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Effect of Magnetic Field on Rayleigh Bénard Marangoni Convection in a Relatively Hotter or Cooler Layer of Liquid with Insulating Boundaries

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ABSTRACT— ***The effect of the uniform vertical magnetic field acting opposite to gravity on the onset of steady Rayleigh-Bénard-Marangoni convection in horizontal layer of an electrically conducting liquid is investigated, using the modified linear stability theory. The upper surface of liquid layer is free where surface tension gradients arise on account of variation of temperature and the lower boundary surface is rigid, each subject to the constant heat flux condition. Both mechanisms namely, surface tension and buoyancy causing instability are taken into account. The Galerkin method is used to obtain the eigenvalue equation which is then computed numerically. Results of this analysis indicate that the critical eigenvalues in the presence of a uniform magnetic field are greater in a relatively hotter layer of liquid than a cooler one under identical conditions otherwise. The asymptotic behaviour of both the Rayleigh and Marangoni numbers for large values of the Chandrasekhar number is also obtained. During the course of this analysis, we also correct the inaccuracies in the work of earlier authors.***

Keywords---- Buoyancy, Convection, Insulating, Linear stability, Stationary, Surface tension.

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1. INTRODUCTION

Quantitative disagreement between theory and experiment has indicated that gravity was present in Bénard's[1, 2] experiments as well as other experiments on convection in a liquid layer with free surface in a laboratory on the earth. Therefore, Nield [3] studied the combined effect of both surface tension and buoyancy on the onset of convection, using the linear stability theory given by Rayleigh [4], and established that as thickness of the fluid layer decreases the surface tension effect becomes more dominant. Further, Nield [5] established that the buoyancy and surface tension effects are perfectly coupled when the heat flux across each boundary is kept constant. The stabilizing nature of the magnetic field, a fact, that has already been established by Chandrasekhar [6, 7] for the buoyancy driven convection, and by Nield [8] for the combined surface tension and buoyancy driven convection. The effect of the magnetic field on the onset of pure Marangoni convection in an electrically conducting liquid layer heated from below has been discussed by a number of authors notably by Maekawa & Tanasawa [9] and Wilson [10], Hashim and Wilson [11]. Recently, the effect of uniform vertical magnetic field on the onset of surface tension and buoyancy driven convection has been studied by Gupta and Dhiman [12] for the case wherein lower boundary is conducting and the upper one is insulating, using the modified linear stability analysis of Banerjee et al. [13].

In this paper, we investigate the effect of a uniform vertical magnetic field on the onset of combined surface tension and buoyancy driven convection in a relatively hotter or cooler layer of liquid subject to constant heat flux condition at both lower and upper boundaries, using the modified linear stability analysis. The Galerkin method is used to obtain the eigenvalue equation analytically. The numerical results obtained for a wide range of the parameters are presented. The results of this analysis indicate that the uniform magnetic field suppresses convection more effectively in a relatively hotter layer of liquid than the cooler one, irrespective of whether the two mechanisms namely, surface tension and buoyancy causing instability act individually or simultaneously. The two mechanisms causing instability are found to reinforce each other and are perfectly coupled in the absence of magnetic field. It is found that the two mechanisms causing instability no longer remain perfectly coupled for increasing intensity of the magnetic field. Further, it is interesting to note that this situation occurs when the critical wave number is zero. Further, the asymptotic behaviour of both the Rayleigh and Marangoni numbers for large value of the Chandrasekhar number is also obtained. A detail description of the marginal stability curves showing the influence of the magnetic field on the onset of convection in a relatively hotter or cooler layer of liquid is also presented. During the course of this analysis, we found that Isa et al. [14] made a small but significant error while choosing the velocity trial function as a fourth order polynomial not justifying the requirement of selecting the lowest order polynomial (Finlayson [15]) satisfying the three boundary conditions, when surface tension effects are present under the influence of magnetic field. In this case, we shall correct the inaccuracies in the results of Isa et al. [14].

