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Random Fuzzy Complementarity Problem

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Abstract

In this paper, we introduce and study a random fuzzy complementarity problem in Hilbert spaces. We define an iterative algorithm for finding the approximate solutions of this class of complementarity problem and establish the convergence of iterative sequences generated by proposed algorithm. Our result can be viewed as generalization of many known corresponding results. One example is constructed in support of our problem.

Keywords: Complementarity problem; Random fuzzy mapping; Algorithm; D-Lipschitz continuity; Relaxed Cocoercivity.

MSC: 90C33; 28E10; 46S40.

1. Introduction

The uncertainty in human-centered and information society is a precise combination of fuzziness and randomness. In 1979, *Zadeh* [1] indicated that it is important to understand the structure of evidence and developed a conclusion reached on the basis of evidence and reasoning. Information and uncertainty, these two elements taken together constitute the ground of many problems today: complexity, to admit traditionally prevailing logic as centering around the issue of complexity. *Zadeh*[1] provided us a useful theory of fuzziness, vagueness and has a significant influence on the orientation of science and engineering.

Complementarity theory was introduced by *Lemke*[2] and *Cottle* and *Dantzig* [3] in early sixties. Further it has been observed that this theory has wide applications in mechanics, physics, nonlinear programming, optimization and control, transportation equilibrium, contact problems in elasticity, fluid flow in porous media and many other branches in mathematical and engineering sciences, see for example [4–6]. In 1993, *Chang* and *Huang*[7] introduced and studied a new class of complementarity problem for fuzzy mappings in $\mathbb{R}^n$, which generalized the work of *Noor*[8, 9].

The concept of random fuzzy mapping was first introduced by *Huang*[10] and this notion is used for the study of many random variational inequality (inclusion) problems. For more details, we refer to [6, 11–13] and references therein.

The aim of this paper is to introduce and study a random fuzzy complementarity problem in Hilbert spaces. An iterative algorithm is defined to compute the approximate solutions of random fuzzy complementarity problem. Most of the complementarity problems are solved by using strongly monotonicity condition, but in this paper, we are solving random fuzzy complementarity problem without using strongly monotonicity condition.

2. Preliminaries

Let H be a Hilbert space with norm $\|\cdot\|$ and inner product $\langle\cdot,\cdot\rangle$, respectively. If $K \subset H$ is a closed convex cone, we denote by K^*, the polar cone of K, i.e.,

$$K^* = \{u \in H : \langle u, v\rangle \geq 0, \;\; \forall v \in K\}.$$

Let $\mathcal{F}(H)$ be a collection of all fuzzy sets over H. A mapping $F : H \longrightarrow \mathcal{F}(H)$ is called a fuzzy mapping on H. If F is a fuzzy mapping on H, then $F(x)$ (denote it by F_x, in the sequel) is a fuzzy set on H and $F_x(y)$ is degree of membership or membership function of y in F_x. Let $N \in \mathcal{F}(H)$, $q \in [0,1]$, then the set

$$(N)_q = \{x \in H : N(x) \geq q\}$$

is called a q-cut set of N.

Throughout this paper, we denote by (Ω, Σ) a measurable space, where Ω is a set and Σ is a σ-algebra of subsets of Ω and by $\mathcal{B}(H), 2^H, CB(H)$ and $D(\cdot,\cdot)$ the class of Borel σ-fields in H, the family of nonempty subsets of H, the family of all nonempty closed bounded subsets of H and the Hausdorff metric on $CB(H)$, respectively.

Definition 2.1. *A multivalued mapping $T : \Omega \longrightarrow 2^H$ is said to be measurable, if for any $B \in \mathcal{B}(H)$, $T^{-1}(B) = \{t \in \Omega : T(t) \cap B \neq \emptyset\} \in \Sigma$.*

Definition 2.2. *A mapping $u : \Omega \longrightarrow H$ is called a measurable selection of a multivalued measurable mapping $T : \Omega \longrightarrow 2^H$, if u is measurable and for any $t \in \Omega$, $u(t) \in T(t)$.*

Definition 2.3. *A fuzzy mapping $F : \Omega \longrightarrow \mathcal{F}(H)$ is called measurable, if for any $\alpha \in (0,1]$, $(F(\cdot))_\alpha : \Omega \longrightarrow 2^H$ is a measurable multivalued mapping.*

Definition 2.4. *A fuzzy mapping $F : \Omega \times H \longrightarrow \mathcal{F}(H)$ is called a random fuzzy mapping, if for any $x \in H$, $F(\cdot, x) : \Omega \longrightarrow \mathcal{F}(H)$ is a measurable fuzzy mapping.*

Clearly, the random fuzzy mappings include multivalued mappings, random multivalued mappings and fuzzy mappings as special cases.

