Formulation of Conservation Laws of Current and Energy for Neutrino Field in Tensor Formalism

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ABSTRACT---- ***In previous works, using Cartan map, Weyl´s equation for neutrino has been written in vector form, in the form of non-linear Maxwell´s like equations, through isotropic complex vector $\vec{F} = \vec{E} + i\vec{H}$. The vector $\vec{F} = \vec{E} + i\vec{H}$ satisfies non-linear condition $\vec{F}^2 = 0$, equivalent to two conditions for real quantities $\vec{E}^2 - \vec{H}^2 = 0$ and $\vec{E}.\vec{H} = 0$, obtained by separating real and imaginary parts in the equality $\vec{F}^2 = 0$. It has been proved that the vectors $\vec{E}$ and $\vec{H}$ have the same properties as the vectors $\vec{E}$ and $\vec{H}$, components of electromagnetic field. In particular, the solution of these non-linear equations for free particle as well fulfils Maxwell´s equations for vacuum (with zero at the right side). This allows us to interpret neutrino field as a new form of electromagnetic field. In this work, in studying the properties of this new electromagnetic field, we derived its laws of conservation of current and energy.***

Keywords---- Conservation law, current, energy, neutrino field.

1. INTRODUCTION

In previous works, Weyl´s equation for neutrino has been written in tensor form through complex isotropic vector $\vec{F} = \vec{E} + i\vec{H}$, satisfying non-linear condition $\vec{F}^2 = 0$. This non-linear condition is equivalent to two conditions for real quantities $\vec{E}^2 - \vec{H}^2 = 0$ and $\vec{E}.\vec{H} = 0$, obtained by equating to zero separately real and imaginary parts of equality $\vec{F}^2 = 0$. The quantities $\vec{E}$ and $\vec{H}$ have the same properties as components of tensor $F_{\mu\nu}$ of electromagnetic field. For example, under relativistic Lorentz transformations they are transformed like components of electric and magnetic fields. It has been proved that, Weyl´s equation for neutrino written in tensor form takes the form of non –linear Maxwell´s like equations and its solution for free particle as well fulfils Maxwell´s equations for vacuum (with zero at the right side).

In this work, in order to understand these non-linear equations, we shall investigate the laws of conservation of current and energy for neutrino field as formulated in this new tensor formalism.

2. RESEARCH METHOD

In this work, we shall investigate the law of conservation of current and the law of conservation of energy for neutrino field in tensor formalism. To derive these two laws of conservation, we shall use the general mathematical method, used in derivation of the laws of conservation in classical field theory. Weyl´s equation for neutrino will be written in tensor form, in the form of non-linear Maxwell's like equations for "electromagnetic field" $(\vec{E}, \vec{H})$. We shall apply to these non-linear Maxwell's like equations the method often used in deriving the laws of conservation of current and energy for electromagnetic field by using the well known theorems and identities of vector analysis.

3. THE LAW OF CONSERVATION OF CURRENT

In previous works, Weyl´s equation for neutrino

$$p_0\xi = (\vec{p}\vec{\sigma})\xi \qquad (1)$$

has been written in tensor form, through isotropic complex vector

$\vec{F} = \vec{E} + i\vec{H}$ as follows

$$D_0\vec{F} = i\vec{D} \times \vec{F} - (\vec{D}F_i)v_i\ . \qquad (2)$$

Where $D_0 = i\frac{\partial}{\partial t}$

$$\vec{D}=-\frac{i}{2}\vec{\nabla} \qquad (3)$$

$$\vec{v}=\frac{\vec{E}\times\vec{H}}{|\vec{E}|^2}\,.$$

Here we use the natural system of units in which c=ħ=1.

Separating real and imaginary parts in equations (2) we obtain

$$\begin{cases} \mathrm{rot}\vec{E}+\frac{\partial\vec{H}}{\partial t}=v_i(\vec{\nabla}H_i) \\ \mathrm{rot}\vec{H}-\frac{\partial\vec{E}}{\partial t}=-v_i(\vec{\nabla}E_i) \\ \mathrm{div}\vec{E}=-v_i\frac{\partial E_i}{\partial t} \\ \mathrm{div}\vec{H}=-v_i\frac{\partial H_i}{\partial t} \end{cases} \qquad (4)$$

Acting by operator div on the second equation of the system (4) and using the vector identity $\mathrm{div}(\mathrm{rot}\vec{A})=0$ for any vector $\vec{A}$, we obtain

$$\frac{\partial}{\partial t}\mathrm{div}\vec{E}=\mathrm{div}\left(v_i(\vec{\nabla}E_i)\right). \qquad (5)$$