2. FORMULATION OF THE PROBLEM

We consider an infinite horizontal liquid layer of viscous, incompressible and electrically conducting fluid of uniform thickness d heated from below, in the presence of a uniform vertical magnetic field $\vec{H}$ acting opposite to the gravity $\vec{g}$. The lower boundary surface of the layer of liquid is rigid and the upper surface is free non-deformable where surface tension gradients arise on account to variation of temperature with the upper free surface open to the ambient air, where surface tension gradients arise due to temperature perturbations. We choose a Cartesian coordinate system of axes with the x and y axis in the plane of the lower surface and the z-axis along the vertically upward direction so that the fluid is confined between the planes at $z = 0$ and $z = d$. A temperature gradient is maintained across the layer by maintaining the lower boundary at a constant temperature T_0 and the upper boundary at T_1 ($< T_0$). The surface tension on the upper free surface of the fluid is regarded as a function of temperature only which is given by the simple linear law $\tau = \tau_1 - \sigma(T - T_1)$ where the constant τ_1 is the unperturbed value of τ at the unperturbed surface temperature $T = T_1$ and $-\sigma = (\partial\tau/\partial T)_{T=T_1}$ represents the rate of change of surface tension with temperature, evaluated at temperature T_1, and surface tension being a monotonically decreasing function of temperature, σ is positive. We wish to examine the stability of this configuration under the joint action of buoyancy and surface tension in the light of modified linear stability analysis.

Following Banerjee et al [13], we can write modified linearized perturbation equations under uniform magnetic field in the relevant contextas

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\nabla^2 w = g\alpha\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\theta + \frac{\mu_e H}{4\pi\rho_0}\nabla^2\frac{\partial}{\partial z}h_z, \quad (1)$$

$$(1-\alpha_2\,\mathrm{T}_0)\left(\frac{\partial\theta}{\partial t} - \beta\, w\right) = \kappa\nabla^2\theta, \quad (2)$$

$$\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)h_z = H\frac{\partial w}{\partial z}, \quad (3)$$

where w is the perturbation velocity, θ is the perturbation temperature, h_z is the z-component of the perturbation from the uniform vertical magnetic field $\vec{H}$ and ρ is the density of fluid. The kinematic viscosity ν, the thermal diffusivity κ, the gravitational acceleration g, the magnetic permeability μ_e, the magnetic resistivity η, the temperature gradient β which is maintained and are each assumed to be constant. Further, that coefficient α_2 (due to variation in the temperature) is a constant that ranges from 0 to 10^{-4} for the liquid with which we are most concerned.

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and t denotes time.

In seeking solutions of the Eqs. (1), (2) and (3), we must satisfy certain boundary conditions. The boundary conditions at the lower rigid and thermally insulating surface $z = 0$ are

$$w = 0, \quad \frac{\partial w}{\partial z} = 0, \quad \frac{\partial\theta}{\partial z} = 0, \quad h_z = 0. \qquad (4a, b, c, d)$$

The boundary conditions at the upper free surface $z = d$ are

$$w = 0, \quad \rho\nu\frac{\partial^2 w}{\partial z^2} = \sigma\nabla_1^2\theta, \quad \frac{\partial\theta}{\partial z} = 0, \quad h_z = 0, \quad (5a, b, c, d)$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

We now suppose that the perturbations w, θ and h_z are of the form

$$[w, \theta, h_z] = [w(z), \theta(z), h_z(z)]\exp[i(a_x x + a_y y) + pt],$$

where $a = \sqrt{a_x^2 + a_y^2}$ is the wave number of the disturbance and p is a time constant (which can be complex). We now introduce the non-dimensional quantities using d, d^2/ν, d/ν and $\beta d\nu/\kappa$ as the appropriate scales for length, velocity, time and temperature respectively and putting

$$a_* = ad, \quad w_* = \frac{wd}{\nu}, \quad \theta_* = \frac{\theta\kappa R a_*^2}{\beta d\nu}, \quad p_* = \frac{pd^2}{\nu}, \quad h_z = \frac{h_z\eta}{\nu H}.$$

We now let x, y and z stand for co-ordinates in the new units and omitting asterisk for simplicity, Eq. (1)-(3) and boundary conditions (4a, b, c, d)-(5a, b, c, d) can be reduced to the following non-dimensional form

$$\left(D^2-a^2\right)\left(D^2-a^2-p\right)w+Q\left(D^2-a^2\right)Dh_z=\theta, \quad (6)$$

$$\left(D^2-a^2-(1-\alpha_2 T_0)pP_r\right)\theta=-Ra^2\left(1-\alpha_2 T_0\right)w, \quad (7)$$

$$\left(D^2-a^2-pP_m\right)h_z=-Dw, \quad (8)$$

$$w=0, \quad Dw=0, \quad D\theta=0, \quad h_z=0, \quad \text{at} \quad z=0, \quad (9a, b, c, d)$$