Let $F, G : \Omega \times H \longrightarrow \mathcal{F}(H)$ be the random fuzzy mappings satisfying the following condition (C):

(C) : There exists two mappings $a, b : H \longrightarrow [0,1]$ such that

$$(F_{t,x})_{a(x)} \in CB(H), \quad (G_{t,x})_{b(x)} \in CB(H).$$

By using the random fuzzy mappings F and G, we can define two random multivalued mappings $\widetilde{F}$ and $\widetilde{G}$ as:

$$\widetilde{F} : \Omega \times H \longrightarrow CB(H), \;\; x \mapsto (F_{t,x})_{a(x)}, \;\; \forall (t,x) \in \Omega \times H,$$

$$\widetilde{G} : \Omega \times H \longrightarrow CB(H), \;\; x \mapsto (G_{t,x})_{b(x)}, \;\; \forall (t,x) \in \Omega \times H,$$

where $F_{t,x} = F(t, x(t))$.

In the sequel, $\widetilde{F}$ and $\widetilde{G}$ are called random multivalued mappings induced by the random fuzzy mappings F and G, respectively.

Given mappings $a, b : H \longrightarrow [0,1]$, random fuzzy mappings $F, G : \Omega \times H \longrightarrow \mathcal{F}(H)$, random mappings $g, S, T : \Omega \times H \longrightarrow H$. We consider the following problem:

Find measurable mappings $x, u, v : \Omega \longrightarrow H$ such that for all $t \in \Omega$, $x(t) \in H$, $F_{t,x(t)}(u(t)) \geq a(x(t))$, $G_{t,x(t)}(v(t)) \geq b(x(t))$, $g(t, x(t)) \in K$, $S(t, u(t)) + T(t, v(t)) \in K^*$ such that

$$\langle g(t, x(t)), S(t, u(t)) + T(t, v(t)) \rangle = 0. \tag{2.1}$$

Problem (2.1) is called random fuzzy complementarity problem.

If $F, G : H \longrightarrow CB(H)$ are multivalued mappings and $g, S, T : H \longrightarrow H$ are single-valued mappings, then the problem (2.1) is equivalent to find $x \in H$, $u \in F(x)$, $v \in G(x)$ such that

$$g(x) \in K,\ S(u) + T(v) \in K^* \text{ and } \langle g(x), S(u) + T(v) \rangle = 0. \tag{2.2}$$

Problem (2.2) is called generalized complementarity problem, see e.g. [14,15].

Further we remark that for appropriate and suitable choice of operators involved in the formulation of problems (2.1) and (2.2), many complementarity problems can be obtained as special cases of problems (2.1) and (2.2) previously studied, see e.g. [3,16–20].

Example 2.1. Let us consider a “continuum of players" recognized with an interval Ω of the straight line. The fuzzy alliances of players are identified with the measurable functions from Ω to $[0,1]$.

Each player can be related with its action $g(t, \cdot)$ where $g : \Omega \times L \longrightarrow \mathbb{R}^n$, L is a nonempty subset of $\mathbb{R}^n$, and each fuzzy alliance $c(t)$ with its action $\int_\Omega g(t, x)c(t)dt$. Assume that

(i) $\forall x \in L,\ t \mapsto g(t, x) \in L^1$;

(ii) $x \mapsto g(t, x)$ is continuous for almost all t;

(iii) $\sup_{x \in L} \sup_{i=1,\cdots,n} |g_i(t, x)| \leq g_0(t)$, where $g_0 \in L^1$.

Then the problem of fuzzy game is to find $x \in L$ and $c \in L^\infty(\Omega, [0,1])$ such that

$$\int_\Omega g(t, x)c(t)dt = 0.$$

The above problem can be derive from problem (2.1). For this, we can take $H = \mathbb{R}^n$, the set Ω as above and Σ is σ-algebra generated by open subsets of real numbers and $c(t) = (S + T)(t, u)$. Define $g, S, T : \Omega \times H \longrightarrow H$ by

$$\langle g(t, x), (S + T)(t, u) \rangle = \int_\Omega g(t, x)(S + T)(t, u)dt$$

with the condition $u = v : \Omega \longrightarrow \mathbb{R}^n$.

3. Random Iterative Algorithm

The following results are needed to prove the main result.

