Unknown Values Estimation in Incomplete Fuzzy Soft and Interval Valued Fuzzy Soft Matrices

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ABSTRACT---- *In day to day problems, incomplete information due to unknown values in data is the cause of the loss of information which leads to uncertainly, ambiguity and vagueness. There are many reasons for unknown values in data, like errors in data collection, lack of data information, inappropriate technique and illegibility of data which cause incompleteness in data. Thus, to estimate the unknown values in data of various information systems is an important area of research. In this communication, the definitions of incomplete Fuzzy soft and interval-valued fuzzy soft matrices are given with application in numerical problems. An algorithm is proposed for unknown values estimation in an incomplete fuzzy soft matrix and applied in solving a numerical problem. An application of incomplete fuzzy soft matrix after inserting the unknown values in dimension reduction is studied. The unknown values in the incomplete interval-valued fuzzy soft matrix are also estimated and applied in multi-criteria decision-making method.*

Keywords and Phrases---- Fuzzy soft set, Interval-valued Fuzzy soft set, Estimation, Fuzzy Soft Matrices and Dimension Reduction
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1. INTRODUCTION

Zadeh [27] defined a fuzzy set to describe the uncertainty due to ambiguity and impreciseness in the environment of vagueness and applied to solve the complex real life problems. Subsequently, a number of generalizations of the fuzzy set were developed applied in many fields fo science and engineering. In view of the fact that fuzzy set has inadequacy of parameterization, Molodtsov [19] evolved soft set theory and that is considered a better theory in comparison to any other theory, like fuzzy set, rough set, and statistical approach, to deal with parametric data.

Theoretical as well as applied research in soft set theory has attracted the attention of many scientists from different fields. Maji et al. [21] applied soft set theory in decision-making problems. Later on, their work was improved by Chen et al. [1], Kong et al. [14], and Ma et al.[23] sequentially. Besides these developments many other researchers, namely Celik and Yamak [4], Jun et al. [12], Kalaichelvi and Malini [17], Kalayathankal and Singh [16], Tanay and Kandemir [25] used soft set hybridization with fuzzy set theory for handling the uncertainty in vagueness of data related to daily life.

The theory of soft set was extended by Maji et al. [20, 22] to fuzzy soft set theory. A computational tool based on a fuzzy soft set was introduced by Kong et al. [15]. That improved the traditional decision-making process and proved more convenient in case attributes were changed across the decision process. The theory of soft and fuzzy soft sets has proved useful in many fields such as forecasting, role mining, data analysis, simulation, and evaluation of sound quality.

There is a very convenient representation of soft and fuzzy soft sets in the form of Matrices. The advantage of representing a fuzzy soft set in the matrix form is that the data is easily stored and interpreted in a computer. Kandasamy et al. [13] initially introduced the concept of a fuzzy matrix. Cayman and Enginoglo [2] defined a soft matrix and multiplication of soft matrices. They further used the multiplication of soft matrices to describe a soft max-min decision-making method. Hooda and Jain [8] defined and characterized the information measures on fuzzy matrices and described their binary relations. Fuzzy soft matrix and its application in decision-making problems were introduced and studied by Cayman et al. [3].

It is easy to handle matrices when complete information is given, but the problem arises when some of the data values are missing due to mishandling, inaccurate measurement, or inappropriate tools for collecting information, etc. This leads to more uncertainty and ambiguity. Missing data occurs almost in all scientific fields, even in well-designed and controlled studies. Due to missing data, the statistical power of the study is reduced that leads to biased estimation and invalid conclusion. Thus, it becomes the utmost essential to estimate the unknown values in data instead of ignoring them.

A data analysis approach for soft sets with incomplete information was described by Zou and Xiao[28] . Their method is based on a weighted average of all possible choice values of the objects. The weight of each possible choice value from the probability of belonging of an object with a parameter is calculated. This method to compute the summation of all parameters' values for each object has a drawback as the unknown entries remain unknown. They also described an average probability method to predict the missing data in a fuzzy soft set. Actually, that method is based on the probability of an object belonging to a parameter, which is considered as the degree of belongingness of the object to that parameter.

Deng and Wang [5] defined an object parameter approach to estimate the missing values in the incomplete fuzzy soft set. In that technique, complete information from the relationship among various objects of a certain parameter and among the values of different parameters about a concerned object was considered. The new concept of complete distance between two objects and relative dominance degree between two parameters was introduced to explore the unknown information in a more precise and reasonable way.

Later on, Dong and Xiao[6] defined Dempster-Shafer's fuzzy soft set by combining Dempster-Shafer theory and fuzzy soft set. A decision-making algorithm for the incomplete fuzzy soft set was also described. A data filling approach using the relations between parameters and the association degree to measure the relation was explained by Qin et al.[24]. However, Khan et al. [18] also defined another data filling approach for unknown values estimation in soft sets. They mainly focussed on the reliability of association between parameters instead of probabilities.

