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Application of Hurwitz-Radon Matrices in Shape Coefficients

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ABSTRACT— ***Computer vision needs suitable methods of shape representation and contour reconstruction. Method of Hurwitz-Radon Matrices (MHR), invented and described by the author, is applied in reconstruction and interpolation of curves in the plane. Reconstructed curves represent the shape and contour of the object. Any point of the contour can be calculated by MHR method and then parameters of the object, used in shape coefficients, are computed: length of the contour, area of the object, Feret's diameters. Proposed method is based on a family of Hurwitz-Radon (HR) matrices. The matrices are skew-symmetric and possess columns composed of orthogonal vectors. The operator of Hurwitz-Radon (OHR), built from these matrices, is described. The shape is represented by the set of nodes. It is shown how to create the orthogonal and discrete OHR and how to use it in a process of shape representation and reconstruction. MHR method is interpolating the curve point by point without using any formula or function.***

Keywords— shape coefficients, curve interpolation, contour reconstruction, length estimation, area estimation, Hurwitz-Radon matrices.

1. INTRODUCTION

A significant problem in object recognition and computer vision [1] is that of appropriate shape representation and reconstruction. Classical discussion about shape representation is based on the problem: contour versus skeleton. This paper is voting for contour which forms boundary of the object. Contour of the object, represented by contour points, consists of information which allows us to describe many important features of the object as the shape coefficients [2].

A digital curve (open or closed) may be represented by chain code (Freeman's code). Chain code depends on selection of the started point and transformations of the object. So Freeman's code is one of the method how to describe and to find contour of the object. An analog (continuous) version of Freeman's code is the curve α - s. Another contour representation and reconstruction is based on the Fourier coefficients calculated in Discrete Fourier Transformation (DFT). These coefficients are used to fix similarity of the contours with different sizes or directions. If we assume that a contour is built from the segments of a line and fragments of circles or ellipses, Hough transformation is applied to detect the contour lines. Also geometrical moments of the object are used during the process of object shape representation [3]. MHR method requires to detect specific points of the object contour, for example in compression and reconstruction of monochromatic medical images [4]. Contour is also applied in the shape decomposition [5]. Many branches of medicine, for example computed tomography [6], need suitable and accurate methods of contour reconstruction [7]. Also industry and manufacturing are looking for the methods connected with geometry of the contour [8]. So suitable shape representation and precise reconstruction or interpolation [9] of the object contour is a key factor in many applications of computer analysis and image processing.

2. CONTOUR REPRESENTATION

The shape can be represented by the object contour, i.e. curves that create each part of the contour. One curve is described by the set of nodes (xi,yi) $\in$ R2 (contour points) as follows in proposed method:

1. nodes (interpolation points) are settled at local extrema (maximum or minimum) of one of coordinates and at least one point between two successive local extrema;
2. nodes (x_i,y_i) are monotonic in coordinates x_i ($x_i < x_{i+1}$ for all i) or y_i ($y_i < y_{i+1}$);
3. one curve (one part of the contour) is represented by at least five nodes.

Condition 1 is done for the most appropriate description of a curve. So we have m curves C_1, C_2, ... C_m that build whole contour and each curve is represented by the nodes according to assumptions 1-3.

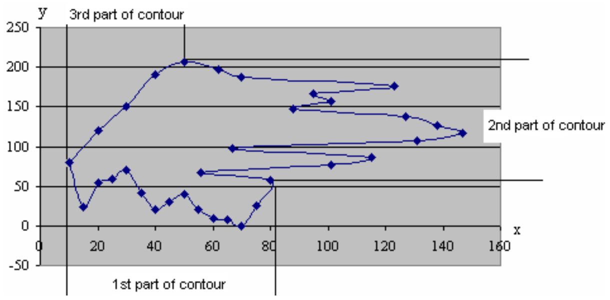


Fig. 1. A contour consists of three parts (three curves and their nodes).

Fig. 1 is an example for m = 3: first part of the contour C1 is represented by the nodes monotonic in coordinates xi, second part of the contour C2 is represented by the nodes monotonic in coordinates yi and third part C3 could be represented by the nodes either monotonic in coordinates xi or monotonic in coordinates yi. Number of the curves is optional and number of the nodes for each curve is optional too (but at least five nodes for one curve). Representation points are treated as interpolation nodes. How accurate can we reconstruct whole contour using representation points? The contour reconstruction is possible using novel MHR method.

