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A Fuzzy-Based Approach to E-learning Evaluation

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ABSTRACT—*In this paper a fuzzy-based methodology is illustrated, able to implement educational taxonomies and returning a rich linguistic evaluation. This approach is suitable for not structured answer scripts and uses a BL-algebra-based calculus.*

Keywords – Student Evaluation, Fuzzy Sets, BL-algebra, Linguistic Variables.

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1. INTRODUCTION

The increasingly fast penetration of PCs and electronic devices inside all aspects of the daily life, along with the exponential growth of Internet users, represent the most evident changes occurred in our lives. This revolution has deeply affected the world of education and has fostered the birth of E-learning techniques and methodologies that support distance learning in order to improve the relationship teacher-student. To support the new needs arising from recent advances in communication technology, new approaches are devised in order to promote virtual classes and learning communities, to stimulate students in using educational portals and to develop courses that certify the learning level.

It is avowedly recognized the opportunity of new evaluation methods, endowed with a lot more details than the methodologies giving only numerical results, as proposed by several researchers. In particular, Bloom [2] has suggested that the activity of learning evaluation should not be demanded solely to the teacher, but should be rationalized by means of suitable methodologies. Bloom's taxonomies has been widely applied and investigated [1, 12, 13, 17, 18]. In fact, his taxonomies allows to achieve a well formulated assessment that takes into account both *Knowledge* (i.e., knowledge retrieval, terminology, classifications, abstract representations, and so on) and *Abilities and intellectual skills* (e.g., understanding, transposition, interpretation, extrapolation, synthesis). In such way the student is able to single out the learning aspects that should be improved. This aspect is emphasized in the context of distance learning, where the student lacks a personal touch with the teacher and disorientation can easily arise and deeply affect the overall learning activity. It is apparent that this problem should be tackled by advanced evaluation tools that give appropriate assessment regarding all aspects of tutoring activity, such as the one presented in this paper arises purpose. So the student can focus his/her attention onto the aspects deserving further study. It is worth noting that several authors [3, 9, 10, 14, 15, 16] have already fruitfully applied fuzzy sets for evaluating students' learning achievement.

The paper is organized as follows: Section 2 presents some basic definitions. A summary of the evaluation methodology is given in Section 3. Then the features of a suitable BL-algebra and the operation of composition are presented in Sections 4. The problem of linguistic approximation, vital for the readability of the results, is tackled in Section 5. Next section deals with the steps involved in the proposed methodology. In Section 7 a case study is discussed. Finally, points currently under investigation are briefly illustrated. It is worth emphasizing that the formal properties of the BL-algebra have been extensively investigated in [7], now the attention is focused on applicative and computational aspects.

2. BASIC DEFINITIONS

Let A be a nonempty classical set. A *fuzzy set*s on A [11, 19] is a function s: A → [0, 1]. If a ∈A then s(a) is said the *membership degree* of a to A. A *triangular fuzzy number* x=[a, b, c] on [0, 1] is a fuzzy set whose membership function is a triangle whose vertices are the points (a, 0), (b, 1) and (c, 0). In the sequel the following extended operations are used on the class of the [0,1]-triangular fuzzy numbers: i) α*[a,b,c]=[α*a, α*b, α*c] (*product of a real number*); ii) [a, b, c] + [d, e, f] = [a+d, b+e, c+f] (*sum*).

A *fuzzy partition* on A is a class of fuzzy set A_i on [0,1] so that for each x in A it is true that $\Sigma A_i(x) = 1$.

A *type-2 fuzzy set* s_2 on A is a function s_2: A → $[0, 1]^{[0,1]}$. An example of type-2 fuzzy set on the numbers (t_1, …,t_k) is the following:

$$s_2(A) = t_1/a_1 + t_2/a_2 + ... + t_n/a_n.$$

In this paper this equivalent notation is used: $s_2(A) = a_1^{t_1} + a_2^{t_2} + ... + a_n^{t_n}$.

Each word in natural language is an example of *linguistic variable* (lv, for short) [21]. Like numeric variables, a lvassumes values; these values are called *linguistic terms* (lt, for short). In this way, for example, the word *Age*(lv) could assume the following values (lts):young, old, not very young, old enough, not young but not too old. Formally, a lv is aquintuple (*v*, *T*, *X*, *g*, *m*) in which *v* is the name of the variable, *T* is the set of linguistic terms of *v* whose values range over a universal set *X*(in the example could be the set of years [0-120]), *g* is a syntactic rule for generating linguistic terms, and *m* is a semantic rule that assigns to each linguistic term $t \in T$ its *meaning*, m(t), which is a fuzzy set on *X*.In this paper, the terms of lvs are represented through type-2 fuzzy sets. In case $m(t) \in [0,1]^{[0,1]}$, they are triangular numbers on [0, 1].