Lemma 3.1. [21] *Let $T : \Omega \longrightarrow CB(H)$ be a D-continuous random multivalued mapping. Then for any measurable mapping $w : \Omega \longrightarrow H$, the multivalued mapping $T(\cdot, w(\cdot)) : \Omega \longrightarrow CB(H)$ is measurable.*

Lemma 3.2. [21] *Let $S, T : \Omega \longrightarrow CB(H)$ be two measurable multivalued mappings, $\varepsilon > 0$ be a constant and $v : \Omega \longrightarrow H$ be a measurable selection of S. Then there exists a measurable selection $w : \Omega \longrightarrow H$ of T such that for all $t \in \Omega$,*

$$\|v(t) - w(t)\| \leq (1 + \varepsilon)\, D(S(t), T(t)).$$

Lemma 3.3. [6, 22] *If $K \subset H$ is a closed convex set and $z \in H$ is a given point, then $u \in K$ satisfying the inequality*

$$\langle u - z, v - u \rangle \geq 0, \quad \forall v \in K,$$

if and only if $u = P_K z$, where P_K is the projection operator of H onto K.

Note that the projection operator P_K is nonexpansive.

Lemma 3.4. *The set of measurable mappings $x, u, v : \Omega \longrightarrow H$ is a random solution of random fuzzy complementarity problem (2.1) if and only if for all $t \in \Omega$, $x(t) \in H$, $u(t) \in \widetilde{F}(t, x(t))$, $v(t) \in \widetilde{G}(t, x(t))$ satisfying the following random fuzzy variational inequality problem:*

$$\langle g(t, y(t)) - g(t, x(t)), S(t, u(t)) + T(t, v(t)) \rangle \geq 0, \tag{3.1}$$

for all $y(t) \in H$, $g(t, y(t)) \in K$.

Proof. Let $x(t) \in H$, $u(t) \in \widetilde{F}(t, x(t))$, $v(t) \in \widetilde{G}(t, x(t))$ be the solution of random fuzzy complementarity problem (2.1). Then we have $g(t, x(t)) \in K$, $S(t, u(t)) + T(t, v(t)) \in K^*$ such that

$$\langle g(t, x(t)), S(t, u(t)) + T(t, v(t)) \rangle = 0.$$

Then for any $y(t) \in H$, $g(t, y(t)) \in K$, we can write

$$\begin{aligned} & \langle g(t, y(t)) - g(t, x(t)), S(t, u(t)) + T(t, v(t)) \rangle \\ = & \langle g(t, y(t)), S(t, u(t)) + T(t, v(t)) \rangle \\ & - \langle g(t, x(t)), S(t, u(t)) + T(t, v(t)) \rangle \\ = & \langle g(t, y(t)), S(t, u(t)) + T(t, v(t)) \rangle \\ \geq & \ 0. \end{aligned}$$

This implies that $x(t) \in H$, $u(t) \in \widetilde{F}(t, x(t))$, $v(t) \in \widetilde{G}(t, x(t))$ are the solutions of the random fuzzy variational inequality problem(3.1), for some $y(t) \in H$ and $g(t, y(t)) \in K$.

Conversely, Suppose that $x(t) \in H$, $u(t) \in \widetilde{F}(t, x(t))$, $v(t) \in \widetilde{G}(t, x(t))$ satisfy random fuzzy variational inequality problem(3.1), for some $y(t) \in H$ and $g(t, y(t)) \in K$.

Since $0 \in K$ and $2g(t, x(t)) \in K$, taking $g(t, y(t)) = 0$ and $g(t, y(t)) = 2g(t, x(t))$ in (3.1), in turn, we obtain

$$\langle g(t, x(t)), S(t, u(t)) + T(t, v(t)) \rangle \leq 0, \tag{3.2}$$

$$\langle g(t, x(t)), S(t, u(t)) + T(t, v(t)) \rangle \geq 0. \tag{3.3}$$

Combining (3.2) and (3.3), we have

$$\langle g(t, x(t)), S(t, u(t)) + T(t, v(t)) \rangle = 0,$$

which is random fuzzy complementarity problem (2.1).

It remains to show that $S(t, u(t)) + T(t, v(t)) \in K^*$. Putting $g(t, y(t)) = g(t, x(t)) + w(t)$ in (3.1) for some $w(t) \in K$, we have

$$\langle g(t, x(t)) + w(t) - g(t, x(t)), S(t, u(t)) + T(t, v(t)) \rangle \geq 0,$$

thus, we have

$$\langle w(t), S(t, u(t)) + T(t, v(t)) \rangle \geq 0,$$

which implies that $S(t, u(t)) + T(t, v(t)) \in K^*$. This completes the proof. □

Lemma 3.5. *The set of measurable mappings $x, u, v : \Omega \longrightarrow H$ is a random solution of problem* (2.1) *if and only if for all $t \in \Omega,\ x(t) \in H,\ u(t) \in \widetilde{F}(t, x(t)),\ v(t) \in \widetilde{G}(t, x(t))$ and*

$$g(t, x(t)) = P_K[g(t, x(t)) - \rho(t)\{S(t, u(t)) + T(t, v(t))\}],$$

where $\rho : \Omega \longrightarrow (0, \infty)$ is a measurable function and P_K is the projection of H on K.

