Second Order Tensor Wave Equations for Neutrinos

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ABSTRACT--- ***In previous works, with the use of Cartan map, Weyl´s equation for neutrino has been written in tensor form, in the form of non-linear Maxwell´s like equations, through isotropic complex vector $\vec{F} = \vec{E} + i\vec{H}$. It has been proved, that complex vector $\vec{F} = \vec{E} + i\vec{H}$ satisfies non-linear condition $\vec{F}^2 = 0$, equivalent to two conditions for real quantities $\vec{E}^2 - \vec{H}^2 = 0$ and $\vec{E}.\vec{H} = 0$, obtained by equating to zero separately real and imaginary parts in the equality $\vec{F}^2 = 0$. It has been proved, that the vectors $\vec{E}$ and $\vec{H}$ have the same properties as those of the strengths of electric and magnetic fields.***

In this work, in order to investigate and to understand these neutrino waves, we derived the corresponding second order wave equations.

Keywords--- Second order, tensor, wave equation, neutrinos

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1. INTRODUCTION

In previous works, Weyl´s equation for neutrino has been written in tensor form, in the form of non-linear Maxwell´s like equations through isotropic complex vector $\vec{F} = \vec{E} + i\vec{H}$, satisfying non-linear condition $\vec{F}^2 = 0$. The last condition is equivalent to two conditions for real quantities $\vec{E}^2 - \vec{H}^2 = 0$ and $\vec{E}.\vec{H} = 0$, obtained by equating to zero separately real and imaginary parts in equality $\vec{F}^2 = 0$. It has been proved, that the vectors $\vec{E}$ and $\vec{H}$ have the same properties as those of the strengths of electric and magnetic fields. For example, under Lorentz relativistic transformations, they are transformed as components of a second rank tensor $F_{\mu\nu}$. In addition, the solution of these non-linear equations for free particle as well fulfils Maxwell´s equations for vacuum (with zero at the right side).

In this work, in order to investigate and to understand these neutrino waves, we shall derive the corresponding second order tensor wave equations.

2. RESEARCH METHOD

In previous works, using Cartan map, Weyl´s equation for neutrino has been written in tensor form, in the form of non-linear Maxwell's like equations through isotropic complex vector $\vec{F} = \vec{E} + i\vec{H}$. In this work, we shall use the general method and theorems of vector analysis to derive the second order tensor wave equations satisfied by the fields $\vec{E}$ and $\vec{H}$.

3. SECOND ORDER WAVE EQUATIONS FOR NEUTRINO IN TENSOR FORMALISM

In previous works , using Cartan map, Weyl´s equation for neutrino

$$p_0\xi=(\vec{p}\vec{\sigma})\xi \qquad (1)$$

has been written in tensor form, through isotropic complex vector $\vec{F}=\vec{E}+i\vec{H}$ as follows

$$D_0\vec{F}=i\vec{D}\times\vec{F}-(\vec{D}F_i)v_i\ . \qquad (2)$$

Where $D_0=i\frac{\partial}{\partial t}$

$$\vec{D}=-\frac{i}{2}\vec{\nabla} \qquad (3)$$

$$\vec{v}=\frac{\vec{E}\times\vec{H}}{|\vec{E}|^2}\ .$$

Here we use the natural system of units in which c=ħ=1.

Separating real and imaginary parts in equations (2), we obtain

$$\begin{cases} rot\vec{E}+\frac{\partial\vec{H}}{\partial t}=v_i(\vec{\nabla}H_i) \\ rot\vec{H}-\frac{\partial\vec{E}}{\partial t}=-v_i(\vec{\nabla}E_i) \\ div\vec{E}=-v_i\frac{\partial E_i}{\partial t} \\ div\vec{H}=-v_i\frac{\partial H_i}{\partial t} \end{cases} \qquad (4)$$

Now we shall derive the second order wave equations, that satisfy the vectors $\vec{E}$ and $\vec{H}$.