3. SKETCHING THE EVALUATION METHODOLOGY

Each educational taxonomy contains a series of intellectual operations (categories) that can be requested to the student in relation to any content. Each objective or behavioural category belonging to a taxonomy can be treated as a linguistic variable. Let us consider the Bloom's taxonomy. Its pedagogical objectives are defined in three levels. First, one has the verbal description of the considered behavioural category, then a set of related objectives is given, finally examples of concrete behaviour are given, they consist of exercises, questions and tests.

Let us consider, for example, the category "*understanding*". It can be considered as a triple {description, definition, objectives}. The first component (description) consists, in turn, of {translation, interpretation, extrapolation}.

In the operative arrangement for the taxonomy, each sub-category allows to obtain and express the *operative objectives* through a list of verbs and a list of objects.

For instance, one can associate with *interpretation* the objective "to be capable of interpreting relations" and "to be capable of explaining theories". Each capacity generates linguistic terms that allow to describe the learning process.In the same way, but in a simpler form, the variable *Evaluation* will be used in the sequel.

By using linguistic variables, the evaluation model can contain categories belonging not only to the cognitive sector, but also to the affective and psychological ones.In this way the available information increases a lot.

The planning of a course or a didactic unit finds expression through couples (contents, objectives) that are recognized by the pedagogical taxonomy. Table 1 presents an example of didactic specification matrix, where for each content (C_i) are reported the didactic objectives (O_j).

Table 1: An example of taxonomic matrix

	O_1= Knowing	O_2=Understanding	O_3=Applying	O_4= Analyzing	O_5=Summarizing
C_1 =ProcedureA					
C_2 =Principle B					
C_3 =Content C					
C_4 =Content D					
C_5 =Element E					
C_6 =Element F					

Following this reasoning, a couple s = (s_1, s_2) has been put at the basis of the fuzzy evaluation method presented in this paper, where:

s_1 = Evaluating questions = (QxK, EVal(Q), M_Q, EVal(K), M_K, $MEVal_{QK}$)

s_2 = Evaluating mastery = (CxO, EVal(O), M_O, EVal(C), M_C, $MEVal_{CO}$).

The t-uples$_1$ formalizes an approach to the assessment of n questions Q_i with respect to m evaluation criteria K_j. So the componentQxK represents the matrix (Questions) x (Evaluation criteria), (Table 2) that contains for each question Q_i his/her assessment criteria. In each cell of the matrix (i, j) the linguistic evaluation is reported by the teacher for the couple (Q_i, K_j). Each row and each column of the matrix are type-2 fuzzy sets.

Table 2: Matrix (Questions) x (Evaluation criteria)

	K_1	K_2	...	K_j	...	K_m
Q_1	e_{11}	…	...	...	...	…
Q_2	...	...	...	...	...	...
...	...	...	...	...	...	...
Q_i	...	...	...	e_{ij}	...	...
...	...	...	...	...	...	...
Q_n	e_{n1}	...	...	...	...	e_{nm}

The componentEVal(Q) gives a *global analytical evaluation for each question Q_i* belonging to the table QxK. This evaluation takes into account the teacher's assessments present in the matrix; it is expressed through a fuzzy set whose domain is Q and whose values are in the lv *Evaluation*.

The elementM_Q is the *final concise linguistic evaluation referred to all the questions*: it is obtained by EVal(Q) through an extended average operation on the fuzzy set.

The component EVal(K) gives a *global analytical assessment for each criterion K_j* in the table QxK. Again, this evaluation takes into account all teacher's assessments, reported in the table. The component EVal(K)is a fuzzy set whose domain is K and whose values are in the set of terms of the lv *Evaluation*.

The component M_k is the *linguistic final evaluation about the competence attained with respect to the whole set of evaluation criteria*. It is obtained through an operation of extended average on the fuzzy set.

Finally $MEVal_{QK}$is the *overall concise linguistic evaluation* obtained fromEVal(K) and EVal(Q). It is the term that summarizes the assessments given by the teacher in the matrix QxK.

Symmetrically, the t-uples$_2$regards didactic objectives chosen in the planning of a course which are present in the question proposed to the student. For didactic objective, one means a content which must be learnt by the student.The component CxO is the matrix of the didactic specifications (Contents) x (Objectives) (Tab. 3); it holds the content C_i that the student must learn and the abilities O_j that he/she should acquire for each content. Each element (i, j) contains the student linguistic evaluation for the couple (C_i, O_j). Again each row and each column contains a type-2 fuzzy set: from them one obtains the linguistic elements in the table. Each value e_{ij} is given by the teacher, and it belongs to the set of linguistic terms chosen for the linguistic variable *Evaluation*.

Table 3: Matrix (Contents) x (Didactic objectives)

	O_1	O_2	...	O_j	...	O_m
C_1	e_{11}	…	...	...	...	…
C_2	...	...	...	...	...	...
...	...	...	...	...	...	...
C_i	...	...	...	e_{ij}	...	...
...	...	...	...	...	...	...
C_n	e_{n1}	...	...	...	...	e_{nm}

The meaning of the elements present in the t-uple s_2 is similar to the corresponding elements of s_1. The only difference is that they involve mastery evaluation.