$$w=0, \quad D^2w=\Gamma\theta, \quad D\theta=0, \quad h_z=0, \quad \text{at} \quad z=1. \quad (10a, b, c, d)$$

where $Q=\mu_e H^2 d^4/4\pi\rho\eta\nu$ is the Chandrasekhar number, $R=g\alpha\beta d^4/\kappa\nu$ is the Rayleigh number, $P_r=\nu/\kappa$ is the thermal Prandtl number, $P_m=\nu/\eta$ is the magnetic Prandtl number. $\Gamma=M/R=\sigma/\rho g\alpha d^2$ with $M=\sigma\beta d^2/\rho\kappa\nu$ as the Marangoni number, characterizes the strength of surface tension relative to buoyancy.

We restrict our analysis to the case when the marginal state is stationary so that the marginal state is characterized by setting $p=0$ and h_z is eliminated from the resulting equations, we obtain

$$\left[\left(D^2-a^2\right)^2-QD^2\right]w=\theta, \quad (11)$$

$$(D^2-a^2)\theta=-Ra^2(1-\alpha_2 T_0)w. \quad (12)$$

In terms of new variables, the non-dimensional form of boundary conditions (9a, b, c)–(10a, b, c) can be written as

$$w(0)=0, \quad Dw(0)=0, \quad D\theta(0)=0, \quad (13a, b, c)$$

$$w(1)=0, \quad D^2w(1)=\Gamma\theta(1), \quad D\theta(1)=0. \quad (14a, b, c)$$

The Eqs. (11)-(12) together with boundary conditions (13a, b, c)-(14a, b, c) constitute an eigenvalue problem of order six.

3. Solution Of The Problem

The single term Galerkin method is convenient for solving the present problem. Accordingly, the unknown variables w and θ are written as

$w=Aw_1$ and $\theta=B\theta_1$, (15)

in which A and B are constants and w_1 and θ_1 are the trial functions, which are chosen suitably satisfying the boundary conditions (13a, b, c)-(14a, b, c). Multiplying Eq. (11) by w and Eq. (12) by θ, integrating the resulting equations with respect to z from 0 to 1 using the boundary conditions (13a, b, c)-(14a, b, c). Substituting for wand θ from (15) and eliminating A and B from resulting system of equations, we obtain the following eigenvalue equation

$$\begin{vmatrix} \left\langle \left(D^2w\right)^2+(2a^2+Q)\left(Dw\right)^2+a^4\left(w\right)^2 \right\rangle & \langle w\theta\rangle-\Gamma Dw(1)\theta(1) \\ Ra^2\left(1-\alpha_2 T_0\right)\langle w\theta\rangle & \left\langle \left(D\theta\right)^2+a^2\left(\theta\right)^2 \right\rangle \end{vmatrix}=0 \quad (16)$$

In Eq. (16), $\langle --\rangle$ denotes integration with respect to z between z = 0 and z =1 and suffixes have been dropped for simplicity while writing the Eq. (16). The eigenvalue Eq. (16) may be put in the following form

$$R=\frac{\left\langle \left(D^2w\right)^2+(2a^2+Q)\left(Dw\right)^2+a^4\left(w\right)^2 \right\rangle \left\langle \left(D\theta\right)^2+a^2\left(\theta\right)^2 \right\rangle}{a^2\left(1-\alpha_2 T_0\right)\langle w\theta\rangle\left[\ \langle w\theta\rangle-\Gamma Dw(1)\theta(1)\right]} \quad (17)$$

We select the trial function

$$w=z^2\left(1-z\right)\left[\frac{\Gamma}{4}+\frac{1}{24}\left(\frac{3}{2}-z\right)\right] \quad \text{and} \quad \theta=1, \quad (18)$$

such that they satisfy all the boundary conditions (13a, b, c)-(14a, b, c). It is important to remark here that above choice of the velocity trial function given by (18) is found to be useful for cases in which the two mechanisms (buoyancy and surface tension) causing instability act individually or simultaneously. Substitution of trial functions given by (18) into the eigenvalue Eq. (17), we get

$$R\left(\frac{1}{320}+\frac{\Gamma}{48}\right) = \frac{1}{(1-\alpha_2 T_0)}\left[1+\frac{\left(2a^2+Q\right)}{30}\left\{\frac{\left(\Gamma+\frac{1}{8}\right)^2+\frac{1}{448}}{\left(\Gamma+\frac{1}{12}\right)^2+\frac{1}{180}}\right\}+\frac{a^4}{420}\left\{\frac{\left(\Gamma+\frac{7}{48}\right)^2+\frac{5}{6912}}{\left(\Gamma+\frac{1}{12}\right)^2+\frac{1}{180}}\right\}\right] \quad (19)$$

