

# New Constructive Approach to Solve Problems of Integers' Divisibility

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**ABSTRACT---** *This paper aims at introducing a new constructive approach to solve problems in elementary number theory. It starts with a comprehensive analysis on present approaches to solve problems related with divisible features of consecutive integers, which include consecutive positive integers, consecutive positive odd integers and consecutive positive even integers; then it detailly demonstrates advantages and disadvantages of the present-applied approaches in their deducing process, especially the conflicts in proving the almost same-stated statements; in the end the paper puts forward a new constructive approach and uses it to have a new proof for the three fundamental theorems: for any positive integer  $n$  and among  $n$  consecutive positive integers there exists one and only one that can be divisible by  $n$ ; for any positive odd integer  $p$  and among  $p$  consecutive positive odd integers there exists one and only one that can be divisible by  $p$ ; for a positive even integer  $w$  and among  $w$  consecutive positive even integers, there exist exactly two that can be divisible by  $w$ . The new constructive proof is valuable for more extensive utilities in elementary number theory.*

**Keywords----** Mathematical proof;Elementary number theory;Divisibility; Consecutive Integers;Residue system; Constructive Proof.

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## 1. INTRODUCTION

Problems of integer's divisibility always play a role in kinds of mathematical competitions and in elementary number theory. The elementary number theory provides two canonical approaches to solve such problems. One is based on the Euclidean division and the other is based on the residue system, as summarized in K H Rosen's and M B Nathanson's books [1] and [2], in Daniel Sutantyo's thesis [3] and in the chapter 11 of book [4].

It seems a common expectation in mathematics and other science that people might find a unitary resolution for a class of similar problems, and this trends are said to be a mandatory process of systematization, as hinted in Peter J. Eccles's and Antonena Cupilari's books [5] and [6].

In one of my recent researches, I come across a question that needs to prove one of  $p$  consecutive odd numbers must be divisible by  $p$ , where  $p$  is an odd number. To find the proof, I have a look at bibliographies and the Internet forums but have found none. Meanwhile, I find that a question, which wants to prove that one of  $n$  consecutive integers must be divisible by  $n$ , has been frequently asked in the Internet forums. For example, Martin Sleziak asks the question in techQues [7] and StackExchange [8], and some other people have asked the same question in Chinese Baidu as listed in [9] to [12]. Since this question is very close to my question, I put them together and make them two statements, which are list by the following statement 1 and 2, and intend to find their proofs. And to the idea of systematization, I add another statement 3 to form the following three almost same-stated statements.

**Statement 1.** Let  $n$  be a positive integer; then among  $n$  consecutive positive integers there exists one and only one that can be divisible by  $n$ .

**Statement 2.** Let  $p$  be a positive odd integer; then among  $p$  consecutive positive odd integers there exists one and

only one that can be divisible by  $p$ .

**Statement 3.** Let  $w$  be a positive even integer; then among  $w$  consecutive positive even integers there exists one and only one that can be divisible by  $w$ .

At first, I collected many typical approaches that people adopted to prove the statement 1 and classify them into certain classes. I intended to derive a unitary method from the classes to prove all the three statements, however I have failed to do so because the statement 3 is false by my later proof. In addition, the approaches that are available for proving the statements 1 and 2 are not available for proving the statement 3. Thereupon, I have to find a new approach that can show and prove the correct assertions for all the three statements and fortunately I find out one. This paper presents my studies.

The paper is formed by four sections. The section 1 is the introduction and background, as stated above; the section 2 presents necessary lemmas and preliminaries for the later sections; section 3 makes a comprehensive analysis and comparison on the approaches I collect from books and the Internet forums, pointing out the advantages and disadvantages of each approach in proving the three statements; section introduces three theorems derived from the three statements and their classical proofs; section 5 introduces the constructive approach I put forward and uses it to prove the three theorems.

## 2. LEMMAS AND PRELIMINARIES

We need the following lemmas for later parts' proofs.

**Lemma 1.** Let  $a_1, a_2, \dots, a_n$  be  $n$  positive consecutive integers; then the absolute value of the difference of any two  $a_i, a_j (1 \leq i, j \leq n; i \neq j)$  is smaller than  $n$ .

**Lemma 2** The absolute value of the difference of any two in  $p$  consecutive odd positive integers is no more than  $2(p-1)$ ; the absolute value of the difference of any two in  $w$  consecutive even positive integers is no more than  $2(w-1)$ .