The elements involved in the evaluation process, are summarized in Table 4.

Table 4: Summary of the elements involved in the evaluation method.

s_1 = Evaluation of questions		s_2 = Evaluation of mastery	
QxK	matrix of the didactic specifications Q x K	**CxO**	matrix of the didactic specifications C x O
EVal(K)	global analytic assessment for each criterion K_j	**EVal(O)**	global analytic assessment for each mastery
M_K	linguistic final evaluation about the competence attained with respect to the whole set of evaluation criteria	**M_O**	linguistic final evaluation of the reached competence
EVal(Q)	global analytical evaluation for each question Q_i	**EVal(C)**	global analytical evaluation for each content
M_Q	final concise linguistic evaluation	**M_C**	final concise linguistic evaluation
$MEVal_{QK}$	overall concise linguistic evaluation	**$MEVal_{CO}$**	overall concise linguistic evaluation

The fuzzy-based method allows, for example, to obtain an output similar to that presented in Table 5 starting from a matrix like that of Table 2:

Table 5: An example of possible results

EVal(Q)	EVal(K)	M_Q	M_K	$MEVal_{QK}$
More than Good on Question1, Good on Question4, Very Fair on Question5, Fair on Question 3	*Very good as regards the criterion Analysis competence, More than good for the criterion Optimal strategy choosing competence*	Good	Almost Fair	More than Fair

4.THE BL-ALGEBRA ON THE SPACE OF TYPE-2 FUZZY SETS

In order to make the paper self-consistent the basic features of the algebraic structure supporting this approach are recalled. Consider:

- **U** = $\{x_1, x_2, \ldots, x_n\}$: a finite crisp set;
- **T** : the class of totally ordered triangular fuzzy numbers on the interval [0,1];
- **[0, 0, 0]** and **[1, 1, 1]** special numbers;
- **∏(U)** : the set of classical partitions on U;
- **F(U)** : the class of the type-2 fuzzy sets on **∏(U)** whose membership functions are elements of **T**;
- ***A*** : a set of attributes compatible with U.

Definition 1: Given $A \in \boldsymbol{A}$, the *representation of* A is an element r(A) of **S(U)** = **F(U)**$\cup$ { $\mathbf{U}^{[0,\,0,\,0]}$, $\mathbf{U}^{[1,\,1,\,1]}$ } defined as follows:

r(A) = $a_n^{\alpha_n} a_{n-1}^{\alpha_{n-1}} \ldots a_2^{\alpha_2} a_1^{\alpha_1} (k, d_n, d_{n-1}, \ldots, d_2, d_1)$, where

1) $\{a_n, a_{n-1}, \ldots, a_2, a_1\} \in \prod(U)$;
2) $\alpha_i \in T$, $\forall i = 1, \ldots, n$;
3) $\alpha_n > \alpha_{n-1} > \ldots > \alpha_2 > \alpha_1$;
4) $(k, d_n, d_{n-1}, \ldots, d_2, d_1)$ is a (n+1)-ple of natural numbers called *outfit*, that satisfies the following constraints: j) if k= 1 then d_i=1 for any i:1,...,n; jj) if k>1 the t-uple $(d_n, d_{n-1}, \ldots, d_2, d_1)$ is symmetric with respect to the central values.

or **r(A)** = $U^{[0,\,0,\,0]}(1,1)$;

or **r(A)** = $U^{[1,\,1,\,1]}(0,1)$.

From the definition follows that the sub-string $a_n^{\alpha_n} a_{n-1}^{\alpha_{n-1}} \ldots a_2^{\alpha_2} a_1^{\alpha_1}$ is a type-2 fuzzy set. The subsets $a_n, a_{n-1}, \ldots, a_1$ are called *first parts* and the fuzzy numbers $\alpha_n, \alpha_{n-1}, \ldots, \alpha_1$ are called *second parts.*

Given $A \in \boldsymbol{A}$, the representationof A gives the evaluation $\alpha_i \in T$ for any $a_i \in \prod(U)$ with respect to the meaning of A. In the sequel, for the sake of simplicity, A is used instead of r(A), when this does not create confusion.

The *outfit* contains the following information about the related string: *j*) the value k denotes the number of strings necessary for its generation; *jj*) the values d_i denote the number of first parts whose composition has generated the i-th first cluster, i.e. the i-th element of the partition present in the string. So $(k, d_n, \ldots, d_1) = (1, 1, 1, \ldots, 1)$ if and only if the related type-2 fuzzy set is not the product of other elements of **S(U)** through the operation of composition introduced below.