4. Numerical Results And Discussion

The numerical calculations are carried out using the symbolic algebraic package Mathematica, for assigned values of the parameters Γ, $\alpha_2 T_0$ and Q. We seek theminimum of R as a function of the wave number a to obtain values of the critical Rayleigh number R_c and corresponding critical wave number a_c. Validation of the computer program is achieved through verification of existing results obtained by Gupta and Surya [16].

Case I: When buoyancy is sole agency causing instability.

When $\Gamma \to 0$ (or $M \to 0$) implies that in the absence of surface tension effect, buoyancy is the sole agency causing instability. In this case, we obtain R from theeigenvalue Eq. (19) in terms of a, $\alpha_2 T_0$ and Q as

$$R=\frac{320}{(1-\alpha_2 T_0)}\left\{1+\frac{2a^2}{21}+\frac{19a^4}{4536}+\frac{Q}{21}\right\}. \quad (20)$$

For given values of parameter $\alpha_2 T_0$ and Q, it follows from Eq. (20) that the minimum of R exists at $a=0$. Thus the critical Rayleigh number R_c is given by

$$R=\frac{320}{(1-\alpha_2 T_0)}\left\{1+\frac{Q}{21}\right\}. \quad (21)$$

The numerical values of R_c are calculated for various values of $\alpha_2 T_0$ and Q, using Eq. (21) and are presented in Table 1.

TABLE 1

Values of R_c for various values of Q when $\alpha_2 T_0$ = 0, 0.3 and 0.5.

Q	$\alpha_2 T_0=0$	$\alpha_2 T_0=0.3$	$\alpha_2 T_0=0.5$
	R_c	R_c	R_c
0	320.000	457.143	640.000
1	335.238	478.912	670.476
10	472.381	674.830	944.762
10^2	1843.810	2634.010	3687.620
10^3	15558.100	22225.900	31116.200
10^4	152701.000	218144.000	305401.000
10^6	15.24×10^6	21.77×10^6	30.48×10^6
10^8	15.24×10^8	21.77×10^8	30.48×10^8

When Q = 0, we observe from Table 1 that values of R_c obtained here for various values of $\alpha_2 T_0$ agree precisely with corresponding values obtained Gupta and Surya [16]. Further, for a prescribed value of $\alpha_2 T_0$, Table 1 shows that an increase in the value of Q leads to a greater value of R_c indicating that the magnetic field strength has stabilizing effect on the onset of convection. On the other hand, for a prescribed value of Q, it is observed that an increase in the value of $\alpha_2 T_0$ leads to a greater value of R_c indicating that hotter the liquid layer more the postponement of the onset of instability. It is interesting to note that value of the critical wave number a_c is zero**.**

The $(R,\ \alpha_2 T_0)$ curves corresponding to neutral stability are plotted in Fig. 1, using the relation (21), for various values of Q. From Fig. 1, we observe that R increases with increase in Q indicating that the magnetic field strength has the stabilizing effect on the onset of buoyancy driven convection. Further, each curve corresponding to a fixed valueof Q in the Fig. 1 illustrates that a relatively hotter layer of liquid is more stable thanthe cooler one under almost identical conditions.

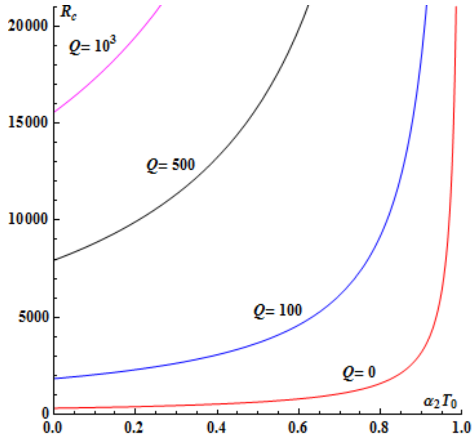


Fig. 1. Variation of R_c as a function of $\alpha_2 T_0$ for various values of Q.