**Proof.** Actually, let  $a_1 = 2k+1, a_2 = 2k+3, \dots, a_p = 2k+1+2(p-1)$  ( $k \in \mathbf{Z}^+$ , where  $\mathbf{Z}^+$  is the set of positive integers); obviously, the biggest one is  $a_p$  and the smallest one is  $a_1$ . Hence the biggest difference of the two is  $a_p - a_1 = 2(p-1)$ . The second part of the lemma is proven by the same way.

**Lemma 3** If  $r_1, r_2, \dots, r_m$  form a complete residue system of a integer  $m$  and  $(\alpha, m) = 1$ , then  $\alpha r_i + b (i = 1, \dots, m)$  also form complete residue system of  $m$  for an arbitrary integer  $b$ .

**Proof.** See the proof of the theorem 4.6 in the K H Rosen's book [1].

**Proposition1** Let  $p$  be an odd positive integer; then  $p$  consecutive odd positive integers form a complete residue system of  $p$ . Let  $w$  be an even positive integer; then  $w$  consecutive even positive integers *can not* form a complete residue

system of  $m$ .

**Proof.**  $p$  consecutive odd positive integers  $a_1 = 2k + 1, a_2 = 2k + 3, \dots, a_p = 2k + 1 + 2(p - 1)$  ( $k \in \mathbf{Z}^+$ ) can be rewritten by

$$a_1 = 2k + 1 + 0 \times 2, a_2 = 2k + 1 + 1 \times 2, \dots, a_p = 2k + 1 + (p - 1) \times 2 \quad (1)$$

Note that  $\{0\}, \{1\}, \dots, \{p - 1\}$  form the minimal non-negative complete residue system of  $p$ . Hence the proposition is just the case that takes  $b = 2k + 1$  and  $\alpha = 2$  in the lemma 3. To prove the second part, we denote  $r_i (i = 1, 2, \dots, w)$  to be the remainders of the  $w$  consecutive even positive integers  $a_i$  divided by  $w$ , respectively. Then it holds

$$a_i = q_i w + r_i, 0 \leq r_i \leq w - 1; i = 1, 2, \dots, w; q_i \geq 0$$

Since  $a_i$  and  $w$  are all positive even integers,  $r_i$  is certainly an even integer. Because there are only  $w/2$  even integers between 0 and  $w - 1$ , by the *pigeonhole principle* there must exist  $i, j (1 \leq i, j \leq w, i \neq j)$  such that  $r_i = r_j$ , which shows that  $a_1, a_2, \dots, a_w$  can not form a complete residue system of  $w$ . Hence the proposition 1 holds.

### 3. CURRENT APPROACHES AND THEIR TRAITS IN PROVING THE STATEMENTS

Literature searches show that there are four typical approaches that are adopted to prove the statement 1 while there are few reports on the statements 2 and 3. The first one of the four is proposed by Amin Witno when he proves the Proposition 1.3 in his textbook [13] of Philadelphia University; we call this approach **approach I** in this paper. The second approach, which is called **approach II**, is the mathematical deduction and is proposed by robjohn in his thread to answer the questions in [7] and [8]. The third one is to use the principle of the residue system as posed in [7], [8] and [11], which we call **approach III**. And the fourth one, which we call **approach IV**, is posed by wingwf2000 in [14] and by another anonymous Chinese old person in [15].

#### 3.1 The Approaches I and II

Referring to Amin Witno's proof and robjohn's deduction process, one knows that the approaches I and II are available for proving the existence that the statement 1 states. However, they are not a better choice for proving the uniqueness the statement 1 states. Therefore the approaches I and II are not recommended to prove any of the statements 1, 2 and 3.

#### 3.2 The Approach III

Because any  $n$  consecutive positive integers form a complete residue system of the integer  $n$ , the statement 1 can be easily proved true by the approach III. In addition, by the proposition 1, the statement 2 can also be easily proved true by the approach III. However, the proposition 1 also asserts that the approach III can not prove if the statement 3 is true or false. Since most textbooks of elementary number theory illustrate the approach III circumstantially, we omit the detail here.

### 3.3 The Approach IV

Compared with the approach III, the approach IV is a little more independent because it adopts a proof by contradiction and the *pigeonhole principle*. And like that of the approach III, the approach IV is available for proving the truth of the statements 1 and 2 but it cannot prove if the statement 3 is true or false. For readers to know the approach in detail, I present its proofs for the statements 1 and 2 here.