Proof. Let for all $t \in \Omega,\ x(t) \in H,\ u(t) \in \widetilde{F}(t, x(t)),\ v(t) \in \widetilde{G}(t, x(t))$ are the solutions of problem (2.1). Then by Lemma 3.4, it is the solution of problem (3.1). Then we have,

$$\Big\langle g(t, x(t)) - [g(t, x(t)) - \rho(t)\{S(t, u(t)) + T(t, v(t))\}], g(t, y(t)) - g(t, x(t)) \Big\rangle \geq 0,$$

for some $y(t) \in H$ and $g(t, y(t)) \in K$, which is equivalent to, using Lemma 3.3,

$$g(t, x(t)) = P_K[g(t, x(t)) - \rho(t)\{S(t, u(t)) + T(t, v(t))\}],$$

which is required result. This completes the proof. □

Based on Lemma 3.4, we propose the following random iterative algorithm to compute the approximate solutions for problem (2.1).

Algorithm 3.1. Suppose that $F, G : \Omega \times H \longrightarrow \mathcal{F}(H)$ be the random fuzzy mappings satisfying the condition (C). Let $\widetilde{F}, \widetilde{G} : \Omega \times H \longrightarrow CB(H)$ be D-Lipschitz continuous random multivalued mappings induced by F and G, respectively and $g, S, T : \Omega \times H \longrightarrow H$ be continuous random mappings. For any given measurable mapping $x_0 : \Omega \longrightarrow H$, the multivalued mappings $\widetilde{F}(\cdot, x_0(\cdot)), \widetilde{G}(\cdot, x_0(\cdot)) : \Omega \longrightarrow CB(H)$ are measurable by Lemma 3.1. Hence there exist measurable selections $u_0 : \Omega \longrightarrow H$ of $\widetilde{F}(\cdot, x_0(\cdot))$, $v_0 : \Omega \longrightarrow H$ of $\widetilde{G}(\cdot, x_0(\cdot))$ by *Himmelberg* [23]. Let

$$x_1(t) = x_0(t) - g(t, x_0(t)) + P_K[g(t, x_0(t)) - \rho(t)\{S(t, u_0(t)) + T(t, v_0(t))\}],$$

where $\rho(t)$ is same as in Lemma 3.5.

It is easy to see that $x_1 : \Omega \longrightarrow H$ is measurable. By Lemma 3.2, there exist measurable selections $u_1 : \Omega \longrightarrow H$ of $\widetilde{F}(\cdot, x_1(\cdot))$, $v_1 : \Omega \longrightarrow H$ of $\widetilde{G}(\cdot, x_1(\cdot))$ such that for all $t \in \Omega$

$$\|u_0(t) - u_1(t)\| \leq D\big(\widetilde{F}(t, x_0(t)), \widetilde{F}(t, x_1(t))\big),$$

$$\|v_0(t) - v_1(t)\| \leq D\big(\widetilde{G}(t, x_0(t)), \widetilde{G}(t, x_1(t))\big).$$

Let

$$x_2(t) = x_1(t) - g(t, x_1(t)) + P_K[g(t, x_1(t)) - \rho(t)\{S(t, u_1(t)) + T(t, v_1(t))\}],$$

then $x_2 : \Omega \longrightarrow H$ is measurable. Continuing the above process inductively, we can obtain the following random iterative sequences $\{x_n(t)\}$, $\{u_n(t)\}$ and $\{v_n(t)\}$ for solving problem (2.1) as follows:

$$\begin{aligned} x_{n+1} = x_n(t) - g(t, x_n(t)) + P_K[g(t, x_n(t)) - \rho(t)\{S(t, u_n(t)) + T(t, v_n(t))\}], \quad (3.4) \\ u_n(t) \in \widetilde{F}(t, x_n(t)), v_n(t) \in \widetilde{G}(t, x_n(t)) \text{ and} \\ \|u_n(t) - u_{n+1}(t)\| \leq D\big(\widetilde{F}(t, x_n(t)), \widetilde{F}(t, x_{n+1}(t))\big), \\ \|v_n(t) - v_{n+1}(t)\| \leq D\big(\widetilde{G}(t, x_n(t)), \widetilde{G}(t, x_{n+1}(t))\big), \end{aligned}$$

for any $t \in \Omega$ and $n = 0, 1, 2, \cdots$. □

If $F, G : H \longrightarrow CB(H)$ are D-Lipschitz continuous multivalued mappings and $g, S, T : H \longrightarrow H$ are single-valued mappings, then from Algorithm (3.1), we can obtain the following Algorithm.