Recently, Hooda and Barak [10] described an algorithm and applied in the estimation of the missing data in the design of experiments and contingency table. They have also studied the estimation of missing values in fuzzy matrices and extended application to interval-valued fuzzy matrices (refer to Hooda and Barak [11]). An algorithmic approach for predicting unknown information in incomplete fuzzy soft and interval-valued fuzzy soft sets was also given by Surjit et al. [7]. Wang et al. [26] also studied incomplete fuzzy soft set and its application to decision making.

In the present communication, a data filling approach to estimate unknown values in fuzzy soft and interval-valued fuzzy soft matrices is described and its application in numerical problems is illustrated. In section 2, some basic concepts used in the development of the paper were defined with examples. In section 3, an algorithm for unknown values estimation for a fuzzy soft matrix is proposed and illustrated with an example. In section 4, the application of fuzzy soft matrix in data dimension reduction is explained. An algorithm is proposed in section 5 for estimation of unknown values in an incomplete interval-valued fuzzy soft matrix and solved a numerical problem for illustration. A decision making method is also described by using of interval-valued fuzzy soft matrix in section 6. Discussion and conclusion are given in section 7 and the references are enumerated at the end of the paper.

2.   PRELIMINARIES

In this section some basic concept which are used further for the development of the paper are defined.

Definition 2.1[9]: Fuzzy Set
Let X is a non-empty set of universe, a function $\mu_A(x): X \to [0, 1]$ defines a fuzzy set A on X and usually written as
$$A = \{(x_i, \mu_A(x_i)): \mu_A(x_i) \in [0, 1]; \forall\, x_i \in X\}, \text{ where}$$

$\mu_A(x_i)$ is called membership function which satisfies the following conditions:

$$\mu_A(x_i) = \begin{cases} 0, & \text{if } x_i \notin A \text{ and there is no ambiguity} \\ 1, & \text{if } x_i \in A \text{ and there is no ambiguity} \\ 0.5, & \text{there is } \max \text{ ambiguity whether } x_i \in A \text{ or } x_i \notin A \end{cases}$$

Definition 2.2[21]: Soft Set
Let us consider 'U' as the universal set and T be the parametric set, then the pair (π, T) is defined as soft set over U iff π is a function of T to the power set of U i.e.

$$\pi : T \to P(U)$$

Example2.2. Let $U = (t_1, t_2, t_3, t_4)$ be the set of four televisions and 'E' be the parametric set namely, $E = (e_1(clear\ screen), e_2(cheap), e_3(good\ body))$. Then, the soft sets (π, E) over U is given by

$$(\pi, E) = \{ F(e_1) = (t_1, t_4),\ F(e_2) = (t_2, t_3),\ F(e_3) = (t_1, t_3)\}$$

Table 1: Tabular Representation of Soft Set

U/E	e_1	e_2	e_3
t_1	1	0	1
t_2	0	1	0
t_3	0	1	1
t_4	1	0	0

Definition 2.3[7]: Fuzzy Soft Set
Let X is a set of universe under discussion; E is the set of parameters and A⊆ E. Let F(X) be the set of all fuzzy subsets of X, then the pair (F, A) is called fuzzy soft set of X, where F is a mapping from A to F(X).

Example2.3. Let us consider C = $\{c_1, c_2, c_3\}$ be the set of three courses and E = $\{e_1 = job\ oriented, e_2 = affiliated, e_3 = cheaper\}$ be the set of parameters and let B =$\{e_1, e_2\} \subset E$, then

$$(F, B) = \begin{cases} F(e_1) = \{c_1/0.5,\ c_2/0.9,\ c_3/0.7 \\ F(e_2) = \{c_1/0.6,\ c_2/0.8,\ c_3/0.4 \end{cases};$$

(F, B) is a fuzzy soft set over C to give the "quality of the courses".

Definition 2.4[11]: Fuzzy Matrix
If a matrix holds an additional property such that all entries belong to closed unit interval [0, 1], then it is called Fuzzy Matrix.

Example 2.4

$$\begin{bmatrix} 0.2 & 0.6 & 0.5 \\ 0.4 & 0 & 0.9 \\ 0.3 & 0.7 & 1 \\ 0.1 & 0.2 & 0.8 \end{bmatrix}$$

is a fuzzy matrix of order 4×3.

