

# A Study on Anti Fuzzy Interior Ideals of Ternary Semigroups

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**ABSTRACT---** *In this paper, we introduce the definitions of anti fuzzy interior ideal and anti fuzzy characteristic interior ideals in ternary semigroups. Also we investigate some of their basic properties.*

**Keywords---** Ternary semigroups, anti fuzzy interior ideals, anti fuzzy characteristic interior ideals.

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## 1. INTRODUCTION

The concept of fuzzy set was initiated by L. Zadeh [17]. The study of fuzzy algebraic structures started with the introduction of the concepts of fuzzy groups in the pioneering paper of Rosenfeld [13]. Kuroki [ 8, 9, 10,11] defined a fuzzy semigroup and various kinds of fuzzy ideals in semigroups and characterized them. M. Santiago and S. Bala developed the theory of ternary semigroups[14]. Recently, S. Kar and P. Sarkar defined fuzzy left (right, lateral) ideals of ternary semigroups and characterize regular and intra-regular ternary semigroups by using the concept of fuzzy ideals of ternary semigroups[4,5]. Biswas in [1] introduced the concept of anti fuzzy subgroups of groups. M. Khan, M. Iqbal and M. Nouman studied the concept of anti fuzzy ideals in LA- Semigroups[7]. W. K. Jeong in [3] defined anti fuzzy prime ideals in BCK-algebra. In [15], M. Shabir and Y. Nawaz introduced the concept of anti fuzzy ideals in semigroups and characterized different classes of semigroups by the properties of their anti fuzzy ideals. In [6], which is the continuation of the work carried out by M. Shabir and Y. Nawaz , the authors characterized regular, intra-regular and left (right) quasi-regular semigroups by the properties of their anti fuzzy ideals. M. Shabir and N. Rehman in [16] defined various kinds of anti fuzzy ideals in ternary semigroups and characterized different classes of ternary semigroups by the properties of these ideals.

In this paper, we introduce the definitions of anti fuzzy interior ideal and anti fuzzy characteristic interior ideals in ternary semigroups. Also we investigate some of their basic properties.

## 2. PRELIMINARIES

A ternary semigroup is a nonempty set  $S$  together with a ternary operation  $(a, b, c) \rightarrow abc$  satisfying  $(abc)de = a(bcd)e = ab(cde)$  for all  $a; b; c; d; e \in S$  (cf [14]). A non-empty subset  $A$  of a ternary semigroup  $S$  is called a ternary subsemigroup of  $S$  if  $A^3 = AAA \subseteq A$ . By a left (right, lateral) ideal of a ternary semigroup  $S$  we mean a non-empty subset  $A$  of  $S$  such that  $SSA \subseteq A$  ( $ASS \subseteq A$ ,  $SAS \subseteq A$ ). If a non-empty subset  $A$  of  $S$  is a left and right ideal of  $S$ , then it is called a two sided ideal of  $S$ . If a non-empty subset  $A$  of a ternary semigroup  $S$  is a left, right and lateral ideal of  $S$ , then it is called an ideal of  $S$ . A non-empty subset  $A$  of a ternary semigroup  $S$  is called an interior ideal of  $S$  if  $SSASS \subseteq A$ . It is clear that every ideal of  $S$  is an interior ideal. An element  $a$  of a ternary semigroup  $S$  is called regular if there exists an element  $x \in S$  such that  $axa = a$ . A ternary semigroup  $S$  is called regular if every element of  $S$  is regular. It is obvious that for a regular ternary semigroup the concept of ideal and interior ideal coincide.

A function  $f$  from  $S$  to the closed interval  $[0, 1]$  is called a fuzzy subset of  $S$  [17]. The ternary semigroup  $S$  itself is a fuzzy subset of  $S$  such that  $S(x) = 1$  for all  $x \in S$ , denoted also by  $C_S$ . Let  $f$  and  $g$  be two fuzzy subsets of  $S$ . Then the inclusion relation  $f \subseteq g$  is defined by  $f(x) \leq g(x)$  for all  $x \in S$ .  $f \cap g$  and  $f \cup g$  are fuzzy subsets of  $S$  defined by  $(f \cap g)(x) = \min\{f(x), g(x)\} = f(x) \wedge g(x)$ ,  $(f \cup g)(x) = \max\{f(x), g(x)\} = f(x) \vee g(x)$ , for all  $x \in S$ . let  $f$ ,  $g$  and  $h$  be three fuzzy subsets of a ternary semigroup  $S$ . The product  $f \circ g \circ h$  is defined by

$$(f \circ g \circ h)(x) = \begin{cases} \bigvee_{x=abc} \{f(a) \wedge g(b) \wedge h(c)\}, \\ 0 & \text{otherwise.} \end{cases}$$

A non-empty fuzzy subset  $f$  in a ternary semigroup  $S$  is called a fuzzy ternary subsemigroup of  $S$  if  $f(xyz) \geq f(x) \wedge f(y) \wedge f(z)$  for all  $x, y, z \in S$  and is called a fuzzy left (resp. lateral, right) ideal of  $S$  if  $f(xyz) \geq f(z)$  (resp.  $f(xyz) \geq f(y), f(xyz) \geq f(x)$ ) for all  $x, y, z \in S$ . If  $f$  is a fuzzy left ideal, fuzzy lateral ideal and fuzzy right ideal of  $S$ , then  $f$  is called a fuzzy ideal of  $S$ . It is clear that  $f$  is a fuzzy ideal of a ternary semigroup  $S$  if and only if  $f(xyz) \geq f(x) \vee f(y) \vee f(z)$  for all  $x, y, z \in S$ , and that every fuzzy left (lateral, right) ideal is a fuzzy ternary subsemigroup of  $S$ .