#### 3.3.1 Proof of the Statement 1 by Approach IV

First is the proof of the existence. By the **Euclidean division**, the remainder of an integer divided by  $n$  must be one of  $0, 1, 2, \dots, n-1$ . Now suppose none of the  $n$  consecutive positive integers could be divided by  $n$ ; then the  $n$  remainders must be in  $n-1$  integers of  $1, 2, \dots, n-1$ . By the pigeonhole principle, there are at least two of them that are equal. Let  $a_i$  and  $a_j$  ( $1 \leq i, j \leq n; i \neq j$ ) be the two integers that have the same remainder  $r$  ( $0 < r \leq n-1$ ) when divided by  $n$ ; then there exist  $q_i$  and  $q_j$  such that  $a_i = q_i n + r, a_j = q_j n + r$  and  $q_i \neq q_j$  (otherwise a contradiction that  $a_i = a_j$  occurs). Hence it yields  $a_i - a_j = (q_i - q_j)n$ , which means  $|a_i - a_j| = |(q_i - q_j)n| \geq n$ . That is the contradiction to the lemma 1. The only case that **avoids** the contradiction is that none of the remainders is equal to another. Then again by the pigeonhole principle, there must be one remainder is 0, which means there must be one of the  $n$  consecutive positive integers divisible by  $n$ .

Next is the proof of the uniqueness. Suppose there be two integers, say  $a_i$  and  $a_j$  ( $1 \leq i, j \leq n; i \neq j$ ), which can be divisible by  $n$  in the  $n$  consecutive positive integers. Then

$$a_i = q_i n, a_j = q_j n$$

It is clear that this will lead to a contradiction that  $|a_i - a_j| = |(q_i - q_j)n| \geq n$  unless  $q_i = q_j$  or  $a_i = a_j$ . Therefore the uniqueness gets proved.

#### 3.3.2 Proof of the Statement 2 by Approach IV

Assume  $a_i$  ( $i = 1, 2, \dots, p$ ) are the  $p$  consecutive positive odd integers and none of them can be divisible by  $p$ ; then it holds

$$a_i = q_i p + r_i, 0 < r_i \leq p-1; i = 1, 2, \dots, p$$

By the pigeonhole principle, there must exist  $j, l$  ( $0 \leq j, l \leq p-1; j \neq l$ ) such that  $r_j = r_l$ . Consequently

$$a_j - a_l = (q_j - q_l)p \tag{2}$$

Since  $a_i$  and  $p$  are all odd integers, the assumption that  $r_j = r_l$  results in that  $q_j$  and  $q_l$  have the same parity, which means  $|q_j - q_l| \geq 2$ . This leads to  $|a_j - a_l| = |(q_j - q_l)p| \geq 2p$ . A contradiction! The only case that avoids the contradiction is that  $r_j \neq r_l$  for any  $i, l$  ( $0 \leq j, l \leq p-1; j \neq l$ ). Then the pigeonhole principle tells us that there

must be one and only one  $k(0 \leq k \leq p-1)$  such that  $r_k = 0$ , as the statement 2 says.

### 3.4 A Comparison to the Approach III and IV

By now it is clear that both the approaches III and IV are available for proving the statements 1 and 2. The difference between the two is that the former is by the aid of the complete residue system while the later is derived from the Euclidean division, the proof by contradiction and the pigeonhole principle. The common behavior of the two is that neither is appreciative to prove or disprove the statement 3 because the condition in statement 3 provides no complete residue system for applying the approach III according to the proposition 1 and it is difficult to draw a contraction when applying the approach IV( Readers can realize this assertion in the proof of the following theorem 3).

## 4. THEOREMS DERIVED FROM THE STATEMENTS

By now it is clear that the statements 1 and 2 are true and their truth can be proved by either the approaches III or the approach IV. Hence we derive the following theorems.

**Theorem 1** Let  $n$  be a positive integer; then among  $n$  consecutive positive integers there exists one and only one that can be divisible by  $n$ .

**Theorem 2** Let  $p$  be a positive odd integer; then among  $p$  consecutive positive odd integers there exists one and only one that can be divisible by  $p$ .