Definition 2.5[3]: Fuzzy Soft Matrix

Let $R_S \in FS(U)$. Then a fuzzy relation form is defined by
$$R_S = \{(\mu_{R_S}(u, x)/(u, x): (u, x) \in U \times E\},$$
where U is the universal set, E is the set of parameters, S is the subset of E and the membership function μ_{R_S} is given by
$$\mu_{R_S}: U \times E \to [0,1]\ i.e.\ \mu_{R_S}(u, x) = \mu_{\gamma_S(x)}(u).$$

If $a_{ij} = \mu_{R_S}(u_i, x_j)$, for each i and j, then a fuzzy soft matrix is defined as
$$[a_{ij}]_{m \times n} = \begin{bmatrix} x_{11} & x_{12} \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} \cdots & x_{mn} \end{bmatrix},$$
and is called $m \times n$ fuzzy soft matrix of the fuzzy set R_S over U.

Definition 2.6[26]: Incomplete Fuzzy Soft Set

If there are missing values in tabular form of fuzzy soft set, it is called as incomplete fuzzy soft set and unknown values are marked as the sign" $*$ ".

Example2.5. Let $U = (u_{ij})_{6 \times 5}$ is an IFSS containing six objects $H = (u_1, u_2, u_3, u_4, u_5, u_6)$ and five parameters $E = (e_1, e_2, e_3, e_4, e_5)$. The matrix representation of the incomplete fuzzy soft set is given in table 2.

Table 2: Incomplete Fuzzy Soft Matrix

U/E	e_1	e_2	e_3	e_4	e_5
u_1	0.2	0.6	0.2	0.4	0.3
u_2	*	0.4	0.3	0.3	0.6
u_3	0.7	*	0.4	0.5	0.1
u_4	0.7	0.3	0.2	0.2	*
u_5	0.4	*	0.3	*	0.3
u_6	0.8	0.2	0.1	0.1	0.8

Definition 2.7[11]: Interval Valued Fuzzy Soft Matrix

An interval valued fuzzy soft matrix of order $m \times n$ is defined as $H = (h_{ij})_{m \times n}$, where $h_{ij} = [h_{ij}^{\,-}, h_{ij}^{\,+}]$ is $(i,j)^{th}$ element of H is given as subinterval of $[0,1]$ representing a value of an object corresponding to another object arranged in a rectangular form.

Example2.6. Consider an interval valued fuzzy soft matrix H given below:

$$H = \begin{bmatrix} [0.5, 0.7] & [0.3, 0.7] & [0.1, 0.2] & [0.1, 0.3] \\ [0.7, 0.9] & [0.2, 0.6] & [0.5, 0.9] & [0.2, 0.7] \\ [0.3, 0.5] & [0.4, 0.6] & [0.6, 0.7] & [0.6, 0.8] \end{bmatrix}$$

3. ALGORITHM FOR UNKNOWN VALUES ESTIMATION IN INCOMPLETE FUZZY SOFT MATRIX(IFSM)

Let $U = (H, E) = (u_{ij})_{m \times n}, i = 1\ to\ m, j = 1\ to\ n$ be an incomplete fuzzy soft set, where $H = (h_1, h_2, \dots, h_m)$ be the set of objects and $E = (e_1, e_2, \dots, e_n)$ be the set of parameters. The missing entries are denoted by sign '*' in matrix representation of U. Then, to estimate missing values, the following algorithm is applied:

1. Firstly, calculate mean value X_j for each parameter $e_j; j = 1\ to\ n$ as
   $X_j = \sum_{i=1}^{n} u_{ij} / |H_j|$, where $H_j = \{i / u_{ij} \neq *, 1 \leq i \leq m\}$.

2. Compute average distance denoted by $D_{avg}(X_j)$ for each entry $u_{ij}; i = 1\ to\ m, j = 1\ to\ n$ as $(X_j) = \sum_{i=1}^{m} |X_j - u_{ij}| / |H_j| \ \forall j$,
   where $H_j = \{i / u_{ij} \neq *, 1 \leq i \leq m\}$.

3. Next compute distance deviation$\{d_{X_j^+}, d_{X_j^-}\}$ on either side of the mean as
   $$d_{X_j^+} = X_j + D_{avg}(X_j)$$
   $$d_{X_j^-} = X_j - D_{avg}(X_j).$$

4. Compare $u_{ij}, j = 1\ to\ n$ for each $h_i, i = 1\ to\ m$ with mean value $X_j, j = 1\ to\ n, u_{ij} \neq *, 1 \leq i \leq m$. Let n_L and n_G denotes the number of count for which $u_{ij} < X_j$ and $u_{ij} \geq X_j$ respectively for corresponding parameter $e_j, j = 1\ to\ n$.