3. CONTOUR RECONSTRUCTION

The following question is important in mathematics and computer sciences: is it possible to find a method of curve interpolation in the plane without building the interpolation polynomials or other functions? Our paper aims at giving the positive answer to this question. In comparison MHR method with Bézier curves, Hermite curves and B-curves (B-splines) or NURBS one unpleasant feature of these curves must be mentioned: a small change of one characteristic point can make big change of whole reconstructed curve. Such a feature does not appear in MHR method. The methods of curve interpolation based on classical polynomial interpolation: Newton, Lagrange or Hermite polynomials and the spline curves which are piecewise polynomials [10]. Classical methods are useless to interpolate the function that fails to be differentiable at one point, for example the absolute value function f(x) = |x| at x=0. If point (0;0) is one of the interpolation nodes, then precise polynomial interpolation of the absolute value function is impossible. Also when the graph of interpolated function differs from the shape of polynomials considerably, for example f(x) = 1/x, interpolation is very hard because of existing local extrema of polynomial. Lagrange interpolation polynomial for function f(x) = 1/x and nodes (5;0.2), (5/3;0.6), (1;1), (5/7;1.4), (5/9;1.8) has one minimum and two roots.

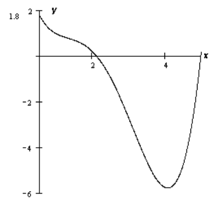


Fig. 2. Lagrange interpolation polynomial for nodes (5;0.2), (5/3;0.6), (1;1), (5/7;1.4), (5/9;1.8) differs extremely from the shape of function $f(x) = 1/x$.

We cannot forget about the Runge's phenomenon: when the interpolation nodes are equidistance then high-order polynomial oscillates toward the end of the interval, for example close to -1 and 1 with function f(x) = 1/(1+25x2) [11]. Method of Hurwitz – Radon Matrices (MHR), described in this paper, is free of these bad features. The curve or function in MHR method is parameterized for value $\alpha \in [0;1]$ in the range of two successive interpolation nodes.

3.1 The Operator of Hurwitz-Radon

Adolf Hurwitz (1859-1919) and Johann Radon (1887-1956) published the papers about specific class of matrices in 1923, working on the problem of quadratic forms. Matrices Ai, i = 1,2…m satisfying

$$A_jA_k+A_kA_j = 0,\ A_j^2 = -I \text{ for } j \neq k;\ j,\ k = 1,2...m$$

are called *a family of Hurwitz - Radon matrices*. A family of Hurwitz - Radon (HR) matrices has important features [12]: HR matrices are skew-symmetric ($A_i^T = -A_i$) and reverse matrices are easy to find ($A_i^{-1} = -A_i$). Only for dimension $N = 2$, 4 or 8 the family of HR matrices consists of $N - 1$ matrices. For $N = 2$ we have one matrix:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

For $N = 4$ there are three HR matrices with integer entries:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix},\quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix},\quad A_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}.$$

For $N = 8$ we have seven HR matrices with elements 0, ±1 [4].

So far HR matrices are applied in electronics [13]: in Space-Time Block Coding (STBC) and orthogonal design [14], also in signal processing [15] and Hamiltonian Neural Nets [16].

If one curve is described by a set of representation points $\{(x_i,y_i),\ i = 1, 2, \ldots, n\}$ monotonic in coordinates x_i, then HR matrices combined with the identity matrix I_N are used to build the orthogonal and discrete Hurwitz - Radon Operator (OHR). For nodes (x_1,y_1), (x_2,y_2) OHR M of dimension $N = 2$ is constructed:

$$B = (x_1 \cdot I_2 + x_2 \cdot A_1)(y_1 \cdot I_2 - y_2 \cdot A_1) = \begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix}\begin{bmatrix} y_1 & -y_2 \\ y_2 & y_1 \end{bmatrix},\quad M = \frac{1}{x_1^2 + x_2^2}B,$$

$$M = \frac{1}{x_1^2 + x_2^2}\begin{bmatrix} x_1y_1 + x_2y_2 & x_2y_1 - x_1y_2 \\ x_1y_2 - x_2y_1 & x_1y_1 + x_2y_2 \end{bmatrix}. \quad \textbf{(1)}$$