Algorithm 3.2. For any given $x_0 \in H$, $u_0 \in F(x_0)$ and $v_0 \in G(x_0)$, we can obtain the following iterative sequences $\{x_n\}$, $\{u_n\}$ and $\{v_n\}$ for solving problem (2.2) as follows:

$$\begin{aligned} x_{n+1} = x_n - g(x_n) + P_K[g(x_n) - \rho\{S(u_n) + T(v_n)\}], \\ u_n \in F(x_n),\ \|u_n - u_{n+1}\| \leq D\big(F(x_n), F(x_{n+1})\big) \text{ and} \\ v_n \in G(x_n),\ \|v_n - v_{n+1}\| \leq D\big(G(x_n), G(x_{n+1})\big), \end{aligned}$$

where ρ is a constant and $n = 0, 1, 2, \cdots$. □

4. Convergence Result

Definition 4.1. *A random mapping $g : \Omega \times H \longrightarrow H$ is said to be Lipschitz continuous, if there exists a measurable function $\lambda_g : \Omega \longrightarrow (0, \infty)$ such that*

$$\|g(t, x_1(t)) - g(t, x_2(t))\| \leq \lambda_g(t)\|x_1(t) - x_2(t)\|,$$

for all $x_1(t),\ x_2(t) \in H$ and $t \in \Omega$.

Definition 4.2. *A random multivalued mapping $T : \Omega \times H \longrightarrow CB(H)$ is said to be D-Lipschitz continuous, if there exists some measurable function $\lambda_T : \Omega \longrightarrow (0, \infty)$ such that*

$$D\big(T(t, x_1(t)), T(t, x_2(t))\big) \leq \lambda_T(t)\|x_1(t) - x_2(t)\|,$$

for all $x_1(t),\ x_2(t) \in H$ and $t \in \Omega$.

Definition 4.3. *A random mapping $g : \Omega \times H \longrightarrow H$ is said to be (α, ξ)-relaxed cocoercive, if there exist measurable functions $\alpha, \xi : \Omega \longrightarrow (0, \infty)$ such that*

$$\langle g(t, x_1(t)) - g(t, x_2(t)), x_1(t) - x_2(t)\rangle \geq -\alpha(t)\|g(t, x_1(t)) - g(t, x_2(t))\|^2 + \xi(t)\|x_1(t) - x_2(t)\|^2,$$

for all $x_1(t),\ x_2(t) \in H$ and $t \in \Omega$.

Theorem 4.1. *Let H be a real Hilbert space and $S, T : \Omega \times H \longrightarrow H$ be Lipschitz continuous random mappings with constants $\lambda_S(t)$ and $\lambda_T(t)$, respectively. Let $F, G : \Omega \times H \longrightarrow \mathcal{F}(H)$ be the random fuzzy mappings satisfying condition (C) and $\widetilde{F}, \widetilde{G} : \Omega \times H \longrightarrow CB(H)$ be the random multivalued mappings induced by F and G, respectively. Suppose that $\widetilde{F}$ and $\widetilde{G}$ are D-Lipschitz continuous mappings with constants $\lambda_{\widetilde{F}}(t)$ and $\lambda_{\widetilde{G}}(t)$, respectively, and $g : \Omega \times H \longrightarrow H$ is $(\alpha(t), \xi(t))$-relaxed cocoercive and Lipschitz continuous random mapping with constant $\lambda_g(t)$. If the following condition holds:*

$$\left|\rho(t) - \frac{1-\lambda_g(t)}{\lambda_S(t)\lambda_{\widetilde{F}}(t) + \lambda_T(t)\lambda_{\widetilde{G}}(t)}\right| < \frac{\sqrt{\left(1-\lambda_g(t)\right)^2 - 2\left(\lambda_g(t) + \alpha(t)\lambda_g^2(t) - \xi(t)\right)}}{\lambda_S(t)\lambda_{\widetilde{F}}(t) + \lambda_T(t)\lambda_{\widetilde{G}}(t)} \tag{4.1}$$

provided that $\left(1-\lambda_g(t)\right) > \sqrt{2\left(\lambda_g(t) + \alpha(t)\lambda_g^2(t) - \xi(t)\right)}$, then there exist measurable mappings $x, u, v : \Omega \longrightarrow H$ such that (2.1) holds. Moreover, $x_n(t) \to x(t)$, $u_n(t) \to u(t)$ and $v_n(t) \to v(t)$, where $\{x_n(t)\}$, $\{u_n(t)\}$ and $\{v_n(t)\}$ are the random sequences obtained by Algorithm (3.1).

