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# Some Results of Twin Prime

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**ABSTRACT---** ***If p and q are twin prime numbers and $p = q + 2$ then sum of p and q is always divisible by 12, when p+q≥12 .***

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## 1. INTRODUCTION

Since the dawn of mathematics the prime number over the year people have sought mainly in vain for patterns among the primes motivation many of these fundamental question. How many primes are there less than or equal to given number x and after operation are divisible are not. This paper focuses on the some proof for the sum of twin prime is divisible and this ideas and steps of the proof the given here.

## 2. MATERIALS AND METHOD

A number $p$ is said to be prime number if

(1). $p > 1$

(2). $p$ has no positive divisor except 1 and $p$

Twin prime: the prime pair $(p, q)$ are twin primes of the form $(p, p + 2)$ where $p$ and $q$ are prime $q = p + 2$

Triple prime: the triple prime$(p, q, r)$ are those primes triple of the form $(p, p + 2, p + 4)$ or $(p, p + 2, p + 6)$ where $p, q, r$ are prime

Quadratic equation: the name of quadratic comes from quad meaning square because the variable get squared like $x^2$ it is also called an equation of degree 2.

Standard form of quadratic equation $ax^2 + bx + c = 0$ where $a, b, c$ are known values $a$ can't be zero $x$ is the variable or unknown.

Formula; $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ x is called roots or sometimes "zeros"

In this formula $b^2 - 4ac$ is called discriminant because it can discriminate between the possible types of answer. when

$b^2 - 4ac$ is positive we get two real solution. When $b^2 - 4ac$ is zero we get just one real solution (both answer are same). When it is negative we get complex solution.

Mathematical induction: mathematical induction is a method of mathematical proof typically used to establish a given statement for all natural number. It is form of direct proof and it is done only in two steps the first step known as the base case is to prove the given statement for the first natural number the second step known as the inductive step is to prove that the given statement for anyone natural number.

1. Base case- prove that the statement holds for the first natural number $n = 1$
2. Inductive step- prove that if the statement holds for some natural number $n$ then the statement holds for $n + 1$

Divisibility: when we say $d$ divides $n$, $(d \mid n)$ whenever $n = cd$ for some $c$ we also say that $n$ is multiple of $d$ that $d$ is a divisor of $n$ or that $d$ is a factor of $n$

Divisibility establish a relation between any two integer with the following elementry properties

(a). $n \mid n$ (reflexive property)

(b). $d \mid n$ and $n \mid m$ implies $d \mid m$ (transitive property)

(c). $d \mid n$ and $d \mid m$ implies $d \mid (an + bm)$ (linearity property)

(d). $d \mid n$ implies $ad \mid an$ (multiplicative property)

(e). $ad \mid an$ and $a \neq 0$ implies $d \mid n$ (cancellation law)

(f). $1 \mid n$ (1 divide every integer)

(g). $n \mid 0$ (every integer divides zero)

(h). $0 \mid n$ implies $n = 0$ (zero divides only zero)

(i). $d \mid n$ and $n \neq 0$ implies $|d| \leq |n|$ (comparison property)

(j). $d \mid n$ and $n \mid d$ implies $|d| = |n|$

(k). $d \mid n$ and $d \neq 0$ implies $(n \mid d) \mid n$

Greatest common divisor (G.C.D): gcd of two or more integer when at least one of them is not zero is the largest positive integer that divides the numbers without a remainder ex. Gcd of 8 and 12 is 4.

Gcd is also known as the greatest common factor (gcf), highest common factor(hcf), greatest common measure (gcm), or highest common divisor.

Basic concept of congruency: if two numbers $a$ and $b$ have the property that their difference $a - b$ is integrally divisible by a number $m$ then the mathematically written as $a \equiv b(modm)$ the number $m$ is called modulus.

Definition: let $a$ be an integer and $n$ a positive integer greater than 1 we define a mod n to be the remainder $r$ when $a$ is divided to $n$ that is $r = a(\,mod\ n) = a - \left[\frac{a}{n}\right] n$.

We may also know that “$r$ is equal to a reduced modulo $n$".

Arithmetic function: in number theory an arithmetic function or arithmetical or number theoretic function is a real or complex valued function $f(n)$ defined on the set of positive integer.

An arithemetic function a is if $a(mn) = a(m) + a(n)$ for every natural number $m$ and $n$ is called completely additive.

If $a(mn) = a(m)a(n)$ for every natural number $m$ and $n$ is called completely multiplicative.

Function: $\tau(n)$ , $\varphi(n)$, $\sigma(n)$, are some function

$$\tau(n) = \sum_{d \mid n} 1$$

$$\varphi(n) = \sum_{1 \leq k < n} 1$$

gcd $(k, n) = 1$

$$\sigma(n) = \sum_{d|n} d$$

Notation: $\sum_p f(p)$ and $\prod_p f(p)$ mean that the sum or product is overall prime number.

$$\sum_p f(p) = f(2) + f(3) + f(5) + \cdots \dots \dots \dots$$

$$\prod_p f(p) = f(2)f(3)f(5) \dots \dots \dots \dots \dots \dots ..$$

similarly $\sum_{d|n} f(d)$ and $\prod_{d|n} f(d)$ mean that sum or product is over all positive divisor of $n$ including 1 and $n$

ex. If $n = 12$ then

$$\sum_{d|12} f(d) = f(1) + f(2) + f(3) + f(4) + f(6) + f(12)$$

$$\prod_{d|12} f(d) = f(1)f(2)f(3)f(4)f(6)f(12)$$

The notation can be combined;

$\sum_{p|n} f(p)$ and $\prod_{p|n} f(p)$ mean that the sum or product is over all prime divisor of $n$ ex. If $n = 18$

$$\sum_{p|18} f(p) = f(2) + f(3)$$

$$\prod_{p|18} f(p) = f(2)f(3)$$

## 3. RESULT AND CONCLUSION

Theorem 1.1- Twin prime always satisfy $x^2 - 12nx + 36n^2 - 1 = 0$

For some $n$ but converse is not true. That is if any positive integer satisfy this equation then it is not necessary to be twin prime number.