Matrix M in (1) is found as a solution of equation:

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}. \quad \textbf{(2)}$$

For nodes (x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4), monotonic in x_i, OHR of dimension $N = 4$ is constructed:

$$M = \frac{1}{x_1^2 + x_2^2 + x_3^2 + x_4^2}\begin{bmatrix} u_0 & u_1 & u_2 & u_3 \\ -u_1 & u_0 & -u_3 & u_2 \\ -u_2 & u_3 & u_0 & -u_1 \\ -u_3 & -u_2 & u_1 & u_0 \end{bmatrix} \quad \textbf{(3)}$$

where

$$u_0 = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4,\quad u_1 = -x_1y_2 + x_2y_1 + x_3y_4 - x_4y_3,$$

$$u_2 = -x_1y_3 - x_2y_4 + x_3y_1 + x_4y_2,\quad u_3 = -x_1y_4 + x_2y_3 - x_3y_2 + x_4y_1.$$

Matrix M in (3) is found as a solution of equation:

$$\begin{bmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}. \quad \textbf{(4)}$$

For nodes (x_1,y_1), (x_2,y_2), …, (x_8,y_8), monotonic in x_i, OHR of dimension $N = 8$ is built [17] similarly as (1) or (3). Note that OHR operators M (1)-(3) satisfy the condition of interpolation

$$M \cdot \mathbf{x} = \mathbf{y} \quad \textbf{(5)}$$

for $\mathbf{x} = (x_1,x_2\ldots,x_N)^T \in \boldsymbol{R}^N$, $\mathbf{x} \neq \mathbf{0}$, $\mathbf{y} = (y_1,y_2\ldots,y_N)^T \in \boldsymbol{R}^N$, $N = 2$, 4 or 8.

If one curve is described by a set of nodes $\{(x_i,y_i),\ i = 1, 2, \ldots, n\}$ monotonic in coordinates y_i, then HR matrices combined with the identity matrix I_N are used to build the orthogonal and discrete reverse Hurwitz - Radon Operator (reverse OHR)
M^{-1}. If matrix M in (1)-(3) is described as:

$$M = \frac{1}{\sum_{i=1}^{N} x_i^2}(u_0 \cdot I_N + D),$$

where D with elements u_1, …, u_{N-1}, then reverse OHR M^{-1} is given by:

$$M^{-1} = \frac{1}{\sum_{i=1}^{N} y_i^2}(u_0 \cdot I_N - D). \tag{6}$$

Note that reverse OHR operator (6) satisfies the condition of interpolation

$$M^{-1} \cdot \mathbf{y} = \mathbf{x} \tag{7}$$

for $\mathbf{x} = (x_1,x_2\ldots,x_N)^T \in \boldsymbol{R}^N$, $\mathbf{y} = (y_1,y_2\ldots,y_N)^T \in \boldsymbol{R}^N$, $\mathbf{y} \neq \mathbf{0}$, $N = 2$, 4 or 8.

3.2 Method of Hurwitz-Radon Matrices

Key question looks as follows: how can we compute coordinates of points settled between the interpolation nodes? On a segment of a line every number "c" situated between "a" and "b" is described by a linear (convex) combination $c=\alpha \cdot a+(1 - \alpha) \cdot b$ for

$$\alpha = \frac{b-c}{b-a} \in [0;1]. \tag{8}$$

When the nodes are monotonic in coordinates x_i, the average OHR operator M_2 of dimension $N = 2$, 4 or 8 is constructed as follows:

$$M_2 = \alpha \cdot M_0 + (1-\alpha) \cdot M_1 \tag{9}$$

with the operator M_0 built (1)-(3) by "odd" nodes $(x_1=a,y_1)$, (x_3,y_3), …, (x_{2N-1},y_{2N-1}) and M_1 built (1)-(3) by "even" nodes $(x_2=b,y_2)$, (x_4,y_4), …, (x_{2N},y_{2N}). Having the operator M_2 for coordinates $x_i < x_{i+1}$ it is possible to reconstruct the second coordinates of points (x,y) in terms of the vector C defined with

$$c_i = \alpha \cdot x_{2i-1} + (1-\alpha) \cdot x_{2i} \quad , \quad i = 1, 2,\ldots, N \tag{10}$$

as $C = [c_1, c_2,\ldots, c_N]^T$. The required formula is similar to (5):

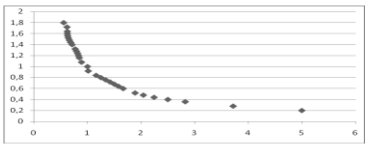
$$Y(C) = M_2 \cdot C \tag{11}$$

in which components of vector $Y(C)$ give the second coordinate of the points (x,y) corresponding to the first coordinate, given in terms of components of the vector C.