5. By using n_L and n_G calculate the probabilistic weights w_L and w_G for each object $h_i, i = 1\ to\ m$ respectively as
   $$w_{L_i} = n_L / (n_L + n_G)$$

$$w_{G_i} = n_G/(n_L + n_G).$$

6. The missing entries for each parameters e_j are computed as

$$u_{ij} = \left(d_{X_j^+} \times w_{G_i}\right) + \left(d_{X_j^-} \times w_{L_i}\right),$$

where $u_{ij} =*, i = 1 \text{ to } m, j = 1 \text{ to } n$.

3.1 Illustration

Let $U = (u_{ij})$ is the FSS containing $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, the set of six houses and the set of parameter is given by $E = \{e_1 = expensive, e_2 = beautiful, e_3 = wooden, e_4 = cheap, e_5 = in\ green\ surrounding, e_6 = modern, e_7 = in\ good\ repair, e_8 = in\ bad\ repair\}$. Let Mr X is interested in buying a house on the basis of his choice of parameters given by $P = \{e_1, e_2, e_3, e_4, e_5\}$. The data is arranged in Matrix form as given in table 3 and it found that some of the entries are missing. So, it becomes necessary to estimate these entries before making decision about the buying of house for Mr X.

The above problem can be written in an Incomplete Fuzzy Soft Matrix as follows:

Table 3: Incomplete Fuzzy Soft Matrix

U/E	e_1	e_2	e_3	e_4	e_5
h_1	0.1	0.5	0.4	0.4	0.3
h_2	*	0.5	0.2	0.3	0.6
h_3	0.1	*	0.3	0.5	0.1
h_4	0.7	0.2	0.2	0.2	0.3
h_5	0.2	0.5	0.3	*	0.3
h_6	0.9	0.2	0.1	0.1	0.8

The entries u_{21}, u_{32}, u_{54} are the missing denoted by '*' and are estimated by using algorithm 3.1 as described below:
1. Mean value X_1 for parameter e_1 is given by
   $X_1 = (0.1 + 0.1 + 0.7 + 0.2 + 0.9)/5 = 0.4$. Similarly, $X_2 = 0.38, X_3 = 0.25, X_4 = 0.3$ and $X_5 = 0.4$.
2. Average distance $D_{avg}(X_1)$ is computed as
$D_{avg}(X_1) = (|0.4 - 0.1| + |0.4 - 0.1| + |0.4 - 0.7| + |0.4 - 0.2| + |0.4 - 0.9|)/5 = (0.3 + 0.3 + 0.3 + 0.2 + 0.5)/5$
   $= 0.32$
   Similarly, remaining average distances are $D_{avg}(X_2) = 0.144, D_{avg}(X_3) = 0.1, D_{avg}(X_4) = 0.12$ and $D_{avg}(X_5) = 0.24$.
3. Now, the distance deviations are given below:
$$d_{X_1^+} = 0.4 + 0.32 = 0.72, d_{X_1^-} = 0.4 - 0.32 = 0.08,$$
$$d_{X_2^+} = 0.38 + 0.144 = 0.524, d_{X_2^-} = 0.38 - 0.144 = 0.236,$$
$$d_{X_3^+} = 0.25 + 0.10 = 0.35, d_{X_3^-} = 0.25 - 0.10 = 0.15,$$
$$d_{X_4^+} = 0.3 + 0.12 = 0.42, d_{X_4^-} = 0.3 - 0.12 = 0.18,$$
$$d_{X_5^+} = 0.4 + 0.24 = 0.64, d_{X_5^-} = 0.4 - 0.24 = 0.16.$$
4. Next n_L and n_G for corresponding parameter e_j are:
   In case of object h_2 for $e_2, u_{22} > X_2(0.5 > 0.38), for\ e_3, u_{23} < X_3(0.2 < 0.25), for\ e_4, u_{24} = X_4(0.3 = 0.3),$
   $for\ e_5, u_{25} > X_5(0.6 > 0.4)$.
   Hence, for $h_2 n_L = 1$ and $n_G = 3$, for object $h_3, n_L = 2, n_G = 2$ and
   for $h_5, n_L = 2, n_G = 2$.
5. Probabilistic weights w_{L_2} and w_{G_2} for object h_2 are given as
$$w_{L_2} = 1/(1 + 3) = 1/4, w_{G_2} = 3/(1 + 3) = 3/4,$$

Thus, for object $h_3, w_{L_3} = 2/(2 + 2) = 2/4, w_{G_3} = 2/(2 + 2) = 2/4,$
and for object $h_5, w_{L_5} = 2/(2 + 2) = 2/4, w_{G_5} = 2/(2 + 2) = 2/4.$