A fuzzy ternary subsemigroup of a ternary semigroup  $S$  is called a fuzzy interior ideal of  $S$  if  $f(xyawz) \geq f(a)$  for all  $x, y, a, w, z \in S$  [12]. It is obvious that every fuzzy ideal of a ternary semigroup  $S$  is a fuzzy interior ideal of  $S$ .

### 3. ANTI FUZZY INTERIOR IDEALS

In this section we define anti fuzzy interior ideal in ternary semigroups and study some basic properties of this notion.

**Definition 3.1** ([15]). A fuzzy subset  $f$  of a ternary semigroup  $S$  is called an anti fuzzy ternary subsemigroup of  $S$  if  $f(xyz) \leq \max \{f(x), f(y), f(z)\}$  for all  $x, y, z \in S$ .

**Definition 3.2** ([15]). A fuzzy subset  $f$  of a ternary semigroup  $S$  is called an anti fuzzy left (right, lateral) ideal of  $S$  if  $f(xyz) \leq f(z)$  ( $f(xyz) \leq f(x), f(xyz) \leq f(y)$ ) for all  $x, y, z \in S$ .

A fuzzy subset  $f$  of a ternary semigroup  $S$  is called an anti fuzzy ideal of  $S$  if it is an anti fuzzy left ideal, anti fuzzy right ideal and anti fuzzy lateral ideal of  $S$ .

**Definition 3.3.** An anti fuzzy ternary subsemigroup  $f$  of a ternary semigroup  $S$  is called an anti fuzzy interior ideal of  $S$  if it satisfies

$$f(xyawz) \leq f(a) \text{ for all } x, y, a, w, z \in S$$

**Definition 3.4** ([15]). For any fuzzy subset  $f$  of a ternary semigroup  $S$  and  $t \in [0,1]$ , the set  $L[f; t] = \{x \in S: f(x) \leq t\}$  is called the lower level cut of  $f$ .

Let  $f, g$  and  $h$  be fuzzy subsets of a ternary semigroup  $S$ . The anti product  $f * g * h$  is defined by

$$(f * g * h)(x) = \begin{cases} \bigwedge_{x=abc} \{f(a) \vee g(b) \vee h(c)\}, \\ 1 & \text{otherwise.} \end{cases}$$

A fuzzy subset  $f$  is called an anti fuzzy ternary subsemigroup of  $S$  if  $f(xyz) \leq f(x) \vee f(y) \vee f(z)$  for all  $x, y, z \in S$ , and is called an anti fuzzy left (lateral, right) ideal of  $S$  if  $f(xyz) \leq f(z)$  ( $f(xyz) \leq f(y), f(xyz) \leq f(x)$ ) for all  $x, y, z \in S$ . A fuzzy subset  $f$  of  $S$  is called an anti fuzzy ideal of  $S$  if it is anti fuzzy left, anti fuzzy lateral and anti fuzzy right ideal of  $S$ . A fuzzy subset  $f$  of  $S$  is called an anti fuzzy idempotent if  $f * f * f = f$ . For a subset  $A$  of  $S$  the anti characteristic function,  $C_{A^c}$  is defined by

$$(C_{A^c})(x) = \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{if } x \notin A \end{cases}$$

**Proposition 3.5.** Let  $S$  be a ternary semigroup, then  $(F(S), *)$  is a ternary semigroup.

**Proof.** Let  $f, g$  and  $h$  be elements in  $F(S)$ . It is clear that  $f * g * h$  belongs to  $F(S)$  [15]. Let  $x$  be any element of  $S$  such that  $x \neq abc$  for any  $a, b, c \in S$ , then we have  $((f * g * h) * l * m)(x) = 1 = (f * (g * h * l) * m)(x) = (f * g * h * l * m)(x)$ . Let the element  $x$  can be written as product of three elements in  $S$  then,

$$\begin{aligned}
 ((f * g * h) * l * m)(x) &= \bigwedge_{x=abc} \{(f * g * h)(a) \vee l(b) \vee m(c)\} \\
 &= \bigwedge_{x=abc} \left\{ \bigwedge_{a=pqr} \{f(p) \vee g(q) \vee h(r)\} \vee l(b) \vee m(c) \right\} \\
 &= \bigwedge_{x=(pqr)bc} \{f(p) \vee g(q) \vee h(r) \vee l(b) \vee m(c)\} \\
 &= \bigwedge_{x=p(qrb)c} \{f(p) \vee g(q) \vee h(r) \vee l(b) \vee m(c)\} = \bigwedge_{x=pwc} \{f(p) \vee \bigwedge_{w=qrb} \{g(q) \vee h(r) \vee l(b)\} \\
 &\vee m(c)\} = \bigwedge_{x=pwc} \{f(p) \vee (g * h * l)(w) \vee m(c)\} = (f * (g * h * l) * m)(x).
 \end{aligned}$$