However, since neither the approach III nor the approach IV can prove if the statement 3 is true or false, we have to find other way to do it. In fact, the following theorem \ref{theorem3} shows that the statement 3 is false.

**Theorem 3** Let  $w$  be an arbitrary positive even integer; then among  $w$  consecutive positive even integers there exist exactly two that can be divisible by  $w$

**Proof.** Without loss of generality, let the  $w$  consecutive positive even integers be given by

$$a_1 = 2k + 0 \times 2, a_2 = 2k + 1 \times 2, \dots, a_w = 2k + (w-1) \times 2, k \in \mathbf{Z}^+$$

Namely

$$a_1 = 2(k + 0), a_2 = 2(k + 1), \dots, a_w = 2(k + (w-1))$$

Note that the above  $w$  consecutive positive even integers contain the set  $S(0, w-1) = \{k, k+1, \dots, k+(w-1) \mid k \in \mathbf{Z}^+\}$ . Since  $w$  is even,  $w/2$  is an integer. Hence we consider the following two subsets

$$S_1 = S(0, \frac{w}{2}-1) = \{k, k+1, \dots, k+(\frac{w}{2}-1) \mid k \in \mathbf{Z}^+\}$$

$$S_2 = S(\frac{w}{2}, w-1) = \{k+\frac{w}{2}, k+\frac{w}{2}+1, \dots, k+(w-1) \mid k \in \mathbf{Z}^+\}$$

Obviously, both  $S_1$  and  $S_2$  contain  $w/2$  consecutive positive integers. By the theorem 1, each of them must exactly have one divisible by  $w/2$ . Consequently, each of the following two sets

$$\{2k, 2k+2, \dots, 2k+(w-2) \mid k \in \mathbf{Z}^+\}$$

$$\{2k+w, 2k+w+2, \dots, 2k+2(w-1) \mid k \in \mathbf{Z}^+\}$$

contain one and only element divisible by  $w$ . That is to say, among  $w$  consecutive positive even integers

$\{2k, 2(k+1), \dots, 2(k+(w-1)) \mid k \in \mathbf{Z}^+\}$ , there exist exact two different ones that can be divisible by  $w$ .

### 5. A NEW CONSTRUCTIVE PROOF

In the previous sections, I have summarized the most-commonly-used approaches to prove the theorems 1,2 and 3. Readers can see that these approaches are all based on classical canonical approaches with some extra deductions and inductions and there has not yet been a unitary approach available for proving all of the three theorems. Then one has to ask if there is such an approach? The answer is YES. This section, we present an approach that can do it. For the limitation of space, we only show the proofs for the theorem 2 and 3.

#### 5.1 Proof of the Theorem 2

Let  $Q = \{1, 3, \dots, p, p+2, \dots, 2p-1, 2p+1, \dots\}$  be the set of all odd positive integers. Choose in  $Q$   $p$  consecutive elements to form a subset  $Q_i^p = \{q_i, q_{i+1}, \dots, q_{i+p-1}\}$ , which is called a  $p$ -section with  $q_i$  being its basis and  $q_{i+p-1}$  being its tail. Obviously,  $Q_1^p = \{1, 3, \dots, p, \dots, 2p-1\}$  is the  $p$ -section with the smallest basis while  $Q_{p+1}^p = \{2p+1, 2p+3, \dots, 3p, \dots, 4p-1\}$  follows it. More generally, it yields

$$Q_1^p = \{1, 3, \dots, p, \dots, 2p-1\}$$

$$Q_{p+1}^p = \{2p+1, 2p+3, \dots, 3p, \dots, 4p-1\}$$

$$Q_{2p+1}^p = \{4p+1, 4p+3, \dots, 5p, \dots, 6p-1\}$$

$$Q_{3p+1}^p = \{6p+1, 6p+3, \dots, 7p, \dots, 8p-1\}$$

.....

$$Q_{mp+1}^p = \{2mp+1, 2mp+3, \dots, (2m+1)p, \dots, 2(m+1)p-1\}$$

.....

and it holds

$$Q_1^p \cap Q_{p+1}^p = \emptyset, Q_{p+1}^p \cap Q_{2p+1}^p = \emptyset, \dots, Q_{mp+1}^p \cap Q_{(m+1)p+1}^p = \emptyset, \dots$$

$$Q = Q_1^p \cup Q_{p+1}^p \cup \dots \cup Q_{mp+1}^p \cup \dots$$

Apparently, the statement 2 is true for any of  $Q_1^p, Q_{p+1}^p, \dots, Q_{mp+1}^p, \dots$ , and the element  $(2m+1)p$  in the  $p$ -section