On the other hand, having the operator M_2^{-1} for coordinates $y_i < y_{i+1}$ it is possible to reconstruct the first coordinates of points (x,y):

$$M_2^{-1} = \alpha \cdot M_0^{-1} + (1-\alpha) \cdot M_1^{-1}, \quad c_i = \alpha \cdot y_{2i-1} + (1-\alpha) \cdot y_{2i} \, ,$$
$$X(C) = M_2^{-1} \cdot C \quad . \tag{12}$$

Contour of the object is constructed with several number of curves. Calculation of unknown coordinates for contour points using (8)-(12) is called by author the method of Hurwitz - Radon Matrices (MHR). Here is the application of MHR method for functions $f(x) = 1/x$ (nodes as Fig. 2) and $f(x) = 1/(1+25x^2)$ with five nodes equidistance in first coordinate: $x_i = -1, -0.5, 0, 0.5, 1$.

a) 

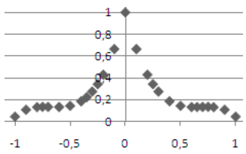
b) 

Fig. 3. Twenty six interpolated points of functions $f(x)=1/x$ (a) and $f(x) = 1/(1+25x^2)$ (b) using MHR method with 5 nodes.

MHR interpolation for function $f(x) = 1/x$ gives better result then Lagrange interpolation (Fig. 2). The same can be said for function $f(x) = 1/(1+25x^2)$.

4. SHAPE COEFFICIENTS

Shape coefficients are the parameters that characterizing and describing the shape of the object. Most of the shape coefficients are calculated using area of the object S and length of the contour L. For example [18]:

1. coefficient R_S

$$R_S = \frac{L^2}{4\pi S},$$

2. coefficient of Feret R_F

$$R_F = \frac{L_h}{L_v},$$

3. coefficients R_{C1} and R_{C2}

$$R_{C1} = 2\sqrt{\frac{S}{\pi}}, \qquad R_{C2} = \frac{L}{\pi},$$

4. coefficient of Malinowska R_M

$$R_M = \frac{L}{2\sqrt{\pi S}} - 1,$$

5. coefficient of Blair-Bliss R_B

$$R_B = \frac{S}{\sqrt{2\pi \sum_i r_i^2}},$$

6. coefficient of Danielsson R_D

$$R_D = \frac{S^3}{\left(\sum_i l_i\right)^2},$$

7. coefficient of Haralick R_H

$$R_H = \sqrt{\frac{\left(\sum_i d_i\right)^2}{c \cdot \sum_i d_i^2 - 1}},$$

8. coefficient of compactness R_C

$$R_C = \frac{S}{S_p},$$

9. coefficient R_{min}/R_{max}

$$\sqrt{\frac{R_{\min}}{R_{\max}}},$$

where:
R_{min} – minimal distance of object contour to center of gravity,
R_{max} – maximal distance of object contour to center of gravity,
S_p – minimal area of the rectangle covering the object,
L_h – maximal horizontal diameter of the object (horizontal Feret's diameter),
L_v – maximal vertical diameter of the object (vertical Feret's diameter),
r_i – distance of object pixel to center of gravity,
i – number of object pixel,
l_i – minimal distance of object pixel to object contour,
d_i – distance of contour pixel to center of gravity,
c – number of contour pixels.