The asymptotic behaviour of R_c critically depends on $\alpha_2 T_0$, for large value of Chandrasekhar number Q. When $Q \to \infty$ asymptotic behaviour of R_c is obtained as

$$R_c \approx \frac{15.2}{(1-\alpha_2 T_0)} Q. \quad (22)$$

Case II: When surface tension is sole agency causing instability.

On substituting $\Gamma = M/R$ on left hand side of the eigenvalue Eq. (19), we find that

$$\frac{R}{320} + \frac{M}{48} = \frac{1}{(1-\alpha_2 T_0)} \left[1 + \frac{(2a^2+Q)}{30} \left\{ \frac{\left(\Gamma + \frac{1}{8}\right)^2 + \frac{1}{448}}{\left(\Gamma + \frac{1}{12}\right)^2 + \frac{1}{180}} \right\} + \frac{a^4}{420} \left\{ \frac{\left(\Gamma + \frac{7}{48}\right)^2 + \frac{5}{6912}}{\left(\Gamma + \frac{1}{12}\right)^2 + \frac{1}{180}} \right\} \right]. \quad (23)$$

When $\Gamma \to \infty$ (or $R \to 0$) implies that in the absence of buoyancy effect, surface tension is the sole agency causing instability. In this case, we obtain M from the eigenvalue Eq. (23) in terms of a, $\alpha_2 T_0$ and Q as

$$M = \frac{48}{(1-\alpha_2 T_0)} \left\{ 1 + \frac{a^2}{15} + \frac{a^4}{420} + \frac{Q}{30} \right\}. \quad (24)$$

For given values of parameter $\alpha_2 T_0$ and Q, it follows from Eq. (24) that the minimum of M exists at $a = 0$. Thus the critical Marangoni number M_c is given by

$$M = \frac{48}{(1-\alpha_2 T_0)} \left\{ 1 + \frac{Q}{30} \right\}. \quad (25)$$

The numerical values of M_c are calculated for various values of $\alpha_2 T_0$ and Q, using Eq. (25) and are presented in Table 2a. When $Q = 0$, we observe from Table 2a that values of M_c obtained here for various values of $\alpha_2 T_0$ agree precisely with corresponding values obtained Gupta and Surya [16].

TABLE 2a

Values of M_c for various values of Q when $\alpha_2 T_0$ = 0, 0.3 and 0.5.

Q	$\alpha_2 T_0 = 0$	$\alpha_2 T_0 = 0.3$	$\alpha_2 T_0 = 0.5$
	M_c	M_c	M_c
0	48.000	68.571	96.000
1	49.600	70.857	99.200
10	64.000	91.429	128.000
10^2	208.000	297.143	416.000
10^3	1648.000	2354.290	3296.000
10^4	16048.000	22925.700	32096.000
10^6	16.00×10^6	22.86×10^6	32.00×10^6
10^8	16.00×10^8	22.86×10^8	32.00×10^8

When $\alpha_2 T_0 = 0$, values of M_c for various corresponding values of Q obtained by us agree precisely with those obtained by Isa et al [17] corresponding to this case when surface tension is the sole agency causing instability in the presence of magnetic field for the basic linear temperature profile. However, values of M_c for various corresponding values of Q obtained by us as well as by Isa et al [17] disagree with those obtained by Isa et al [14] corresponding to this case when surface tension is the sole agency causing instability (R = 0), in the presence of magnetic field for the basic linear temperature profile. A comparison between the corresponding values of M_c for various values of Q when R = 0 obtained by Isa et al. [14] for the basic linear temperature profile and by us in the present analysis when $\alpha_2 T_0 = 0$ is given in the Table 2b. The choice of the velocity trial function given by us in Eq. (18) allows to resolve this disagreement which exists in the literature. It is pointed out that Isa et al [14] made a small but significant error while choosing the velocity trial function as $w = z^2(1-z)(3-2z)$ [their Eq. (19)] which is a fourth order polynomial satisfying three boundary conditions, not justifying the requirement of selecting the lowest order polynomial (a cubic here) particularly in the case when surface tension is the sole agency causing instability (Finlayson [15]). However, their results when buoyancy is the sole agency causing instability (M = 0) are correct, of course, since in this case $D^2 w(1) = 0$ that is, upper boundary becomes stress free.

TABLE 2b

Compared Values of M_c for various values of Q when R = 0.