6. Finally, the missing entries are computed as
$$u_{21} = (0.72 \times 3/4) + (0.8 \times 1/4) = 0.74,$$
$$u_{32} = (0.524 \times 1/2) + (0.236 \times 1/2) = 0.38,$$
$$u_{54} = (0.42 \times 1/2) + (0.18 \times 1/2) = 0.30.$$

Hence, the missing entries are $u_{21} = 0.74, u_{32} = 0.38, u_{54} = 0.30.$

After substituting the estimated values in table 3, we get fuzzy soft matrix as given below

Table 4: Fuzzy Soft Matrix

U/E	e_1	e_2	e_3	e_4	e_5
h_1	0.1	0.5	0.4	0.4	0.3
h_2	**0.74**	0.5	0.2	0.3	0.6
h_3	0.1	**0.38**	0.3	0.5	0.1
h_4	0.7	0.2	0.2	0.2	0.3
h_5	0.2	0.5	0.3	**0.30**	0.3
h_6	0.9	0.2	0.1	0.1	0.8

4.   DIMENSION REDUCTION OF THE DATA

Here application of FSM in dimension reduction of the data arranged in the table 4 is explained. Firstly, we describe an algorithm for data dimension reduction

4.1 Algorithm
The following steps are described which are involve in algorithm for dimension reduction:
1. Input Fuzzy Soft Matrix (H, E).
2. Calculate oriented-object grade $O_i = (\sum_j u_{ij})/|E|$.
3. Calculate oriented-parameter grade $E_j = (\sum_i u_{ij})/|U|$.
4. Calculate threshold value $T = (\sum_{ij} u_{ij})/(|U| \times |E|)$.
5. Remove those rows for which $O_i < T$ and those column for which $E_j > T$.
6. New matrix is the desire dimensionality reduced matrix.

4.2 Application of Algorithm

Considering the fuzzy soft matrix given in Table 4 and applying the algorithm 4.1, the oriented-object grade O_i and oriented-parameter grade E_j are calculated as given in Table 5.

Table 5: Oriented Object and Parameter Grades

U/E	e_1	e_2	e_3	e_4	e_5	O_i
h_1	0.1	0.5	0.4	0.4	0.3	**0.34**
h_2	**0.74**	0.5	0.2	0.3	0.6	**0.46**
h_3	0.1	**0.38**	0.3	0.5	0.1	**0.276**
h_4	0.7	0.2	0.2	0.2	0.3	**0.32**
h_5	0.2	0.5	0.3	**0.30**	0.3	**0.32**
h_6	0.9	0.2	0.1	0.1	0.8	**0.42**
E_j	**0.45**	**0.38**	**0.25**	**0.3**	**0.4**	

The threshold value $T = (\sum_{ij} u_{ij})/(|U| \times |E|)$ is calculated which is 0.356. Now, remove those rows for which $O_i < T$ and those column for which $E_j > T$. The reduce matrix is given in table 6.

Table 6: Reduced Fuzzy Soft Matrix

U/E	e_3	e_4	O_i
h_2	0.2	0.3	**0.46**
h_6	0.1	0.1	**0.42**
E_j	**0.25**	**0.3**	

It may be noted that the data size is reduced by 86% approximately and the maximum object grade value is corresponding to the house h_2. Hence, Mr X will buy house h_2.

5. ALGORITHM TO ESTIMATE MISSING VALUES IN INCOMPLETE INTERVAL VALUED FUZZY SOFT MATRIX

In this section an algorithm for estimating missing values in an incomplete IVFSM is proposed and illustrated with an example.

5.1 Steps of the Algorithm

Let $U = (H, P) = (u_{ij}^-, u_{ij}^+)_{m \times n}, i = 1\ to\ m, j = 1\ to\ n$ be an incomplete interval valued fuzzy soft matrix, where H be the set of 'm'objects and E be the set of 'n'parameters. Each unknown value$(u_{ij}^-, u_{ij}^+)$is denoted by "*". Then steps of algorithm are given below:

a. Calculate mean value $X_j^-\ and\ X_j^+$for lower and upper membership functions respectively for each parameter e_j, where, $j = 1\ to\ n$ as $X_j^- = \sum_{i=1}^m u_{ij}^-/|H_i|$ and $X_j^+ = \sum_{i=1}^m u_{ij}^+/|H_i|$, where $H_i = \{i/u_{ij}^- \neq *, u_{ij}^+ \neq *, 1 \leq i \leq m\}$.

b. Calculate average distance of $u_{ij}^-\ and\ u_{ij}^+$from respective mean $X_j^-\ and\ X_j^+$, denoted by $D_{avg}(X_j^-)\ and\ D_{avg}(X_j^+)$, where $D_{avg}(X_j^-) = \sum_{i=1}^m |X_j^- - u_{ij}^-|/|H_i|$and $D_{avg}(X_j^+) = \sum_{i=1}^m |X_j^+ - u_{ij}^+|/|H_i|$.