$Q_{mp+1}^p (m=0, 1, 2, \dots)$  is a multiple of  $p$ . For the other cases, take any three consecutive  $p$ -sections, say

$$Q_{(m-1)p+1}^p = \{2(m-1)p+1, 2(m-1)p+3, \dots, (2m-1)p, \dots, 2mp-1\}$$

$$Q_{mp+1}^p = \{2mp+1, 2mp+3, \dots, (2m+1)p, \dots, 2(m+1)p-1\}$$

$$Q_{(m+1)p+1}^p = \{2(m+1)p+1, 2(m+1)p+3, \dots, (2m+3)p, \dots, 2(m+2)p-1\}$$

It immediately shows that any new-constructed  $p$ -section form  $Q_{(m-1)p+1}^p$  and  $Q_{mp+1}^p$  must contain  $(2m-1)p$  or  $(2m+1)p$ ; any new-constructed  $p$ -section from  $Q_{mp+1}^p$  and  $Q_{(m+1)p+1}^p$  must contain  $(2m+1)p$  or  $(2m+3)p$ . Consequently, any  $p$ -section in  $Q$  must contain one and only one multiple of  $p$ .

### 5.2 Proof of the Theorem 3

Let  $E = \{2, 4, \dots, w, w+2, \dots, 2w-2, 2w, \dots\}$  be the set of all even consecutive positive integers. Let

$E_i^w = \{e_i, e_{i+1}, \dots, e_{i+w-1}\}$  be a  $w$ -section, which contains  $w$  consecutive elements of  $E$ . Obviously, it holds the follows

$$E_1^w = \{2, 4, \dots, w, \dots, 2w\}$$

$$E_{w+1}^w = \{2w+2, 2w+4, \dots, 3w, \dots, 4w\}$$

$$E_{2w+1}^w = \{4w+2, 4w+4, \dots, 5w, \dots, 6w\}$$

$$E_{3w+1}^w = \{6w+2, 6w+4, \dots, 7w, \dots, 8w\}$$

.....

$$E_{mw+1}^w = \{2mw+2, 2mw+4, \dots, (2m+1)w, \dots, 2(m+1)w\}$$

.....

$$E_1^w \cap E_{w+1}^w = \emptyset, E_{w+1}^w \cap E_{2w+1}^w = \emptyset, \dots, E_{mw+1}^w \cap E_{(m+1)w+1}^w = \emptyset, \dots$$

$$E = E_1^w \cup E_{w+1}^w \cup \dots \cup E_{mw+1}^w \cup \dots$$

Obviously, the theorem 2 holds for any of the  $w$ -sections  $E_1^w, E_{w+1}^w, \dots, E_{mw+1}^w, \dots$ , and the two elements,

$(2m+1)w$  and  $2(m+1)w$ , in  $E_{mw+1}^w (m = 1, 2, \dots)$  are the two multiple of  $w$ . The other case can similarly shows by taking any three consecutive  $w$ -sections as follows

$$E_{(m-1)w+1}^w = \{2(m-1)w+2, 2(m-1)w+4, \dots, (2m-1)w, \dots, 2mw\}$$

$$E_{mw+1}^w = \{2mw+2, 2mw+4, \dots, (2m+1)w, \dots, 2(m+1)w\}$$

$$E_{(m+1)w+1}^w = \{2(m+1)w+2, 2(m+1)w+4, \dots, (2m+3)w, \dots, 2(m+2)w\}$$

Hence any new constructed  $w$ -section form  $E_{(m-1)w+1}^w$  and  $E_{mw+1}^w$  must contain  $(2m-1)w$  and  $2mw$ , or

$2mw$  and  $(2m+1)w$ , or  $(2m+1)w$  and  $2(m+1)w$ ; any new constructed  $w$ -section form  $E_{mp+1}^p$  and  $E_{(m+1)p+1}^p$  must contain  $(2m+1)w$  and  $(2m+1)w$ , or  $(2m+1)w$  and  $(2m+3)w$ , or  $(2m+3)w$  and  $2(m+2)w$ . Consequently, any  $e$ -section in  $E$  contains 2 elements that are the multiple of  $w$ .

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