Proof. From (3.4), for any $t \in \Omega$ and using the non-expansiveness of the projection operator P_K, we have

$$\begin{aligned}
\|x_{n+1}(t) - x_n(t)\| &= \big\|\big(x_n(t) - g(t, x_n(t)) + P_K[g(t, x_n(t)) - \rho(t)\{S(t, u_n(t)) \\
&\quad + T(t, v_n(t))\}]\big) - \big(x_{n-1}(t) - g(t, x_{n-1}(t)) + P_K[g(t, x_{n-1}(t)) \\
&\quad - \rho(t)\{S(t, u_{n-1}(t)) + T(t, v_{n-1}(t))\}]\big)\big\| \\
&\leq \big\|x_n(t) - x_{n-1}(t) - \big(g(t, x_n(t)) - g(t, x_{n-1}(t))\big)\big\| \\
&\quad + \big\|\big(g(t, x_n(t)) - \rho(t)\{S(t, u_n(t)) + T(t, v_n(t))\}\big) \\
&\quad - \big(g(t, x_{n-1}(t)) - \rho(t)\{S(t, u_{n-1}(t)) + T(t, v_{n-1}(t))\}\big)\big\| \\
&\leq \big\|x_n(t) - x_{n-1}(t) - \big(g(t, x_n(t)) - g(t, x_{n-1}(t))\big)\big\| \\
&\quad + \big\|g(t, x_n(t)) - g(t, x_{n-1}(t))\big\| + \rho(t)\big\|S(t, u_n(t)) - \\
&\quad S(t, u_{n-1}(t))\big\| + \rho(t)\big\|T(t, v_n(t)) - T(t, v_{n-1}(t))\big\|.
\end{aligned} \tag{4.2}$$

Since $\widetilde{F}$ and $\widetilde{G}$ are D-Lipschitz continuous mappings, and S, T and g are Lipschitz continuous mappings, we have

$$\begin{aligned}
\|S(t, u_n(t)) - S(t, u_{n-1}(t))\| &\leq \lambda_S(t)\|u_n(t) - u_{n-1}(t)\| \\
&\leq \lambda_S(t)\, D\big(\widetilde{F}(x_n(t)), \widetilde{F}(x_{n-1}(t))\big) \\
&\leq \lambda_S(t)\lambda_{\widetilde{F}}(t)\|x_n(t) - x_{n-1}(t)\|,
\end{aligned} \tag{4.3}$$

$$
\begin{aligned}
\|T(t,v_n(t)) - T(t,v_{n-1}(t))\| &\leq \lambda_T(t)\|v_n(t) - v_{n-1}(t)\| \\
&\leq \lambda_T(t)\, D\big(\widetilde{G}(x_n(t)), \widetilde{G}(x_{n-1}(t))\big) \\
&\leq \lambda_T(t)\lambda_{\widetilde{G}}(t)\|x_n(t) - x_{n-1}(t)\|, \qquad (4.4)
\end{aligned}
$$

$$
\|g(t,x_n(t)) - g(t,x_{n-1}(t))\| \leq \lambda_g(t)\|x_n(t) - x_{n-1}(t)\|. \qquad (4.5)
$$

Using (α,ξ)-relaxed cocoercivity and Lipschitz continuity of g, we have

$$
\begin{aligned}
&\big\|\big(x_n(t) - x_{n-1}(t)\big) - \big(g(t,x_n(t)) - g(t,x_{n-1}(t))\big)\big\|^2 \\
\leq\ & \|x_n(t) - x_{n-1}(t)\|^2 - 2\big\langle x_n(t) - x_{n-1}(t), g(t,x_n(t)) - g(t,x_{n-1}(t))\big\rangle \\
& + \|g(t,x_n(t)) - g(t,x_{n-1}(t))\|^2 \\
\leq\ & \|x_n(t) - x_{n-1}(t)\|^2 + 2\alpha(t)\|g(t,x_n(t)) - g(t,x_{n-1}(t))\|^2 \\
& -2\xi(t)\|x_n(t) - x_{n-1}(t)\|^2 + \|g(t,x_n(t)) - g(t,x_{n-1}(t))\|^2 \\
\leq\ & \|x_n(t) - x_{n-1}(t)\|^2 + 2\alpha(t)\lambda_g^2(t)\|x_n(t) - x_{n-1}(t)\|^2 \\
& -2\xi(t)\|x_n(t) - x_{n-1}(t)\|^2 + \lambda_g^2(t)\|x_n(t) - x_{n-1}(t)\|^2 \\
=\ & \Big[1 - 2\xi(t) + (1 + 2\alpha(t))\lambda_g^2(t)\Big]\, \|x_n(t) - x_{n-1}(t)\|^2.
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
&\big\|\big(x_n(t) - x_{n-1}(t)\big) - \big(g(t,x_n(t)) - g(t,x_{n-1}(t))\big)\big\| \\
\leq\ & \Big(\sqrt{1 - 2\xi(t) + (1 + 2\alpha(t))\lambda_g^2(t)}\Big)\, \|x_n(t) - x_{n-1}(t)\|. \qquad (4.6)
\end{aligned}
$$