Of course, it would be very useful to be able to directly use linguistic terms in the applications. For such purpose, let V be a linguistic variable whose set of linguistic terms is T(V) and M(T(V)) a semantic rule that determines the meaning of any linguistic term λ of every element T(V) belonging to V. Using the semantic rule M, one can write:

$$A = a_n^{\lambda_n} a_{n-1}^{\lambda_{n-1}} \ldots a_2^{\lambda_2} a_1^{\lambda_1} (k, d_n, d_{n-1}, \ldots, d_1), \text{ where } M(\lambda_i) = \alpha_i.$$

The special numbers [0, 0, 0] and [1, 1, 1] correspond to the labels NI and NC respectively. NI stands for “no information” and is used when no information is available about the elements in U in order to state the degree to which an attribute A is satisfied by them; NC stands for the label “not compatible” and is used in case the elements in U are not compatible with the property A. For example, if U = “set of men”, the attribute “*pregnancy*” is incompatible with the elements of U.

The basic idea underlying the use of the BL-algebra for classification purposes relies on a suitable operation for ordered strings, so that the resulting string represents a finer classification of the universe U with respect to the classifications induced by the original strings.

It is worth noting that in these strings each element plays two roles, related to its *value* and its *position* within the string, and this situation reminds us what happens in positional numbering systems. In fact the absolute value of the generic element a_i is the set of objects of U represented by the element, while the value relative to its position is given by the fuzzy number α_i.

In such way the operation of algebraic composition of ordered strings can be considered as a variant of the well known algorithm for the multiplication of natural numbers. However, one has to take into account that the two parts each string is composed of are not homogeneous, since the base is a crisp set whereas the exponent is a fuzzy set.

So, given a finite set U and two strings A and B on U the operation of composition ◊between A and B is carried out by applying the operator * to the first parts (crisp subsets of U) of A and B and the operator ° to the second parts (fuzzy sets) of A and B.

Definition 2:If $A = a_n^{\alpha_n} a_{n-1}^{\alpha_{n-1}} \ldots a_2^{\alpha_2} a_1^{\alpha_1} (k_A, d_{A,n}, \ldots, d_{A,2}, d_{A,1})$ and
$B = b_m^{\beta_m} b_{m-1}^{\beta_{m-1}} \ldots b_2^{\beta_2} b_1^{\beta_1} (k_B, d_{B,n}, \ldots, d_{B,2}, d_{B,1})$ are strings then the *composition*◊:S(U)×S(U)→S(U) is defined as follows:

$$C = A \lozenge B = c_{m+n-1}^{\gamma_{m+n-1}} \ldots c_2^{\gamma_2} c_1^{\gamma_1} (k_A + k_B, d_{Cm+n-1}, \ldots, d_{C2}, d_{C1})$$

where

- first parts $c_i = \bigcup_{\substack{h+k=i+1 \\ 1 \le h \le n \\ 1 \le k \le m}} (a_h \cap b_k), \ i = 1, \ldots, m+n-1$
- second parts

$$\gamma_i = \frac{1}{(k_A + k_B) d_{Ci}} \sum_{\substack{k+j=i+1 \\ 1 \le k \le n \\ 1 \le j \le m}} d_{Ak} d_{Bj} (k_A \alpha_K + k_B \beta_J), \quad i = 1, \ldots, m+n-1$$

where the values of $d_{C,i}$ are:

$$d_{Ci} = \sum_{\substack{k+j=i+1 \\ 1 \le k \le n \\ 1 \le j \le m}} d_{Ak} d_{Bj}, \ i = 1, \ldots, m+n-1$$

Now, it is easy to understand the role played by the outfit string.

In the outfit of C=A ◊ B, $(k_A + k_B, d_{C,m+n-1}, \ldots, d_{C,2}, d_{C,1})$:

- k_A, k_B store the number of attributes utilized to get A and B;

- the indices d_{Ci} represent the number of sets whose union produces the i-th class of C;
- γ_i represent the mean of the triangular numbers where each number has a weight related to its computational "history".

It is possible to show that this algebraic structure can be characterized as a BL-algebra [7]. Its applicability to different areas has been investigated in [4, 5, 6, 8].

5. THE LINGUISTIC APPROXIMATION

The triangular numbers obtained during the computational process are really poor as regards expressivity and readability. For this reason it is important to introduce an algorithm for linguistic approximation so that one gets a readable set of linguistic terms. In fact, situations can arise where not all the fuzzy numbers on [0,1] can be associated with a linguistic label: in order that any string could be represented in linguistic form suitable approximations have to be carried out so that the fuzzy values are translated into linguistic terms.

The linguistic approximation function LAis defined as follows: $\mathbf{LA_K[f] = e_f}$ where f is the fuzzy number that should receive a new linguistic label and e_f is the term attached to f. The algorithm involves the parameter*k* (the number of labels generated between two basic ones) that allows to create complex and precise linguistic terms.