Acting on the first equation of the system (4) by operator rot, we obtain

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E}=-\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H})+\vec{\nabla}\times(v_i\vec{\nabla}H_i). \qquad (5)$$

Using the well known vector identity

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E}=\vec{\nabla}(\vec{\nabla}\vec{E})-\vec{\nabla}^2\vec{E}\ , \qquad (6)$$

and using the second equation of the system (4), equation (5) can be written in the form

$$\vec{\nabla}(\vec{\nabla}\vec{E})-\vec{\nabla}^2\vec{E}=-\frac{\partial}{\partial t}\left[\frac{\partial\vec{E}}{\partial t}-v_i(\vec{\nabla}E_i)\right]+\vec{\nabla}\times(v_i\vec{\nabla}H_i). \qquad (7)$$

Applying the third equation of the system (4), we obtain

$$-\vec{\nabla}\left(v_i\frac{\partial E_i}{\partial t}\right)-\vec{\nabla}^2\vec{E}=-\frac{\partial^2\vec{E}}{\partial t^2}+\frac{\partial}{\partial t}(v_i\vec{\nabla}E_i)+\vec{\nabla}\times(v_i\vec{\nabla}H_i), \qquad (8)$$

Or

$$\vec{\nabla}^2\vec{E}-\frac{\partial^2\vec{E}}{\partial t^2}=-\vec{\nabla}\left(v_i\frac{\partial E_i}{\partial t}\right)-\frac{\partial}{\partial t}(v_i\vec{\nabla}E_i)-\vec{\nabla}\times(v_i\vec{\nabla}H_i). \qquad (9)$$

In the same way, we can derive the wave equation for $\vec{H}$.

Acting by operator rot on the second equation of the system (4), we obtain

$$\vec{\nabla}\times\vec{\nabla}\times\vec{H}=\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{E}\right)-\vec{\nabla}\times\left(v_i\vec{\nabla}E_i\right). \qquad (10)$$

Using the first equation of the system (4), we find

$$\vec{\nabla}\left(\vec{\nabla}\vec{H}\right)-\vec{\nabla}^2\vec{H}=\frac{\partial}{\partial t}\left[-\frac{\partial\vec{H}}{\partial t}+v_i\left(\vec{\nabla}H_i\right)\right]-\vec{\nabla}\times\left(v_i\vec{\nabla}E_i\right). \qquad (11)$$

Using the fourth equation of the system (4), we obtain

$$-\vec{\nabla}\left(v_i\frac{\partial H_i}{\partial t}\right)-\vec{\nabla}^2\vec{H}=-\frac{\partial^2\vec{H}}{\partial t^2}+\frac{\partial}{\partial t}\left(v_i\vec{\nabla}H_i\right)-\vec{\nabla}\times\left(v_i\vec{\nabla}E_i\right). \qquad (12)$$

Or

$$\vec{\nabla}^2\vec{H}-\frac{\partial^2\vec{H}}{\partial t^2}=-\vec{\nabla}\left(v_i\frac{\partial H_i}{\partial t}\right)-\frac{\partial}{\partial t}\left(v_i\vec{\nabla}H_i\right)+\vec{\nabla}\times\left(v_i\vec{\nabla}E_i\right). \qquad (13)$$

In the simple case $\vec{v}$=const (for example, in the case of plane wave), equations (9) and (13) reduce to

$$\vec{\nabla}^2\vec{E}-\frac{\partial^2\vec{E}}{\partial t^2}=-2\frac{\partial}{\partial t}\left(v_i\vec{\nabla}E_i\right), \qquad (14)$$

$$\vec{\nabla}^2\vec{H}-\frac{\partial^2\vec{H}}{\partial t^2}=-2\frac{\partial}{\partial t}\left(v_i\vec{\nabla}H_i\right). \qquad (15)$$

4. DISCUSSION AND CONCLUSION

In this work, we investigated the new tensor formalism for description of fermions. We considered the tensor equations for neutrino. In previous works, Weyl´s equation for neutrino has been written in tensor form, in the form of non-linear Maxwell’s like equations for strengths $\vec{E}$ and $\vec{H}$. In this work, we derived the second order tensor wave equations for strengths $\vec{E}$ and $\vec{H}$. We proved that apart the non-linear combinations appearing on the right sides, the obtained equations are similar to those found in Electrodynamics for electric and magnetic fields. This new result again shows the similarity between Weyl´s theory for neutrino and Maxwell’s theory for electromagnetic field.

5. REFERENCES

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