c. Calculate distance deviations $\{d_l Y_j^-, d_l Y_j^+\}\ and\ \{d_u Y_j^-, d_u Y_j^+\}$ for lower and upper membership functions respectively from mean$X_j^-\ and\ X_j^+$as $d_l Y_j^- = X_j^- - D_{avg}(X_j^-), d_l Y_j^+ = X_j^- + D_{avg}(X_j^-),$ and$d_u Y_j^- = X_j^+ - D_{avg}(X_j^+), d_u Y_j^+ = X_j^+ + D_{avg}(X_j^+)$.

d. For each object $H_i(i = 1\ to\ m), u_{ij}^-\ and\ u_{ij}^+\ (j = 1\ to\ n)$ are compared with mean $X_j^-\ and\ X_j^+$respectively $(u_{ij}^- \neq *, u_{ij}^+ \neq *)$. Let $n_L^-\ and\ n_G^-$be the number of count for which $u_{ij}^- < X_j^-$ and $u_{ij}^- \geq X_j^-$for corresponding parameter $e_j(j = 1\ to\ n)$. Similarly,$n_L^+\ and\ n_G^+$be the number of count for which$u_{ij}^+ < X_j^+\ and\ u_{ij}^+ \geq X_j^+$, respectively.

e. Using $(n_L^-, n_G^-)$and $(n_L^+, n_G^+)$the probabilistic weight $(w_{L_i}^-, w_{G_i}^-)$and $(w_{L_i}^+, w_{G_i}^+)$of each object $H_i(i = 1\ to\ m)$are respectively given as $w_{L_i}^- = n_L^-/(n_L^- + n_G^-), w_{G_i}^- = n_G^-/(n_L^- + n_G^-),$ and$w_{L_i}^+ = n_L^+/(n_L^+ + n_G^+), w_{G_i}^+ = n_G^+/(n_L^+ + n_G^+)$.

f. The unknown data for parameter e_j is computed as $u_{ij}^- = (d_l Y_j^- \times w_{L_i}^-) + (d_l Y_j^+ \times w_{G_i}^-)$ and$u_{ij}^+ = (d_u Y_j^- \times w_{L_i}^+) + (d_u Y_j^+ \times w_{G_i}^+)$, where $(u_{ij}^-\ and\ u_{ij}^+)$are unknown entries in IIVFSM (H, P).

5.1 An Example for Illustration

Let us consider $R = \{h_1, h_2, h_3, h_4, h_5\}$ the set of 5 robots and $P = \{e_1(load\ capacity), e_2(speed), e_3(memory\ capacity), e_4(degree\ of\ freedom)\}$ be the set of parameters. An expert evaluates the performance of the robots $h_i(i = 1\ to\ 5)$with respect to the parameter $e_j(j = 1 to 4)$ and arranged in the form of matrix. But it is found that some of the values are missing. Therefore, before selecting an optimum robot the missing

data need to be estimated. The data arranged in the form of incomplete matrix is known as incomplete IVFSM as given below in table 7:

Table 7: Incomplete Interval Valued Fuzzy Soft Matrix

R/P	e_1	e_2	e_3	e_4
h_1	$[0.6, 0.8]$	$*$	$[0.5, 0.7]$	$[0.4, 0.5]$
h_2	$[0.2, 0.5]$	$[0.7, 0.8]$	$[0.4, 0.8]$	$[0.6, 0.9]$
h_3	$[0.5, 0.7]$	$[0.6, 0.8]$	$*$	$[0.8, 0.9]$
h_4	$[0.8, 0.9]$	$[0.5, 0.8]$	$[0.8, 0.9]$	$[0.6, 0.7]$
h_5	$[0.7, 0.9]$	$[0.6, 0.7]$	$*$	$[0.5, 0.7]$

Here the values $(u_{12}^-, u_{12}^+), (u_{33}^-, u_{33}^+)$ and (u_{53}^-, u_{53}^+) are missing, these values are estimated using the algorithm 5.1.

a) Mean values for lower and upper membership function for parameter e_1 are given by X_1^- and X_1^+ respectively as
$X_1^- = (0.6 + 0.2 + 0.5 + 0.8 + 0.7)/5 = 0.56$ and $X_1^+ = (0.8 + 0.5 + 0.7 + 0.9 + 0.9)/5 = 0.76$.   Similarly,   for parameter $e_2, X_2^- = 0.6, X_2^+ = 0.78$.
For parameter $e_3, X_3^- = 0.57, X_3^+ = 0.8, for\ e_4, X_4^- = 0.58, X_4^+ = 0.74$.