Q	Isa et al [14]	Present analysis
	M_c	M_c
0	48.000	48.000
10	70.857	64.000
10^2	276.571	208.000

Further, for a prescribed value of $\alpha_2 T_0$, Table 2a shows that an increase in the value of Q leads to a greater value of M_c indicating that the magnetic field strength has stabilizing effect on the onset of convection. On the other hand, for a prescribed value of Q, it is observed that an increase in the value of $\alpha_2 T_0$ leads to a greater value of M_c indicating that hotter the liquid layer more the postponement of the onset of instability. It is interesting to note that value of the critical wave number a_c is zero.

The $(M, \alpha_2 T_0)$ curves corresponding to neutral stability are plotted in Fig. 2, using the relation (25), for various values of Q. From Fig. 2, we observe that M increases with increase in Q indicating that the magnetic field strength has the stabilizing effect on the onset of buoyancy driven convection. Further, each curve corresponding to a fixed value of Q in the Fig. 2 illustrates that a relatively hotter layer of liquid is more stable thanthe cooler one under almost identical conditions.

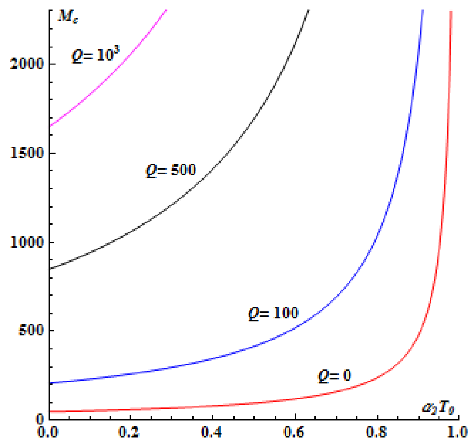


Fig. 2. Variation of M_c as a function of $\alpha_2 T_0$ for various values of Q.

The asymptotic behaviour of M_c critically depends on $\alpha_2 T_0$, for large value of the Chandrasekhar number Q. When $Q \to \infty$ asymptotic behaviour of M_c is obtained as

$$M_c \approx \frac{1.6}{(1-\alpha_2 T_0)} Q. \quad (26)$$

Case III: Combined effect of buoyancy and surface tension.

For given values of Γ (> 0), $\alpha_2 T_0$, and Q, we note that coefficients of both a^2 and a^4 in the eigenvalue Eq. (19) are positive definite. Hence, the true minimum of R exists when a = 0, and the critical Rayleigh number R_c is obtained as

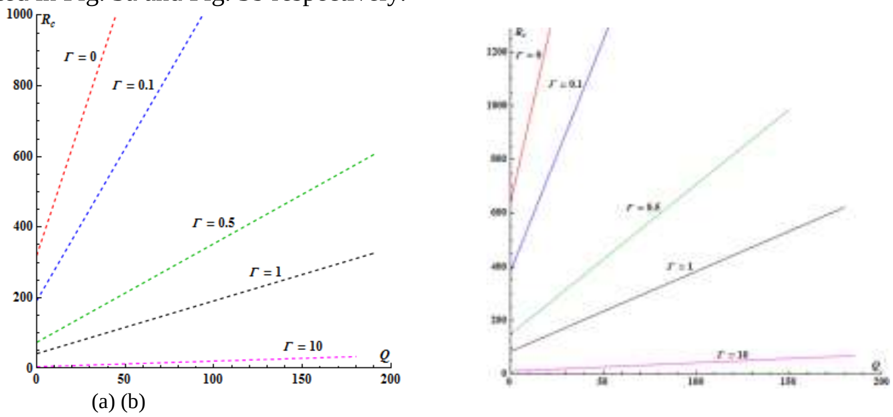
$R_c = \dfrac{960}{(1-\alpha_2 T_0)(3+20\Gamma)}\left[1+\dfrac{Q}{30}\left\{\dfrac{\left(\Gamma+\frac{1}{8}\right)^2+\frac{1}{448}}{\left(\Gamma+\frac{1}{12}\right)^2+\frac{1}{180}}\right\}\right]$. (27) The numerical values of R_c, computed with the aid of Eq. (27), for various values of Γ and Q are presented in Table 3. For a fixed value of Q, we observe from Table 3 that value of R_c decreases with increase in Γ (or decrease in depth d of the liquid layer). In other words, effect of surface tension in the presence of magnetic field causes reduction in the critical Rayleigh number R_c, irrespective of whether the layer of liquid is relatively cooler or hotter. From Table 3, we also observe that for a fixed value of Γ, value of R_c increases with increase in Q indicating the stabilizing effect of the magnetic field, irrespective of whether the layer of liquid is relatively cooler or hotter. Further,we note from Table 3 that an increase in the value of $\alpha_2 T_0$, leads to an increased value of R_c corresponding to given Γ and Q which means that a relatively hotter layer of liquid is more stable than the cooler one.

