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Lagrange Formalism for Neutrino Field in Terms of Complex Isotropic Vectors

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ABSTRACT--- ***In previous works, Weyl´s equation for neutrino has been written in tensor form, in the form of non-linear Maxwell´s like equations, through isotropic complex vector $\vec{F} = \vec{E} + i\vec{H}$. It has been proved, that the complex vector $\vec{F} = \vec{E} + i\vec{H}$ satisfies non-linear condition $\vec{F}^2 = 0$, equivalent to two conditions for real quantities $\vec{E}^2 - \vec{H}^2 = 0$ and $\vec{E}.\vec{H} = 0$, obtained by separating real and imaginary parts in the equality $\vec{F}^2 = 0$. In this work, in the development of this new tensor formalism for description of fermions, we elaborated the Lagrange formalism for neutrino field in tensor formalism, in terms of complex isotropic vectors.***

Keywords--- Neutrino field, Lagrange formalism, tensor, complex isotropic vector

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1. INTRODUCTION

In previous works, using different methods, in particular, via Cartan map, Weyl´s equation for neutrino has been written in tensor form, in the form of non-linear Maxwell´s like equations through complex isotropic vector $\vec{F} = \vec{E} + i\vec{H}$, satisfying non-linear condition $\vec{F}^2 = 0$. The last condition is equivalent to two conditions for real quantities $\vec{E}^2 - \vec{H}^2 = 0$ and $\vec{E}.\vec{H} = 0$, obtained by equating to zero separately real and imaginary parts in equality $\vec{F}^2 = 0$. It has been proved, that the vectors $\vec{E}$ and $\vec{H}$ have the same properties as the vectors $(\vec{E}, \vec{H})$, components of electromagnetic field. For example, under Lorentz relativistic transformations, they are transformed as components of electric and magnetic fields. In addition, it has been proved, that the solution of these non-linear equations for free particle as well fulfils Maxwell´s equations for vacuum (with zero at the right side).

In this work, in the development of this new tensor formalism for description of fermions and fermions fields, we shall elaborate the Lagrange formalism for neutrino field in tensor formalism. We shall find the Lagrange function for the neutrino field in terms of complex isotropic vectors and from this Lagrange function we shall derive expressions for fundamental dynamical variables (energy, momentum, charge, spin) and we shall express them in terms of strengths $\vec{E}$ and $\vec{H}$.

2. RESEARCH METHOD

In previous works, using Cartan map, Weyl´s equation for neutrino has been written in tensor form, in the form of non-linear Maxwell's like equations through complex isotropic vector $\vec{F} = \vec{E} + i\vec{H}$. In this work, using the same method, based on Cartan map, we shall elaborate the Lagrange formalism for neutrino field in tensor formalism, in terms of complex isotropic vectors.

3. VECTOR WAVE EQUATION FOR NEUTRINOS

Weyl's equation for neutrino has the form

$$p_0\xi = (\vec{p}.\vec{\sigma})\xi . \quad (1)$$

With the help of Cartan map, spinor ξ is mapped on isotropic complex vector $\vec{F}=\vec{E}+i\vec{H}$, satisfying non-linear condition $\vec{F}^2=0$. Where $\vec{E}$ and $\vec{H}$ are components of the tensor $F_{\mu\nu}$ of ″electromagnetic field″.

With the help of complex isotropic vector $\vec{F}$, we can also define a current four-vector

$$J_\mu=\begin{bmatrix} J_0 \\ \vec{J} \end{bmatrix}=\begin{bmatrix} |\vec{E}| \\ \frac{\vec{E}\times\vec{H}}{|\vec{E}|} \end{bmatrix}, \tag{2}$$

representing the density of the current for neutrino.

Using Cartan map, Weyl's equation (1) has been written in vector form as follows

$$D^0\vec{F}=i\,\vec{D}\times\vec{F}-(\vec{D}F_i)v_i\,, \tag{3}$$

where

$$D^0=i\,\frac{\hbar}{2}\frac{\partial}{\partial t}\,, \tag{4}$$

$$\vec{D}=-i\,\frac{\hbar}{2}\vec{\nabla}\,, \tag{5}$$

$$\vec{v}=\frac{\vec{J}}{J_0}=\frac{\vec{E}\times\vec{H}}{|\vec{E}|^2}\,. \tag{6}$$

Equation (3), written through $\vec{E}$ and $\vec{H}$, can be represented in the form of a system of non-linear Maxwell's like equations

$$\begin{cases} \mathrm{rot}\vec{E}+\frac{\partial\vec{H}}{\partial t}=v_i(\vec{\nabla}H_i) \\ \mathrm{rot}\vec{H}-\frac{\partial\vec{E}}{\partial t}=-v_i(\vec{\nabla}E_i) \end{cases}. \tag{7}$$

Here and in the following we shall use the natural system of units in which c=ħ=1.