Proof-: we proof that sum of roots for this equation is always divible by 12 when n belongs to $N$.

First we show that basis step of induction that equation is true for n=1

Given quadractic equation $x^2 - 12nx + 36n^2 - 1 = 0$

Putting $n = 1$ in this equation

$x^2 - 12x + 35 = 0$

After solving this equation we have roots $x = 5,7$

$12 | 5 + 7$ always satisfy

Equation is true for $n = 1$, basis step holds.

Next we show that inductive step

Assume $n = k$, $x^2 - 12kx + 36k^2 - 1 = 0$

Assume $n = k$ holds it must be shown that $n = k + 1$ holds that is

For $n = k + 1$ $\quad x^2 - 12(k+1)x + 36(k+1)^2 - 1 = 0$

After that finding roots of this equation

Comparing this equation, $ax^2 + bx + c = 0$ $\quad a = 1, b = -12(k+1), c = 36(k+1)^2 - 1,$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{12(\mathrm{k}+1) \pm \sqrt{12(k+1)^2 - 4(1.36((k+1)^2 - 1)}}{2}$$

$$x = \frac{12(\mathrm{k}+1) \pm \sqrt{144(k+1)^2 - 144(\mathrm{k}+1)^2 + 4}}{2}$$

$$x = \frac{12(\mathrm{k}+1) \pm 2}{2} = 6(\mathrm{k}+1) \pm 1$$

Sum of these roots is also divisible by 12

$= 6(k+1) + 1 + 6(k+1) - 1$

$= 12(k+1)$

$n = k + 1$ ,is also holds that mean this equation is true for every positive integer

Note: Roots of this equation always twin prime if value of $n = \frac{p+q}{12}$ form where $p$ $and$ $q$ are twin prime ex. $n = 1,2,3,5,7,8,10,12,17,18,40,47,58,70,72,77,87,95,107,110,137,138,143,147, \ldots \ldots \ldots \ldots \ldots$

Theorem1.2-: All twin prime number is satisfy this equation

$p \equiv -q(mod12)$ where $p$ and $q$ are twin prime where $q = p + 2$ and $p = 6n - 1, n \in N.$

Proof-: Given $p$ and $q$ are twin prime and $p = 6n - 1, n \in N$ and $q = p + 2$

First we take $q = p + 2$ ———— — —❶

Put the value of $p$ in ❶

$q = (6n - 1) + 2$

$q = 6n + 1$ ————❷

given $\quad p = 6n - 1$ —————❸

adding equation ❷ and ❸

$p + q = 6n + 1 + 6n - 1$ $\quad p + q = 12n$ for every $n \in N,$

after that we write according to congruency rules

$p \equiv -q(mod12)$ $\quad$ proved

Theorem1.3-: If $p$and $q$ a twin prime then $\sum p_i + \sum q_i \equiv 2(mod 12)$

Where $p_i \; and q_i$ are divisor of $p$ and $q$ respectively.

Proof-: given $p$ and $q$ are twin prime so that $q = p + 2$

$p_i \; and q_i$ are divisor of $p$ and $q$ respectively so that

$p_i = 1 \; and \; p$ because $p$ is prime

$q_i = 1 \; and \; q$ because $q$ is prime

Taking equation

$$\sum p_i + \sum q_i \equiv 2(mod 12)$$

$$1 + p + 1 + q \equiv 2(mod 12)$$

$$p + q \equiv 0(mod 12)$$ proved

so that $p + q$ is always divible by 12 when $p$ and $q$ are twin prime number.

Theorem1.4-: if $p$ and $q$ a twin prime then $\sigma(p) + \sigma(q) - 2$ always divisible by 12 where $\sigma(n)$ sum of positive integer of $n$.

Proof-: given $p$ and $q$ are twin prime

We know that sum of twin prime is always divisible by 12

$\sigma(n) = \sum_{d|n} d$

$12 \mid \sigma(p) + \sigma(q) - 2$ ,

Prime number always have only two divisor 1 and itself

So that $\sigma(p) = 1 + p$ , $\sigma(q) = 1 + q$

$12 \mid \sigma(p) + \sigma(q) - 2$ $\quad 12 \mid 1 + p + 1 + q - 2$

$$12 \mid p + q$$ proved

We know that sum of any twin prime is always divisible by 12.

Corollary: $\varphi(p) + \varphi(q) + 2$ *is always divisible by* 12 *if* $p$ *and* $q$ *are prime*.

Proof-: by arithematic function we know that

$\varphi(m) = 0 < n \leq m$ where $(n, m) = 1$

And special case $\varphi(p)$=$p - 1$ where $p$ is prime

$\varphi(q) = q - 1$= where $q$ is prime

$12 \mid \varphi(p) + \varphi(q) + 2$

$12 \mid p - 1 + q - 1 + 2$

$12 \mid p + q$ proved

Corollary-: difference of $\tau(p)$ *and* $\tau(q)$ *is always zero* when $p$ and $q$ are prime.

Proof-: by arithematic function we know that

$\tau(m)$= number of divisors of $m$

If any number is prime then numbers of divisors always 2

So that $\tau(p) - \tau(q) = 0$ proved

## 4. REFERENCES

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