TABLE 3

Values of R_c for various values of Γ and Q when $\alpha_2 T_0$ = 0 and 0.5.

Γ	$\alpha_2 T_0$ = 0			$\alpha_2 T_0$ = 0.5		
	Q = 0	**Q =10^2**	**Q = 10^6**	**Q = 0**	**Q =10^2**	**Q =10^6**
	R_c	**R_c**	**R_c**	**R_c**	**R_c**	**R_c**
0	320.000	1843.810	15.24 ×10 ^{6}	640.000	3687.620	30.48×10 ^{6}
10^{-3}	317.881	1832.560	15.15 ×10 ^{6}	635.762	3665.120	30.29×10 ^{6}
10^{-2}	300.000	1733.910	14.34×10 ^{6}	600.000	3467.820	28.68×10 ^{6}
10^{-1}	192.000	1055.710	86.37×10 ^{5}	384.000	2111.420	17.28×10 ^{6}
0.5	73.846	353.470	27.96×10 ^{5}	147.692	706.940	55.93×10 ^{5}
1	41.739	191.334	14.96×10 ^{5}	83.478	382.668	29.92×10 ^{5}
10	4.729	20.623	158940.000	9.458	41.245	317881.000
10^2	0.479	2.078	15989.800	0.959	4.156	31979.600
10^3	0.048	0.208	1599.940	0.096	0.416	3199.880
10^6	0.000	0.000	1.600	0.000	0.000	3.200

The variation of the critical Rayleigh number R_c with the magnetic field strength Q for various values of Γ when $\alpha_2 T_0 = 0$ and 0.5 are illustrated in Fig. 3a and Fig. 3b respectively.



(a) (b)

Fig. 3. Variation of R_c as a function of Q for various values of Γ when (a) $\alpha_2 T_0 = 0$ (b) $\alpha_2 T_0 = 0.5$

For more clarity, we now discuss the results in terms of the usual parameters R and M when both buoyancy and surface tension effects are present and convectionoccurs at zero wave number, that is, when a = 0. The neutral stability condition (23) may then be put in the form as

$$\frac{R}{R_c}+\frac{M}{M_c}=\frac{1}{(1-\alpha_2 T_0)}\left[1+\frac{Q}{30}\left\{\frac{\left(\Gamma+\frac{1}{8}\right)^2+\frac{1}{448}}{\left(\Gamma+\frac{1}{12}\right)^2+\frac{1}{180}}\right\}\right] \quad (28)$$

Where R_c = 320 is the critical value of the Rayleigh number in the absence of surface tension effect (when $\alpha_2 T_0$ = 0 and Q = 0) and M_c = 48 is the critical value of the Marangoni number in the absence of buoyancy effect (when $\alpha_2 T_0$ = 0 and

Q = 0). The numerical values of M/M_c (normalized) can be computed using the Eq. (28), for prescribed normalized values in R/R_c which value of R equals its critical value corresponding to given Γ, Q and $\alpha_2 T_0$ [cf. Table 3]. The normalized values of R and M for various values of Q when $\alpha_2 T_0$ = 0 and 0.5, are tabulated in Table 4.

TABLE 4

Normalised values of R and M for various values of Γ and Q when $\alpha_2 T_0$ = 0, and 0.5.