Using (4.3) to (4.6), (4.2) becomes

$$
\begin{aligned}
\|x_{n+1}(t) - x_n(t)\| &\leq \Big(\sqrt{1 - 2\xi(t) + (1 + 2\alpha(t))\lambda_g^2(t)}\Big)\, \|x_n(t) - x_{n-1}(t)\| \\
&\quad + \lambda_g(t)\|x_n(t) - x_{n-1}(t)\| + \rho(t)\lambda_S(t)\lambda_{\widetilde{F}}(t)\|x_n(t) - x_{n-1}(t)\| \\
&\quad + \rho(t)\lambda_T(t)\lambda_{\widetilde{G}}(t)\|x_n(t) - x_{n-1}(t)\| \\
&\leq \theta(t)\|x_n(t) - x_{n-1}(t)\|, \qquad (4.7)
\end{aligned}
$$

where

$$
\theta(t) = \sqrt{1 - 2\xi(t) + (1 + 2\alpha(t))\lambda_g^2(t)} + \lambda_g(t) + \rho(t)\lambda_S(t)\lambda_{\widetilde{F}}(t) + \rho(t)\lambda_T(t)\lambda_{\widetilde{G}}(t).
$$

It follows from (4.1) that $\theta(t) < 1$, for all $t \in \Omega$. Therefore $\{x_n(t)\}$ is a cauchy sequence in H. Since H is complete, there exists a measurable mapping $x : \Omega \longrightarrow H$ such that $x_n(t) \to x(t)$, for all $t \in \Omega$. From Algorithm 3.1, it follows that $\{u_n(t)\}$ and $\{v_n(t)\}$ are also cauchy sequences and in view of completeness of H, $u_n(t) \to u(t)$ and $v_n(t) \to v(t)$, for all $t \in \Omega$, where $u, v : \Omega \longrightarrow H$ are measurable mappings. It also follows that $u(t) \in \widetilde{F}(t, x(t))$ and $v(t) \in \widetilde{G}(t, x(t))$, for all $t \in \Omega$. This completes the proof. □

From Theorem 4.1, we can obtain the following theorem.

Theorem 4.2. *Let H be a real Hilbert space and $S, T : H \longrightarrow H$ be Lipschitz continuous mappings with constants λ_S and λ_T, respectively. Let $F, G : H \longrightarrow CB(H)$ be the D-Lipschitz continuous multivalued mappings with constants λ_F and λ_G, respectively, and $g : H \longrightarrow H$ is (α, ξ)-relaxed cocoercive and Lipschitz continuous mapping with constant λ_g. If the following condition holds:*

$$\left|\rho - \frac{1-\lambda_g}{\lambda_S\lambda_F + \lambda_T\lambda_G}\right| < \frac{\sqrt{\left(1-\lambda_g\right)^2 - 2\left(\lambda_g + \alpha\lambda_g^2 - \xi\right)}}{\lambda_S\lambda_F + \lambda_T\lambda_G}$$

provided that $\left(1-\lambda_g\right) > \sqrt{2\left(\lambda_g + \alpha\lambda_g^2 - \xi\right)}$, then there exist $x \in H$, $u \in F(x)$, $v \in G(x)$ which are the solutions of generalized complementarity problem (2.2). Moreover, $x_n \to x$, $u_n \to u$ and $v_n \to v$, as $n \to \infty$, where $\{x_n\}$, $\{u_n\}$ and $\{v_n\}$ are the sequences obtained by Algorithm (3.2).

5. Conclusion

The aim of this work is to introduce and study a random fuzzy complementarity problem in Hilbert spaces. We have initiated and generalized the concept of fuzzy complementarity problem to random fuzzy complementarity problem. An iterative algorithm is suggested to find out a random solution of problem (2.1). We do not use strong monotonicity assumptions for the proof of our main result.