From the formal point of view, one has:

E = $\{e_1, e_2, \ldots, e_n\}$ is a set of linguistic terms of a variable V;
F = $\{f_1, f_2, \ldots, f_n\}$ is the subset of numbers associated with the set **E**;
M is the semantic rule such that **M** $(f_i) = e_i$.

Given e_i and e_{i+1} one can split the interval $[m_i , m_{i+1}]$ (the extremes are the central values of the fuzzy numbers f_i and f_{i+1}, respectively) into a certain number of sub-intervals. The greater is the number, the more precise is the approximation. To represent linguistically these approximate values, one can introduce linguistic modifiers such as much, little, more or less, and so on. Let

- **n** be the number of basic linguistic labels
- **k**be the number of labels approximated for each couple (e_i,e_{i+1})

Then the overall number of linguistic labels is:

$$(n - 1) * k + n$$

It is worth emphasizing that increasing the number of labels deeply affects the computational complexity of the procedure. On the other hand, using few approximate labels the expressive power of the system decreases correspondingly.

Suppose that $\{\alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_k\}$ are the fuzzy numbers associated with the linguistic labels and let β be the fuzzy number to be approximated.

The approximation mechanism provides the generation of $(n - 1) * k$ additional labels. With k=3 a good compromise is achieved between the expressivity of the system and a manageable complexity.

Suppose that $\{ \lambda_1, \lambda_2, \ldots, \lambda_k \}$ are the basic terms, associated with the fuzzy numbers $\{ \alpha_1, \alpha_2, \ldots, \alpha_k\}$ through a semantic rule$M(\alpha_i) = \lambda_i$ and let βbe the fuzzy number to be approximated.

Suppose that the central value of β (denoted by m) takes a value included between those central of α_i and α_{i+1}, namely belongs to the interval $[m_i , m_{i+1}]$.

Thus, the function LA_3 isdefined as follows:
Let $d=(m_{i+1} - m_i)$, then the following approximation is carried out:

I) if $m \in [m_i, m_i + d/10]$ then β is approximated by λ_i ;
II) if $m \in [m_i + (d/10), m_i + (3/10)*d]$ then β is said "next to" λ_i and one writes $N[\lambda_i]$;
III) if $m \in [m_i + (3/10)*d, m_i + (7/10)*d]$ then β is" included between" λ_i and λ_{i+1} one writes $IB[\lambda_i, \lambda_{i+1}]$;
IV) if $m \in [m_i + (7/10)*d, m_i + (9/10)*d]$ then we say that β is "almost" λ_{i+1} and we write $B[\lambda_{i+1}]$;
V) if $m \in [m_i + (9/10)*d, m_{i+1}]$ then β is approximated by λ_{i+1}.

It is worth noting that, in this case, the maximum number of obtainable labels is (4n-3), where *n* stands for the number of basic original labels. Moreover, the linguistic approximation is useful only to facilitate the interpretation of the results, the operations are always carried out on not approximated strings.

Here an example of the linguistic approximation on triangular fuzzy numbers: let's suppose that λ_2= [0.5, 0.7, 0.9] = *Good* and λ_1= [0.4, 0.6, 0.8] = *Fair*. The triangular fuzzy number α = [0.45, 0.63, 0.8] will be approximated as follows:

the middle value m = 0.63∈$[m_1,m_2]$. d = m_2-m_1=0.3. It's in the fourth case of the algorithm, because 0.63 ∈[0.61, 0.67] = $[m_1 + (7/10)*d, m_1 + (9/10)*d]$, thus α ≈ "Almost Good";

6. THE STEPS INVOLVED IN THE EVALUATION METHODOLOGY

The method can be applied to two different kinds of assessments: either to evaluate a set of questions on the basis of a defined set of evaluation criteria or to make an evaluation of students's mastery, on the basis of his/her competence on specific contents and objectives. The steps to be applied are:

Step 1) Define n_{lt} linguistic terms for the variable "Evaluation" (X)

In the following examples and in the case study, these terms are used:
T(X) = { Excellent, Very good, Good, Fair, Poor, Very poor }, *n = 6.*

Step 2) Consider the set T(X) associated with X and define a set M(T(X)) of n triangular fuzzy numbers associated with the linguistic labels

Table 6 shows the terms used in this paper.

Table 6: Fuzzy numbers associated with the linguistic variable *Evaluation*

X* = Evaluation		
Linguistic Labels		**Triangular Fuzzy Numbers**
Excellent	⟶	(0.8,1,1)
Very good	⟶	(0.5,0.7,0.9)
Good	⟶	(0.4,0.6,0.8)
Fair	⟶	(0.2,0.4,0.6)
Poor	⟶	(0.1,0.2,0.4)
Very poor	⟶	(0, 0, 0.2)

The border values (Excellent and Very poor) correspond more or less to the values 1 and 0. However one can modulate the "slope" between these extremes in order to get the desired evaluation grades.