Γ	$\alpha_2 T_0 = 0$						$\alpha_2 T_0 = 0.5$					
	$Q = 0$		$Q = 10^2$		$Q = 10^6$		$Q = 0$		$Q = 10^2$		$Q = 10^6$	
	R/R_c	M/M_c	R/R_c	M/M_c	R/R_c	M/M_c	R/R_c	M/M_c	R/R_c	M/M_c	R/R_c	M/M_c
0	1.000	0.000	1.000	0.000	1.000	0.000	2.000	0.000	2.000	0.000	2.000	0.000
10^{-3}	0.993	0.007	0.994	0.009	0.994	0.010	1.987	0.013	1.988	0.018	1.988	0.019
10^{-2}	0.938	0.063	0.940	0.083	0.941	0.090	1.875	0.125	1.881	0.167	1.882	0.179
10^{-1}	0.600	0.400	0.573	0.508	0.567	0.540	1.200	0.800	1.145	1.015	1.134	0.080
0.5	0.231	0.769	0.192	0.850	0.184	0.874	0.462	1.539	0.383	1.699	0.367	1.748
1	0.130	0.870	0.104	0.920	0.098	0.935	0.261	1.740	0.208	1.840	0.196	1.870
10	0.015	0.985	0.011	0.992	0.010	0.993	0.030	1.970	0.022	1.983	0.021	1.987
10^2	0.002	0.999	0.001	0.999	0.001	0.999	0.003	1.997	0.002	1.998	0.002	1.999
10^3	0.000	1.000	0.000	1.000	0.000	1.000	0.000	2.000	0.000	2.000	0.000	2.000
10^6	0.000	1.000	0.000	1.000	0.000	1.000	0.000	2.000	0.000	2.000	0.000	2.000

For a fixed value of Q, we observe from Table 4 that M/M_c increases with decrease in R/R_c , indicating that the two agencies causing instability reinforce each other, irrespective of whether the layer of liquid is relatively cooler or hotter.The (R, M)-loci corresponding to neutral stability curves for the combined surface tension and buoyancy effects, normalized for critical values R_c and M_c for various values of Q when $\alpha_2 T_0$ = 0 and 0.5, are plotted in Fig. 4. The stable states correspond to the region $R < R_c$ and $M < M_c$. When Q = 0, the curves corresponding to $\alpha_2 T_0$ = 0 (dotted) and 0.5 (thick) in the (R , M) plane as shown in Fig 4 are straight lines represented by

$$\frac{R}{R_c}+\frac{M}{M_c}=\frac{1}{(1-\alpha_2 T_0)}\ . \quad (29)$$

This indicates that there is a maximum reinforcement between the two mechanisms causing instability and the coupling between the two mechanisms is perfect in the absence of magnetic field.

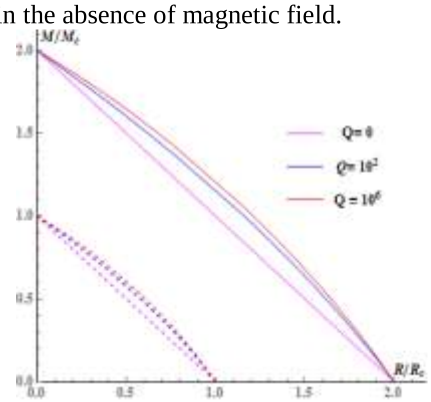


Fig.4. Variation of normalized Marangoni and Rayleigh numbers for various values of Q when $\alpha_2 T_0 = 0$ (dotted curves) and $\alpha_2 T_0 = 0.5$ (thick curve).

As Q increases the locus goes away from the line (29) , showing that the coupling between the two mechanisms causing instabilityremains no longer perfect and coupling between them becomes less tight. Also, Fig. 4 illustrates that a relatively hotter layer of liquid is more stable than a cooler one under identical conditions, irrespective of whether effect of the magnetic field is present or not.

5. Conclusions

The problem of the onset of surface tension and buoyancy driven thermal convection in a liquid layer heated from below in the presence of uniform vertical magnetic field has been studied theoretically, using the modified linear stability analysis. We conclude that

1. The increase in the magnetic field strength always has stabilizing effect on the onset of convection irrespective of whether the two mechanisms causing instability act individually or simultaneously.

2. For large value of the Chandrasekhar number, the asymptotic behavior ofthe critical Rayleigh number (in the absence of surface tension) as well as
the critical Marangoni number (in the absence of buoyancy) found to be significantly dependent on whether the layer of liquid is relatively hotter or cooler.

3. The two mechanisms causing instability reinforce each other and are perfectly coupled in the absence of magnetic field. For large value of the Chandrasekhar number, the coupling between the two mechanisms causing instability remains no longer perfect and it becomes less tight. This situation occurs when the horizontal wave number is zero.

4. The uniform vertical magnetic field suppresses convection relatively more effectively in a relatively hotter layer of liquid than the cooler one.

These qualitative as well as quantitative changes brought in the theory of surface tension and buoyancy driven convection are certainly significant and opens scope of being detected in the laboratory.

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