In the notations of complex isotropic vectors, Maxwell's equations for vacuum take the form

$$\begin{cases} D^0\vec{F}=i\,\vec{D}\times\vec{F} \\ \vec{D}\vec{F}=0 \end{cases}. \tag{8}$$

However, in the general case, the solution of Maxwell's equations does not satisfy isotropic condition $\vec{F}^2=0$, whereas the solution of Weyl's equation (3) always satisfies this condition.

4. LAGRANGE FORMALISM FOR NEUTRINO FIELD IN TERMS OF COMPLEX ISOTROPIC VECTORS

In spinor formalism, Weyl's equation for neutrino can be derived by variation principle from Lagrange function

$$L=\frac{i}{2}\left(\xi\sigma^\mu\,\partial_\mu\xi^*-\partial^\mu\xi\sigma_\mu\xi^*\right). \tag{9}$$

Transforming formula (9) according to Cartan map, we obtain

$$L=\frac{1}{2}\{[D_0\vec{F}-i\vec{D}\times\vec{F}+v_i(\vec{D}F_i)]\vec{F}^*-[D_0\vec{F}^*+i\vec{D}\times\vec{F}^*+v_i(\vec{D}F_i^*)]\vec{F}\}/(\vec{F}\vec{F}^*/2)^{1/2}. \tag{10}$$

Formula (10), written through components of vectors $\vec{F}$ and $\vec{F}^*$takes the form

$$L = \frac{i}{4}\left\{\left[\frac{\partial F_i}{\partial t} + i\varepsilon_{ijk}\frac{\partial F_k}{\partial x_j} - v_j\left(\frac{\partial F_j}{\partial x_i}\right)\right] F_i^* - \left[\frac{\partial F_i^*}{\partial t} - i\varepsilon_{ijk}\frac{\partial F_k^*}{\partial x_j} - v_j\left(\frac{\partial F_j^*}{\partial x_i}\right)\right] F_i\right\}/(F_i F_i^*/2)^{1/2}. \quad (11)$$

In calculating variations in formula (11), expression $\left(\vec{F}\vec{F}^*/2\right)^{1/2}$ will be considered as a constant.

It is well known, that Euler-Lagrange equations have the form

$$\frac{\partial}{\partial x^\mu}\left[\frac{\partial L}{\partial \vec{F}^*,_\mu}\right] - \frac{\partial L}{\partial \vec{F}^*} = 0\ , \quad (12)$$

$$\frac{\partial}{\partial x^\mu}\left[\frac{\partial L}{\partial \vec{F},_\mu}\right] - \frac{\partial L}{\partial \vec{F}} = 0\ . \quad (13)$$

For differentials over fields we obtain expressions

$$\frac{\partial L}{\partial F_i^*} = \frac{i}{4}\left[\frac{\partial F_i}{\partial t} + i\varepsilon_{ijk}\frac{\partial F_k}{\partial x_j} - v_j\left(\frac{\partial F_j}{\partial x_i}\right)\right]/(F_i F_i^*/2)^{1/2}\ , \quad (14)$$

$$\frac{\partial}{\partial t}\left[\frac{\partial L}{\partial F_i^*,_0}\right] = -\frac{i}{4}\frac{\partial F_i}{\partial t}/(F_i F_i^*/2)^{1/2}\ , \quad (15)$$

$$\frac{\partial}{\partial x_j}\left[\frac{\partial L}{\partial F_i^*,_j}\right] = -\frac{i}{4}\left[\frac{\partial F_i}{\partial t} + i\varepsilon_{ijk}\frac{\partial F_k}{\partial x_j} - v_j\left(\frac{\partial F_j}{\partial x_i}\right)\right]/(F_i F_i^*/2)^{1/2} = 0. \quad (16)$$

Replacing formulas (14)-(16) in formula (12), we obtain equation

$$D_0\vec{F} - i\vec{D}\times\vec{F} + v_i\left(\vec{D}F_i\right) = 0\ , \quad (17)$$

coinciding with Weyl's equation in tensor form (3).

Similarly, varying over field $\vec{F}$ and its derivatives, we find equation for the complex conjugate vector $\vec{F}^*$

$$D_0\vec{F}^* + i\vec{D}\times\vec{F}^* + v_i\left(\vec{D}F_i^*\right)\ . \quad (18)$$

5. PRINCIPAL DYNAMICAL VARIABLES

Using Noether's theorem, we can derive from the Lagrange function (10), expressions for fundamental physical dynamical variables, conserved in time.