References

[1] L.A. Zadeh, *Fuzzy Sets and Information Granularity*, Advances in Fuzzy Set Theory and Applications, M. Gupta, R. Ragade and R. Yager (Eds.), Amsterdam: North-Holland Publishing Co., pp. 3-18, 1979.

[2] C.E. Lemke, *Bimatrix Equilibrium Points and Mathematical Programming*, Management Science, Vol. 11, No. 7, pp. 681-689, 1965.

[3] R.W. Cottle and G.B. Dantzing, *Complementary Pivot Theory of Mathematical Programming*, Linear Algebra and its Applications, Vol.1, pp. 103-125, 1968.

[4] Y. Lin and C. Cryer, *An Alternating Direction Implicit Algorithm for the Solution of Linear Complementarity Problems Arising from Boundary Problems*, Applied Mathematics and Optimization, Vol. 13, pp. 1985, 1985.

[5] R.W. Cottle, *Complementarity and Variational Problems*, Symposia Mathematica, Vol. 19, pp. 177-208, 1976.

[6] S.S. Chang, *Variational Inequality and Complementarity Problem Theory with Applications*, Shanghai Scientific and Technological Literature Publishing House, Shanghai, 1991.

[7] S.S. Chang and N.J. Huang, *Generalized Complementarity Problems for Fuzzy Mappings*, Fuzzy Sets and Systems, Vol. 55, pp. 227-234, 1993.

[8] M.A. Noor, *Iterative Methods for a class of Complementarity Problems*, Journal of Mathematical Analysis and Applications, Vol. 133, pp. 366-382, 1988.

[9] M.A. Noor, *Fixed Point Approach for Complementarity Problems*, Journal of Mathematical Analysis and Applications, Vol. 133, pp. 437-448, 1988.

[10] N.J. Huang, *Random Generalized Nonlinear Variational Inclusions for Random Fuzzy Mappings*, Fuzzy Sets and Systems, Vol. 105, pp. 437-444, 1999.

[11] R. Ahmad and F.F. Bazán, *An Iterative Algorithm for Random Generalized Nonlinear Mixed Variational Inclusions for Random Fuzzy Mappings*, Applied Mathematics and Computation, Vol. 167, pp. 1400-1411, 2005.

[12] N.J. Huang, *Random Generalized Set-Valued Implicit Variational Inequalities*, Journal of Liaoning Normal University , Vol. 18, pp. 89-93, 1995.

[13] S.S. Chang and N.J. Huang, *Generalized Random Multivalued Quasi-Complementarity Problem*, Indian Journal of Mathematics, Vol. 35, pp. 305-320, 1993.

[14] M.A. Noor, *On the Nonlinear Complementarity Problems*, Journal of Mathematical Analysis and Applications, Vol.123, pp. 455-460, 1987.

[15] M.A. Noor, *On Complementarity Problems*, Mathematica Japonica, Vol. 34, pp. 83-88, 1989.

[16] R. Ahmad and Q.H. Ansari, *An Iterative Algorithm for Generalized Nonlinear Variational Inclusions*, Applied Mathematics Letters, Vol. 13, pp. 23-26, 2000.

[17] S.S. Chang and Y. Zhu, *On Variational Inequalities for Fuzzy Mappings*, Fuzzy Sets and Systems, Vol. 32, pp. 359-367, 1989.

[18] S.S. Chang and N.J. Huang, *Random Generalized Set-Valued Quasi-Complementarity Problems*, Acta Mathematicae Applicatae Sinica, Vol. 16, pp. 396-405, 1993.

[19] S.S. Chang and Y.G. Zhu, *On the Problems for a class of Random Variational Inequalities and Quasi-Variational Inequalities*, Journal of Mathematical Research and Exposition, Vol. 9, pp. 385-393, 1989.

[20] S.S. Chang and N.J. Huang, *Generalized Multivalued Implicit Complementarity Problems in Hilbert Spaces*, Mathematica Japonica, Vol. 36, pp. 1093-1100, 1991.

[21] S.S. Chang, *Fixed Point Theory with Applications*, Chongqing Publishing House, Chongqing, 1984.

[22] M.A. Noor, *An Iterative Scheme for a Class of Quasi Variational Inequalities*, Journal of Mathematical Analysis and Applications, Vol. 110, pp. 463-468, 1985.

[23] C.J. Himmelberg, *Measurable Relations*, Fundamenta Mathematicae, Vol. 87, pp. 53-72, 1975.