Now it is shown how the student learning level can be represented by the string:

$a_n^{\alpha n}\, a_{n-1}^{\alpha n-1} \ldots\ldots\, a_1^{\alpha 1}$

where the sets $\{a_i\}$ are crisp sets that contain, for example, contents, didactic goals, evaluation criteria, whereas the elements α_i represent fuzzy sets that can be translated into the linguistic terms associated with the variable *Evaluation*.

It is worth emphasizing that by composing two strings one gets a new classification that is influenced by the elements the two strings consist of. Thus, this formal framework is suitable to manage the overall assessment of the student's performance since one can take into account all answer scripts and all didactic goals.

In fact the composition for the second parts generates a new triangular number that embodies and summarizes all available information about student's behaviour and in such way one can get a realistic assessment involving all relevant aspects of learning activity.

6.1. Managing assessment of questions through defined evaluation criteria

Step 3a) Define the questions and the exercises to be processed inorder to get the string:

$$Q = \{ Q_1 , Q_2 , ...,Q_r\}$$

Step 4a) Define specific evaluation criteria:

$$K = \{K_1 , K_2 , ...,K_s\}$$

These stepsarecarried out whenthegoalistheassessmentofquestions (s_1 in Section 3). The elements of the set K include all criteria that can be useful to assess student's performance. There is a large variety of possible didactic goals (e.g., specific learning goals, basic contents to be grasped by the student, and so on).

The first step involves the construction of a matrix:

(Questions) x (Evaluation criteria)

One gets strings that describe the learning level of a content or the mastery attained by the student in a specific ability or the teacher's assessment according to the criterion. From the matrix one can get two types of strings. The first is:

$K_i = [Q_{i1}, ..., Q_{ih}]^{\alpha h}[Q_{i(h+1)}, ... , Q_{is}]^{\alpha s} [Q_{i(h+1)}, …, Q_{is}]^{\alpha n}$
that represents the learning level as regards a specific criterion K_i

This algebraic approach allows to compute, for each particular string, an assessment that summarizes the learning level achieved by the student by considering all evaluation criteria. One gets the following string:
$EVal(Q) = K_1◊ … ◊K_p$.

The result of this composition is called EVal(Q) because its crisp parts are made by Questions; in fact it is obtained by composing the columns of the table.

Vice versa by means of the composition one has:

$Q_i = [K_{i1}, ..., K_{ih}]^{\alpha h}[K_{i(h+1)}, ... , K_{is}]^{\alpha s} [K_{i(h+1)}, …, K_{is}]^{\alpha n}$

and for any question Q_i, one gets an overall evaluation according to each adopted criterion.

By composing the Q_i, one has $EVal(K) = Q_1◊ … ◊Q_r$, and this string represents the global evaluation for all questions taking into account each evaluation criterion. Thanks to the linguistic approximation one finally gets readable and expressive assessments.

Sometimes it can be useful to have a single "word" that expresses the student learning level. One can obtain it by calculating an average of the triangular fuzzy numbers generated for the contents and the objectives assessments:

$$MEval_{QK} = \frac{\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{r} \alpha_j}{p+r}$$

Where: α_i are the triangular numbers of EVal(Q) and α_j are the triangular numbers of EVal(K), p is the number of criteria and r that of questions. If necessary, one can easily obtain also average evaluations only on the whole set of questions (M_Q) or on the overall evaluation criteria (M_K):

$$M_Q = \frac{\sum_{i=1}^{q} \alpha_i}{p} \qquad M_K = \frac{\sum_{j=1}^{r} \alpha_j}{r}$$

6.2. Evaluation of contents and objectives (mastery evaluation)

This approach can be used also to get student's mastery evaluation (s_2 in Section 3). In this case one has the following steps:

Step 3b) Cluster the basic contents to be grasped during the E-learning sessions: $C = \{ C_1, C_2, ..., C_n \}$

Step 4b) Specify the didactic goals $O =\{O_1, O_2, ..., O_m \}$ *associated with the contents* C_i

Then the method proceeds in a similar way, but starting from a different matrix:

(Contents) x (Didactic goals)

In such way one can obtain classifications both with respect to the contents and the goals singled out by the teacher.

For any row one gets a string that describes a content through the related didactic goals and in the meanwhile specifies the learning level shown by the student in the corresponding ability. In general, such a string has the following form:

$C_i = [O_{i1}, ... , O_{ik}]^{\alpha k}[O_{i(k+1)}, ... , O_{ih}]^{\alpha h} [O_{i(h+1)}, ... , O_{in}]^{\alpha n}$

and it represents the mastery level of any activity associated with the specific content C_i . By composing strings of this type one gets: $EVal(O) = C_1◊ … ◊C_p$

that represents, for each didactic goal, the linguistic assessment describing the mastery level of the student. On the other hand, for each column, one can obtain a string that represents the mastery level attained in a specific activity with respect to all contents involved. The corresponding string is:

$O_i = [C_{i1}, ..., C_{ih}]^{\alpha h}[C_{i(h+1)}, ... , C_{is}]^{\alpha s} [C_{i(h+1)}, …, C_{is}]^{\alpha n}$

Also in this case, one can have an overall representation by means of the string:

$EVal(C) = O_1◊ … ◊O_q$ that gives information about the overall abilities achieved by the student in the different contents studied.