b) Average distances $D_{avg}(X_1^-)$ and $D_{avg}(X_1^+)$ are calculated as
$$D_{avg}(X_1^-) = (|0.56 - 0.6| + |0.56 - 0.2| + |0.56 - 0.5| + |0.56 - 0.8| + |0.56 - 0.7|)/5$$
$$= (0.04 + 0.36 + 0.06 + 0.24 + 0.14)/5 = 0.17$$
$$D_{avg}(X_1^+) = (|0.76 - 0.8| + |0.76 - 0.5| + |0.76 - 0.7| + |0.76 - 0.9| + |0.76 - 0.9|)/5$$
$$= (0.04 + 0.26 + 0.06 + 0.14 + 0.14)/5 = 0.13.$$
Similarly, remaining average distances are given by $D_{avg}(X_2^-) = 0.05, D_{avg}(X_2^+) = 0.035$,
$D_{avg}(X_3^-) = 0.157, D_{avg}(X_3^+) = 0.067$ And $D_{avg}(X_4^-) = 0.104, D_{avg}(X_4^+) = 0.128$.

c) Distance deviations $\{d_l Y_1^-, d_l Y_1^+\}$ and $\{d_u Y_1^-, d_u Y_1^+\}$ for lower and upper membership function from respective mean X_1^- and X_1^+ are given below:
$\{d_l Y_1^- = |0.56 - 0.17| = 0.39, d_l Y_1^+ = |0.56 + 0.17| = 0.73\}$ And
$$\{d_u Y_1^- = |0.76 - 0.13| = 0.63, d_u Y_1^+ = |0.76 + 0.13| = 0.89\}.$$
Similarly,
$\{(d_l Y_2^- = 0.55, d_l Y_2^+ = 0.65), (d_u Y_2^- = 0.75, d_u Y_2^+ = 0.82), (d_l Y_3^- = 0.41, d_l Y_3^+ = 0.727), (d_u Y_3^- = 0.73, d_u Y_3^+ = 0.87), (d_l Y_4^- = 0.48, d_l Y_4^+ = 0.68), (d_u Y_4^- = 0.61, d_u Y_4^+ = 0.87)\}.\}$

d) $(n_L^-$ and $n_G^-)$ and $(n_L^+$ and $n_G^+)$ for each object H_i and corresponding to the parameter e_j are given as:
For object $H_1, (n_L^- = 2, n_G^- = 1)$ and $(n_L^+ = 2, n_G^+ = 1)$,
for $H_2, (n_L^- = 2, n_G^- = 2)$ and $(n_L^+ = 1, n_G^+ = 3)$,
for $H_3, (n_L^- = 1, n_G^- = 2)$ and $(n_L^+ = 1, n_G^+ = 2)$,
for $H_4, (n_L^- = 1, n_G^- = 3)$ and $(n_L^+ = 1, n_G^+ = 3)$,
for $H_5, (n_L^- = 1, n_G^- = 2)$ and $(n_L^+ = 2, n_G^+ = 1)$.

e) Probabilistic weight for object H_1, is $(w_{L_1}^- = 0.67, w_{G_1}^- = 0.33)$ and $(w_{L_1}^+ = 0.67, w_{G_1}^+ = 0.33)$, for object $H_2, (w_{L_2}^- = 0.5, w_{G_2}^- = 0.5)$ and $(w_{L_2}^+ = 0.25, w_{G_2}^+ = 0.75)$,   for   object $H_3, (w_{L_3}^- = 0.33, w_{G_3}^- = 0.67)$ and $(w_{L_3}^+ = 0.33, w_{G_3}^+ = 0.67)$, for object H_4, $(w_{L_4}^- = 0.25, w_{G_4}^- = 0.75)$ and $(w_{L_4}^+ = 0.25, w_{G_4}^+ = 0.75)$ and for object $H_5, (w_{L_5}^- = 0.33, w_{G_5}^- = 0.67)$ and $(w_{L_5}^+ = 0.67, w_{G_5}^+ = 0.33)$.

f) Now the missing entries are given by: $u_{12}^- = (0.55 \times 0.67) + (0.65 \times 0.33) = 0.37 + 0.21 = 0.58, u_{12}^+ = (0.75 \times 0.67) + (0.82 \times 0.33) = 0.50 + 0.27 = 0.77, u_{33}^- = (0.41 \times 0.33) + (0.73 \times 0.67) = 0.14 + 0.49 = 0.63, u_{33}^+ = (0.73 \times 0.33) + (0.87 \times 0.67) = 0.24 + 0.58 = 0.82, u_{53}^- = (0.41 \times 0.33) + (0.73 \times 0.67) = 0.14 + 0.49 = 0.63, u_{53}^+ = (0.73 \times 0.67) + (0.87 \times 0.33) = 0.49 + 0.29 = 0.78$.
Therefore, the missing entries are $[0.58, 0.77], [0.63, 0.82], [0.63, 0.78]$.