4.1 Length of the contour

The contour is divided into m curves C_1, C_2, ... C_m. Having nodes (x_1,y_1), (x_2,y_2),…, (x_n,y_n) for each C_i in MHR method, it is possible to compute as many curve points as we want for any parameter $\alpha \in [0;1]$ (8). Assume that k is the number of reconstructed points p together with n nodes ($k = n + p$). So a curve C_i consists of k points that are indexed (x_1',y_1'), (x_2',y_2'),…, (x_k',y_k'), where $(x_1',y_1') = (x_1,y_1)$ and $(x_k',y_k') = (x_n,y_n)$. The length of a curve C_i, consists of k points, is estimated:

$$d(C_i) = \sum_{i=1}^{k-1} \sqrt{(x_{i+1}' - x_i')^2 + (y_{i+1}' - y_i')^2} \,. \qquad \textbf{(13)}$$

Length of whole contour L is computed:

$$L = d(C_1) + d(C_2) + \ldots + d(C_m). \qquad \textbf{(14)}$$

Two examples of estimation a length of the curve via MHR method.
Example 1 The graph of function $f(x) = 1/(1+5x^2)$ reconstructed via MHR method (8)-(11) for $N = 2$ with nodes $x = -1.0, -0.5, 0, 0.5, 1.0$ ($n = 5$) and calculated points $p = 36$ looks as follows (the curve is described by $k = 41$ points):

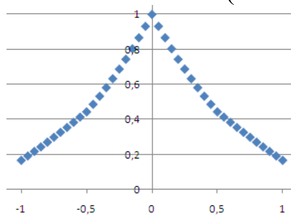


Fig. 4. The curve y = $1/(1+5x^2)$ reconstructed by MHR method for five nodes and 36 calculated points

Length of the curve characterized on Fig.4 and estimated by (13) is $d(C) = 2.643$ whereas precise length is $d(f) = 2.679$. There is no Runge phenomenon on Fig.4 and MHR method preserves the symmetry of the curve.
Example 2 The graph of function $f(x) = 2/x$ reconstructed via MHR (8)-(11) for $N = 2$ with nodes $x = 0.4, 0.7, 1.0, 1.3, 1.6$ ($n = 5$) and calculated points $p = 36$ looks as follows (the curve is described by $k = 41$ points):

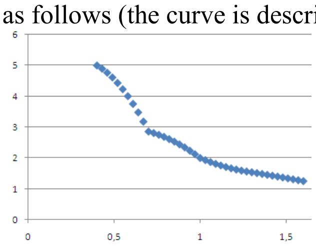


Fig. 5. The curve $y = 2/x$ reconstructed by MHR method for five nodes and 36 calculated points

Length of the curve characterized on Fig.5 and estimated by (13) is $d(C) = 4.050$ whereas precise length is $d(f) = 4.045$.

4.2 Area of the object

Area of the object can be divided horizontally or vertically (Fig.6) into the set of l polygons: triangles and quadrangles (squares, rectangles, trapezoids, rhombuses, parallelograms).

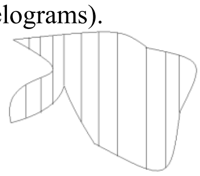


Fig. 6. The object area consists of polygons.

The coordinates of corners for each polygon P_i are calculated by MHR method and then it is easy to estimate the area of P_i. For example P_1 as a trapezoid with the corners (x_1,y_1), (x_1,y_2), (x_2,y_3), (x_2,y_4):

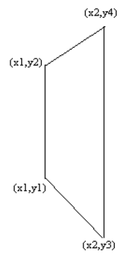


Fig. 7. Trapezoid as a part of the object.

Area of a trapezoid P_1 is computed:

$$s(P_1) = \frac{1}{2}|x_2 - x_1| \cdot (|y_2 - y_1| + |y_4 - y_3|) \cdot$$

It is easy to compute the area of other polygons with given corners: for example a triangle P_2 with sides a, b, c and $p = (a+b+c)/2$:

$$s(P_2) = \sqrt{p(p-a)(p-b)(p-c)}$$

and a rhombus P_3 with diagonals d_1 and d_2:

$$s(P_3) = \frac{1}{2} d_1 \cdot d_2 \cdot$$

Estimation of the object area S is given by a formula:

$$S = \sum_{i=1}^{l} s(P_i). \tag{15}$$

Feret's diameters (horizontal L_h and vertical L_v) are also possible to calculate having a contour of the object. Contour points, computed by MHR method [19], are applied in shape coefficients.