As previously seen, in order to have a single "word" expressing the student's learning level, the following quantity is computed:

$$MEval_{CO} = \frac{\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \alpha_j}{p+q}$$

where: α_i are the triangular numbers of EVal(O)and α_j are the triangular numbers of EVal(C), p is the number of contents and q that of objectives. Again, one can obtain average linguistic evaluations only on Objectives (Mo) or contents knowledge (Mc) :

$$M_O = \frac{\sum_{j=1}^{p} \alpha_j}{p} \qquad M_C = \frac{\sum_{i=1}^{q} \alpha_i}{q}$$

7. A CASE STUDY

Suppose that one aims to assess the final exam for a master in "Italian literature of 13th century". Let us suppose that the following evaluation criteria have been singled out: Ability on summarizing (K_1), Property of language (K_2), Form and style (K_3), Fidelity to specifications (K_4), Richness of contents (K_5).

7.1 Evaluation of not structured answer scripts through evaluation criteria

Suppose that the student has received the marks presented in Table 7 for the first four questions.

Table 7: Marks for not structured answer scripts in function of evaluation criteria

Answer script	K_1	K_2	K_3	K_4	K_5
Q_1	Fair	Very good	Good	Good	Verygood
Q_2	Verygood	Excellent	Fair	Fair	Poor
Q_3	Very good	Good	Good	Fair	Good
Q_4	Very good	Good	Good	Good	Fair

From the table one gets:
$Q_1 = [K_{2,} K_{5,}]^{vg} [K_{3,} K_{4,}]^{g} [K_1]^{f}$,
$Q_2 = [K_2]^{e} [K_1]^{vg} [K_{3,} K_4]^{f} [K_5]^{p}$,
$Q_3 = [K_1]^{vg} [K_2, K_3, K_5]^{g} [K_4]^{f}$,
$Q_4 = [K_1]^{vg} [K_2, K_3, K_4]^{g} [K_5]^{f}$.

The result of the operation of composition among the rows is:

$$EVal(K) = [K_2]^{[0.492\ 0.692\ 0.862]} [K_1]^{[0.445\ 0.644\ 0.823]} [K_3]^{[0.35\ 0.543\ 0.736]} [K_4.K_5]^{[0.304\ 0.494\ 0.691]}$$

For simplicity, the *outfit* is not shown; by applying the linguistic approximation algorithm LA_3, the following new labels are obtained:

[0.304 0.494 0.691] =IB[Fair , Good] = Very Fair
[0.35 0.543 0.736] = B[Good] = Almost Good
[0.445 0.644 0.823] = IB[Good , Very good] = Very Good
[0.492 0.692 0.862] = [Very good] = Very good

So one can translate the evaluation into a linguistic form:

$$EVal(K) = [K_2]^{\text{Very very good}} [K_1]^{\text{Very good}} [K_3]^{\text{Almost good}} [K_4,K_5]^{\text{Very fair}}$$

The fully linguistic interpretation of the string is straightforward:
"*The overall evaluation concerning all questions has shown that the student has exhibited very very good property of language, very good ability on summarizing, almost good for form and style, very fair as it regards richness of contents and for fidelity to specifications*."

Remark:it is worth noting that this assessment fully reflects the data present in the table. For example K_3 is evaluated three times Good and one Fair. By applying the composition one gets a new approximate linguistic term included between Fair and Good, but closer to Good. Similar remarks can be made for other points in this case study.

Table 8 also gives:
$K_1 = [Q_2, Q_3, Q_4]^{vg}[Q_1]^{f}$,
$K_2 = [Q_2]^{e}[Q_1]^{vg} [Q_3,Q_4]^{g}$,
$K_3 = [Q_1, Q_3, Q_4]^{g} [Q_2]^{f}$,
$K_4 = [Q_1, Q_4]^{g}[Q_2, Q_3]^{f}$,
$K_4 = [Q_1]^{vg}[Q_3]^{g} [Q_4]^{f} [Q_2]^{p}$.

The result of the composition of the columns is:

$$EVal(Q) = [Q_1]^{[0.435\ 0.635\ 0.811]} [Q_3.\ Q_4]^{[0.362\ 0.559\ 0.746]} [Q_2]^{[0.327\ 0.52\ 0.711]}$$

By applying the linguistic approximation.one has:

$EVal(Q) = [Q_1]^{\text{Very good}} [Q_3, Q_4]^{\text{Almost good}} [Q_2]^{\text{Very fair}}$

Consequently one gets information about the evaluation of single questions:"The student has been evaluated Very good as regards Question 1. Almost good for Question 4 and Exercise 3.and Very fair for Question 2."
So one can calculate $MEVal_{QK}$ = [0.387. 0.583. 0.768] and by applying LA_3, one has:

$MEVal_{QK}$ = Good.