Energy is determined by the formula

$$E = \int T^{00} d^3x\ , \quad (19)$$

where

$$T^{00} = \frac{\partial L}{\partial \vec{F},0}\vec{F},0 + \frac{\partial L}{\partial F^*,0}\vec{F}^*,0\ . \quad (20)$$

Replacing formula (10) into formula (20), we obtain

$$T^{00} = \frac{i}{4}\left[\vec{F}^*\frac{\partial \vec{F}}{\partial t} - \vec{F}\frac{\partial \vec{F}^*}{\partial t}\right]/\left(\vec{F}\vec{F}^*/2\right)^{1/2}\ . \quad (21)$$

With consideration of expression

$$\vec{F} = \left(\vec{E}^0 + i\vec{H}^0\right) e^{-2i\varepsilon kt + 2i\vec{k}\vec{r}}\ , \quad (22)$$

we find

$$T^{00} = \varepsilon k\,|\vec{E}|\,. \tag{23}$$

Similarly, for momentum we have

$$P^j = \int T^{0j} d^3x\,, \tag{24}$$

where

$$T^{0j} = \frac{\partial L}{\partial \vec{F}_{,0}} \vec{F}_{,j} + \frac{\partial L}{\partial F^*_{,0}} \vec{F}^*_{,j}\,. \tag{25}$$

Replacing expression (10) in formula (25), we find

$$T^{0j} = \frac{i}{4}\left[\left(\vec{F}\vec{\nabla}_j\vec{F}^*\right) - \left(\vec{F}^*\vec{\nabla}_j\vec{F}\right)\right]/\left(\vec{F}\vec{F}^*/2\right)^{1/2}\,. \tag{26}$$

With consideration of expression (22), we obtain

$$\vec{P} = \vec{k}\,|\vec{E}|\,. \tag{27}$$

For charge, we have

$$Q = \int j^0 d^3x\,, \tag{28}$$

where

$$j^0 = i\left(\vec{F}^* \frac{\partial L}{\partial \vec{F}^*_{,0}} - \frac{\partial L}{\partial \vec{F}_{,0}} \vec{F}\right). \tag{29}$$

Using formula (10), we obtain

$$j^0 = \frac{1}{4}\left(\vec{F}\vec{F}^* + \vec{F}^*\vec{F}\right)/\left(\vec{F}\vec{F}^*/2\right)^{1/2}\,. \tag{30}$$

Replacing formula (22) into formula (30), we find

$$j^0 = |\vec{E}|\,. \tag{31}$$

The density of the spin pseudo vector is determined by the formula

$$S_k = \frac{1}{2}\varepsilon_{ijk}\, S_{lm}, \tag{32}$$

where

$$S^0_{lm} = -\frac{\partial L}{\partial F_{i,0}} F_j A^j_{i,lm} - \frac{\partial L}{\partial F^*_{i,0}} F^*_j A^j_{i,lm}\,. \tag{33}$$

Here

$$A^j_{i,lm} = g_{il}\delta^j_m - g_{im}\delta^j_l\,. \tag{34}$$

Replacing formula (10) and formula (34) in formula (33), we find

$$S^0_{lm} = \frac{i}{4}\left(F_l F^*_m - F_m F^*_l\right)/\left(\vec{F}\vec{F}^*/2\right)^{1/2}\,. \tag{35}$$

Thus, from formula (32) and formula (35) we have

$$\vec{S} = i\left(\vec{F} \times \vec{F}^*\right)/\left(\vec{F}\vec{F}^*/2\right)^{1/2}\,. \tag{36}$$

Using formula (22), expression for spin pseudo vector (36) can be rewritten through vectors $\vec{E}$ and $\vec{H}$ as follows

$$\vec{S} = \frac{\vec{E}\times\vec{H}}{|\vec{H}|}\,. \tag{37}$$

6. DISCUSSION AND CONCLUSION

In this work, we elaborated the Lagrange formalism for neutrino field in tensor formalism, in terms of complex isotropic vectors $\vec{F}=\vec{E}+i\vec{H}$. In previous works, Weyl's equation for neutrino has been written in tensor form, in the form of non-linear Maxwell's like equations, through complex isotropic vector $\vec{F}=\vec{E}+i\vec{H}$. In this work, we found the Lagrange function for neutrino field in terms of complex isotropic vector $\vec{F}=\vec{E}+i\vec{H}$. On the basis of Noether's theorem, we derived expressions for fundamental dynamical variables (energy, momentum, charge, spin) in terms of strengths $\vec{E}$ and $\vec{H}$. We proved that expressions for fundamental dynamical variables have the same form as those obtained for electromagnetic field.

7. REFERENCES

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