5. CONCLUSIONS

The method of Hurwitz-Radon Matrices leads to contour interpolation and shape reconstruction depending on the number and location of contour points. No characteristic features of the curve, significant for classical polynomial interpolations or Bezier curves and NURBS, are important in MHR method. MHR gives the possibility of reconstruction a curve consists of several parts, for example closed curve (contour). The only condition is to have a set of nodes for each part of a curve or contour according to assumptions in MHR method. Any point of the contour can be calculated by MHR method and then parameters of the object used in shape coefficients are computed. Contour representation and curve reconstruction by MHR method is connected with possibility of changing the nodes coordinates and reconstruction of new curve or contour for new set of nodes, no matter what shape of curve or contour is to be reconstructed. Main features of MHR method are: accuracy of shape reconstruction depending on number of nodes and method of choosing nodes; reconstruction of curve consists of L points is connected with the computational cost of rank O(L) [19]; MHR method is dealing with local operators: average OHR operators are built by successive 4, 8 or 16 nodes, what is connected with smaller computational costs then using all nodes; MHR is not an affine interpolation [19].

Future works are connected with: geometrical transformations of contour (translations, rotations, scaling)- only nodes are transformed and new curve (for example contour of the object) for new nodes is reconstructed, possibility to apply MHR method to three-dimensional curves and connection MHR method with object recognition.

6. REFERENCES

1. Ballard, D.H.: Computer Vision. Prentice Hall, New York (1982)
2. Tadeusiewicz, R., Flasiński, M.: Image Recognition. PWN, Warsaw (1991)
3. Choraś, R.S.: Computer Vision. Exit, Warsaw (2005)
4. Jakóbczak, D., Kosiński, W.: Application of Hurwitz - Radon Matrices in Monochromatic Medical Images Decompression. In: Kowalczuk, Z., Wiszniewski, B. (eds.) Intelligent Data Mining in Diagnostic Purposes: Automatics and Informatics, pp. 389-398, PWNT, Gdansk (2007)
5. Latecki, L.J., Lakaemper, R.: Convexity Rule for Shape Decomposition Based on Discrete Contour Evolution. Computer Vision and Image Understanding 3(73), 441-454 (1999)
6. Cierniak, R.: Computed Tomography. Exit, Warsaw (2005)
7. Soussen, C., Mohammad-Djafari, A.: Polygonal and Polyhedral Contour Reconstruction in Computed Tomography. IEEE Transactions on Image Processing 11(13), 1507-1523 (2004)
8. Tang, K.: Geometric Optimization Algorithms in Manufacturing. Computer – Aided Design & Applications 2(6), 747-757 (2005)
9. Kozera, R.: Curve Modeling via Interpolation Based on Multidimensional Reduced Data. Silesian University of Technology Press, Gliwice (2004)
10. Dahlquist, G., Bjoerck, A.: Numerical Methods. Prentice Hall, New York (1974)
11. Ralston, A.: A First Course in Numerical Analysis. McGraw-Hill Book Company, New York (1965)
12. Eckmann, B.: Topology, Algebra, Analysis- Relations and Missing Links. Notices of the American Mathematical Society 5(46), 520-527 (1999)

13. Citko, W., Jakóbczak, D., Sieńko, W.: On Hurwitz - Radon Matrices Based Signal Processing. Workshop Signal Processing at Poznan University of Technology (2005)
14. Tarokh, V., Jafarkhani, H., Calderbank, R.: Space-Time Block Codes from Orthogonal Designs. IEEE Transactions on Information Theory 5(45), 1456-1467 (1999)
15. Sieńko, W., Citko, W., Wilamowski, B.: Hamiltonian Neural Nets as a Universal Signal Processor. 28th Annual Conference of the IEEE Industrial Electronics Society IECON (2002)
16. Sieńko, W., Citko, W.: Hamiltonian Neural Net Based Signal Processing. The International Conference on Signal and Electronic System ICSES (2002)
17. Jakóbczak, D.: 2D and 3D Image Modeling Using Hurwitz-Radon Matrices. Polish Journal of Environmental Studies 4A(16), 104-107 (2007)
18. Tadeusiewicz, R., Korohoda, P.: Computer Analysis and Image Processing. FPT, Cracow (1997)
19. Jakóbczak, D.: Curve Interpolation Using Hurwitz-Radon Matrices. Polish Journal of Environmental Studies 3B(18), 126-130 (2009)