7.2 Evaluation of contents and objectives

The mastery level of the student can be also evaluated as regards the following contents: Historical framework (C_1), Cultural-political-ideological trends (C_2), Literary genres (C_3), Rhetorical figures (C_4), Metrics (C_5).
The abilities concerning didactic goals are: Linking historical events (O_1), Relevance of ideological trends (O_2), Linguistic abilities (O_3), Literary genres (O_4), Stylistic aspects (O_5), Singling out allegories (O_6).
The matrix Contents x Didactic goals of Table 8 summarizes the ability concerning not structured answer scripts.

Table 8: Marks for mastery related to didactic goals

	O_1	O_2	O_3	O_4	O_5	O_6
C_1	Verygood	Good	Very good	Fair	Fair	Fair
C_2	Good	Poor	Fair	Fair	Very good	Good
C_3	Fair	Good	Very good	Good	Good	Verygood
C_4	Very good	Good	Fair	Fair	Very good	Excellent
C_5	Excellent	Very good	Good	Good	Excellent	Very good

Each row of the matrix represents a string and thus a classification:

$C_1 = [O_1, O_3]^{vg} [O_2]^{g} [O_4, O_5, O_6]^{f}$
$C_2 = [O_5]^{vg} [O_1, O_6]^{g} [O_3, O_4]^{f} [O_2]^{p}$
$C_3 = [O_3, O_6]^{vg} [O_2, O_4, O_5]^{g} [O_1]^{f}$
$C_4 = [O_5]^{e} [O_1, O_5]^{vg} [O_2]^{g} [O_3, O_4]^{f}$
$C_5 = [O_1, O_5]^{e} [O_2, O_6]^{vg} [O_3, O_4]^{g}$

The mastery level related to didactic goals can be obtained by composing the rows of the table and by linguistically interpreting the string as follows:
$EVal(O) = [O_1, O_5, O_6]^{\text{Almost Very good}} [O_2, O_3]^{\text{Almost Good}} [O_4]^{\text{Very Fair}}$

The overall assessment is:"As regards the abilities: Almost very good in singling out allegories, historical links and stylistic aspects, Almost good in linguistic abilities and ideological trends, Very fair in literary genres."
The level of conceptualization can be obtained by composing the columns of the table:

$O_1 = [C_5]^{e} [C_1, C_4]^{vg} [C_2]^{g} [C_3]^{f}$
$O_2 = [C_5]^{vg} [C_1, C_3, C_4]^{g} [C_2]^{p}$
$O_3 = [C_1, C_3]^{vg} [C_5]^{g} [C_2, C_4]^{f}$
$O_4 = [C_3, C_5]^{g} [C_1, C_2, C_4]^{f}$
$O_5 = [C_5]^{e} [C_2, C_4]^{vg} [C_3]^{g} [C_1]^{f}$
$O_6 = [C_4]^{e} [C_3, C_5]^{vg} [C_2]^{g} [C_1]^{f}$.

The resulting string is:
$EVal(C) = [C_5]^{\text{More than Very good}} [C4]^{\text{Very Good}} [C_3]^{\text{Good}} [C_1, C_2]^{\text{Very Fair}}$

The linguistic assessment is: "As regards the didactic goals: More than very good for the Metrics, Very good for Rhetorical figures, Good for Literary genres, Very Fair for Historical framework and Cultural-political-ideological trends."
Now one can calculate $MEVal_{CO}$ = [0.394, 0.587, 0.765] and by applying LA_3, one has:
$MEVal_{CO}$ = Good.

8. CONCLUDING REMARKS

This paper has presented a method for the evaluation of learning processes based on a BL-algebra whose support set are type-2 fuzzy sets. The methodology presents several aspects deserving further investigation:

- A possible extension of the methodology concerns the introduction of a weighting function. In such way the teacher could associate higher weights to more important questions and, moreover, the teacher could introduce priorities among the didactic goals or the evaluation criteria.
- Another research trend could investigate the formalization of a function which, given the composition of two or more strings, allows to calculate the relevance of some contents in spite of others during the learning process.
- A subsequent step of investigation could involve the expansion of the algebraic structure so that, beginning from the current cognitive state of the student and the final state to be achieved, it could be possible to get the next cognitive state.
- The evaluation methodology could be introduced in a larger context of E-learning evaluation environment, that should adapt the exercises level to the student's knowledge degree.
- The extension of the method to cope with structured answer scripts. The traditional evaluation involves simple formulas that give numerical results. Each assessment is translated into a letter grade. However, if several structured answer scripts are suitably organized, it is possible to carry out assessments involving several answer scripts and achieve an overall linguistic evaluation.

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