current values, the vertical electrode investigated in the previous section is again chosen, and finally with the use of EMTP software, transient voltage is computed as shown in figure 15. As it is seen in this figure, the linear equivalent circuit of the electrode affects the transient voltage considerably at high-current values.

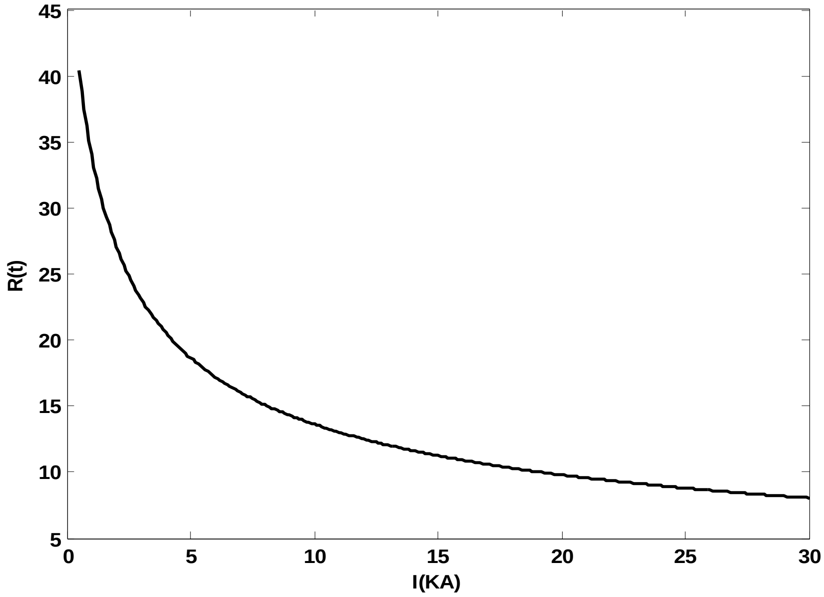


Fig 14. Representing the ionization of soil by the nonlinear resistivity versus lightning current.

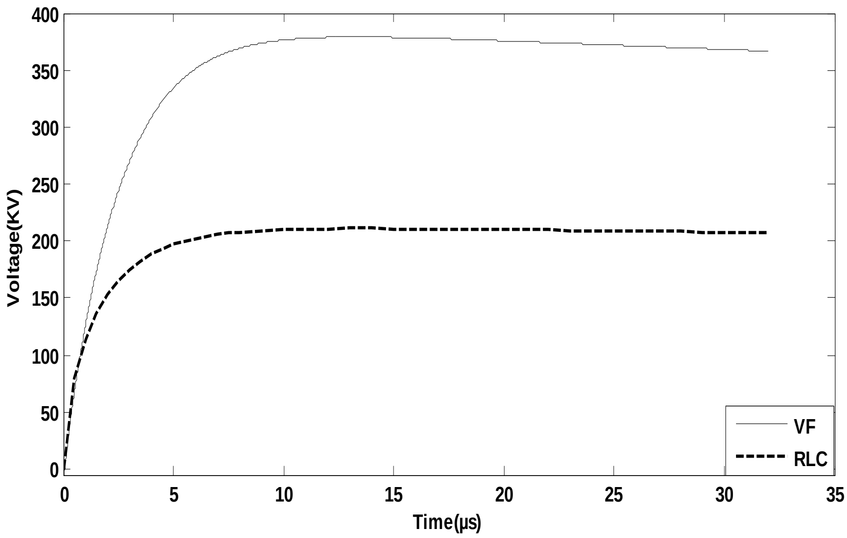


Fig 15. Transient voltage of vertical electrode including ionization effect of soil through importing Yin as equivalent circuit of figures 7(VF), and 2©(RLC) into EMTP software.

5. ACKNOWLEDGEMENT

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