With the third equation of the system (4), equation (5) gives

$$\frac{\partial}{\partial t}\left(v_i\frac{\partial E_i}{\partial t}\right)=-\,\mathrm{div}\left(v_i(\vec{\nabla}E_i)\right). \qquad (6)$$

Denoting by

$$\rho_e=-\left(v_i\frac{\partial E_i}{\partial t}\right)$$

$$\vec{j}_e=-\left(v_i(\vec{\nabla}E_i)\right). \qquad (7)$$

-electric current density, we obtain the continuity equation, expressing the law of conservation of electric current

$$\frac{\partial\rho_e}{\partial t}+\mathrm{div}\vec{j}_e=0. \qquad (8)$$

In the same way, acting by operator div on the first equation of the system (4) we obtain

$$\frac{\partial}{\partial t}\mathrm{div}\vec{H}=\mathrm{div}\left(v_i(\vec{\nabla}H_i)\right). \qquad (9)$$

Or

$$\frac{\partial}{\partial t}\left(-v_i\frac{\partial H_i}{\partial t}\right)=\mathrm{div}\left(v_i(\vec{\nabla}H_i)\right). \qquad (10)$$

Denoting

$$\rho_m=v_i\frac{\partial H_i}{\partial t}$$

$$\vec{j}_m=v_i(\vec{\nabla}H_i). \qquad (11)$$

-magnetic current density, we obtain the law of conservation of magnetic current

$$\frac{\partial\rho_m}{\partial t}+\mathrm{div}\vec{j}_m=0 \qquad (12)$$

4. THE LAW OF CONSERVATION OF ENERGY

With the help of the system (4) we can derive the law of conservation of energy.

Let us multiply the second equation of the system (4) by $\vec{E}$

$$\vec{E}\mathrm{rot}\vec{H} - \vec{E}\frac{\partial\vec{E}}{\partial t} = -\vec{E}\left(v_i\left(\vec{\nabla}E_i\right)\right). \quad (13)$$

Using the vector identity

$$\vec{E}\mathrm{rot}\vec{H} = \vec{H}\mathrm{rot}\vec{E} - \mathrm{div}\vec{E}\times\vec{H}, \quad (14)$$

we obtain

$$\vec{H}\mathrm{rot}\vec{E} - \vec{E}\frac{\partial\vec{E}}{\partial t} - \mathrm{div}\vec{E}\times\vec{H} = -\vec{E}\left(v_i\left(\vec{\nabla}E_i\right)\right). \quad (15)$$

Using the first equation of the system (4)

$$\mathrm{rot}\vec{E} = -\frac{\partial\vec{H}}{\partial t} + v_i\left(\vec{\nabla}H_i\right), \quad (16)$$

we find

$$-\vec{H}\frac{\partial\vec{H}}{\partial t} - \vec{E}\frac{\partial\vec{E}}{\partial t} - \mathrm{div}\vec{E}\times\vec{H} == -\vec{E}\left(v_i\left(\vec{\nabla}E_i\right)\right) - \vec{H}\left(v_i\left(\vec{\nabla}H_i\right)\right). \quad (17)$$

The last equation can be rewritten in the form

$$\frac{\partial w}{\partial t} + \mathrm{div}\vec{s} = \vec{E}\left(v_i\left(\vec{\nabla}E_i\right)\right) + \vec{H}\left(v_i\left(\vec{\nabla}H_i\right)\right). \quad (18)$$

Where

$$w = \frac{1}{2}\left(\vec{E}^2 + \vec{H}^2\right),\ \vec{s} = \vec{E}\times\vec{H}. \quad (19)$$

If $\vec{E}\left(v_i\left(\vec{\nabla}E_i\right)\right) = \vec{H}\left(v_i\left(\vec{\nabla}H_i\right)\right) = 0$ (for example, the case of a plane wave) we obtain the continuity equation

$$\frac{\partial w}{\partial t} + \mathrm{div}\vec{s} = 0. \quad (20)$$

5. DISCUSSION AND CONCLUSION

In this work, we studied the law of conservation of current and the law of conservation of energy for neutrino field in tensor formalism. Starting with the non-linear Maxwell's like equations, equivalent to spinor Weyl´s equation for neutrino and using the general method for derivation of the laws of conservation in electrodynamics, we found expressions (8) and (12) in the form of equation of continuity, expressing the laws of conservation of electric and magnetic currents respectively. We also obtained the general expression (18), expressing the law of conservation of energy. We see that, apart the additional right side, expression (18) coincides with the corresponding expression for electromagnetic field. In particular, when the right side vanishes (for example, in the case of plane wave), this expression leads to the law of conservation of energy for electromagnetic field. Once again, this result proves that neutrino field can be regarded as another form of electromagnetic field, but with half spin.

6. REFERENCES

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