6. MULTI-CRITERIA DECISION MAKING METHOD

To illustrate the application of interval valued fuzzy soft matrix in decision making, we consider in matrix given in table 8

The steps involve in decision making are given below:

1) Input the interval valued fuzzy soft matrix.
2) Calculate the choice value $c_i = [c_i^-, c_i^+]$; where $c_i^- = \sum_{j=1}^n u_{ij}^-$ and $c_i^+ = \sum_{j=1}^n u_{ij}^+$.
3) Compute the score value s_i, given as

$$s_i = \sum_{j=1}^n \{(c_i^- - c_j^-) + (c_i^+ - c_j^+)\}.$$

4) Choose the maximum value of s_i that is the requisite decision. If two or more values of s_i are same, then choose any one of them according to your wisdom.

Now using the above algorithm decision about the choice of robot given in example 5.2 is taken. After estimating the missing values using algorithm 5.1 the complete IVFSM is given in table 8.

Table 8: Interval Valued Fuzzy Soft Matrix

R/P	e_1	e_2	e_3	e_4
h_1	$[0.6, 0.8]$	$[\mathbf{0.58, 0.77}]$	$[0.5, 0.7]$	$[0.4, 0.5]$
h_2	$[0.2, 0.5]$	$[0.7, 0.8]$	$[0.4, 0.8]$	$[0.6, 0.9]$
h_3	$[0.5, 0.7]$	$[0.6, 0.8]$	$[\mathbf{0.63, 0.82}]$	$[0.8, 0.9]$
h_4	$[0.8, 0.9]$	$[0.5, 0.8]$	$[0.8, 0.9]$	$[0.6, 0.7]$
h_5	$[0.7, 0.9]$	$[0.6, 0.7]$	$[\mathbf{0.63, 0.78}]$	$[0.5, 0.7]$

Compute the choice value $c_i = [c_i^-, c_i^+]$ as given in table 9.

Table 9: IVFSM with Choice Value c_i

R/P	e_1	e_2	e_3	e_4	Choice value c_i
h_1	$[0.6, 0.8]$	$[0.58, 0.77]$	$[0.5, 0.7]$	$[0.4, 0.5]$	[2.08, 2.77]
h_2	$[0.2, 0.5]$	$[0.7, 0.8]$	$[0.4, 0.8]$	$[0.6, 0.9]$	[1.9, 3]
h_3	$[0.5, 0.7]$	$[0.6, 0.8]$	$[0.63, 0.82]$	$[0.8, 0.9]$	[2.53, 3.22]
h_4	$[0.8, 0.9]$	$[0.5, 0.8]$	$[0.8, 0.9]$	$[0.6, 0.7]$	[2.7, 3.3]
h_5	$[0.7, 0.9]$	$[0.6, 0.7]$	$[0.63, 0.78]$	$[0.5, 0.7]$	[2.43, 3.08]

Now the score value s_i is given as

$$s_i = \sum_{j=1}^n \{(c_i^- - c_j^-) + (c_i^+ - c_j^+)\}.$$

Thus, $s_1 = -2.76, s_2 = -0.91, s_3 = 1.74, s_4 = 2.99, s_5 = 0.54.$ the maximum score value is $s_4 = 2.99$ that is corresponding to the robot 4. Hence, robot 4 is to be considered for manufacturing purpose.

7. DISCUSSION AND CONCLUSION

The estimation of unknown values described in the present paper is very important technique that can be applied in many areas of research, like agriculture, medical diagnosis, economics, and data management. Recently, Hooda and Barak [10] evolved a technique of estimation of missing values in contingency table and illustrated the same. Hooda and Barak [11]

have also described an interactive technique of missing values estimation in incomplete fuzzy matrix and interval valued fuzzy matrix and have applied the technique in solving a numerical problem.

Surjit Das et al. [7] have also studied an algorithm of estimation of unknown information in incomplete fuzzy soft and interval valued soft sets. In view of the importance and need, we have studied technique to estimate the missing values in incomplete fuzzy soft and interval valued fuzzy soft matrices in the present paper with illustration. The proposed technique makes use of the known information by computing the mean of each parameter and then average distance of each object from the corresponding mean is calculated.

As compared to the approach applied by Deng and Wang [5], our approach is preferably better as we use the weight of probabilities and a pair of deviations to retrieve the missing data. We have also studied the application of incomplete fuzzy soft and interval valued fuzzy soft matrices in decision making problem after estimating the unknown values in data.

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