In similar arguments, we conclude that  $(f * g * h) * l * m = f * (g * h * l) * m = f * g * (h * l * m)$ . Therefore,  $(F(S), *)$  is a ternary semigroup.  $\square$

**Lemma 3.6.** Let  $S$  be a ternary semigroup, a non-empty subset  $A$  of  $S$  is a ternary subsemigroup if and only if  $C_{A^c}$  is an anti fuzzy ternary semigroup of  $S$ .

**Proof.** Let  $A$  be a ternary subsemigroup of  $S$  and  $x, y, z$  be elements of  $S$ . Let  $x, y, z \in A$  then  $xyz \in A$ , so we have,

$$(C_{A^c})(xyz) = 0 = (C_{A^c})(x) \vee (C_{A^c})(y) \vee (C_{A^c})(z).$$

Let  $x \in A$  and  $y, z \in S$  then  $(C_{A^c})(x) = 0$  and  $(C_{A^c})(y) = (C_{A^c})(z) = 1$ , so we have

$$(C_{A^c})(xyz) \leq 1 = (C_{A^c})(x) \vee (C_{A^c})(y) \vee (C_{A^c})(z).$$

Now, assume that both  $x$  and  $y$  are not in  $A$  then  $(C_{A^c})(x) = (C_{A^c})(y) = 1$  and so we have

$$(C_{A^c})(xyz) \leq 1 = (C_{A^c})(x) \vee (C_{A^c})(y) \vee (C_{A^c})(z).$$

Thus for all  $x, y, z \in S$  we have  $(C_{A^c})(xyz) \leq (C_{A^c})(x) \vee (C_{A^c})(y) \vee (C_{A^c})(z)$ , which implies that  $C_{A^c}$  is an anti fuzzy ternary semigroup of  $S$ . Conversely, if the elements  $x, y, z$  are in  $A$  then  $(C_{A^c})(x) = (C_{A^c})(y) = (C_{A^c})(z) = 0$ . But  $(C_{A^c})(xyz) \leq (C_{A^c})(x) \vee (C_{A^c})(y) \vee (C_{A^c})(z) = 0$ , which implies that  $xyz \in A$ . Hence,  $A$  is a ternary sub semigroup of  $S$ .  $\square$

**Theorem 3.7.** A fuzzy subset  $f$  of a ternary semigroup  $S$  is an anti fuzzy interior ideal of  $S$  if and only if  $L[f; t] \neq \emptyset$  is an interior ideal of  $S$ .

**Proof.** Let  $f$  be any an anti fuzzy ideal of  $S$ . By theorem 3.6 ([15]),  $L[f; t]$  is a ternary subsemigroup of  $S$ . Let  $a \in L[f; t]$  and  $x, y, w, z \in S$ . Then  $f(a) \leq t$  and hence  $f(xyawz) \leq f(a) \leq t$ . Thus  $xyawz \in L[f; t]$  and so  $L[f; t]$  is an interior ideal of  $S$ .

Conversely, assume that  $L[f; t] \neq \emptyset$  is an interior ideal of  $S$  for all  $t \in [0,1]$ . By theorem 3.6 ([15]),  $f$  is an anti fuzzy ternary subsemigroup of  $S$ . Suppose there exist  $x, y, a, w, z \in S$  such that  $f(xyawz) > f(a)$ . Choose  $t \in [0,1]$  such that  $f(xyawz) > t \geq f(a)$ . Then  $a \in L[f; t]$  but  $xyawz \notin L[f; t]$ , which is a contradiction, hence  $f(xyawz) \leq f(a)$ . Thus  $f$  is an anti interior ideal of  $S$ .  $\square$

**Theorem 3.8.** A non-empty subset  $A$  of a ternary semigroup  $S$  is an interior ideal of  $S$  if and only if the fuzzy set  $f$  of  $S$  is defined by

$$f(x) = \begin{cases} t & \text{if } x \notin A \\ r & \text{if } x \in A \end{cases}$$

is an anti fuzzy interior ideal of  $S$ , where  $t, r \in [0,1]$  such that  $t \geq r$ .

**Proof.** Suppose  $A$  is an interior ideal of  $S$  and  $x, y, a, w, z \in S$ . If  $a \in A$  then  $xyawz \in A$ , hence  $f(xyawz) = f(a) = r$ . If  $a \notin A$  then  $f(a) = t \geq f(xyawz)$ . Hence and by theorem 3.11 ([15]),  $f$  is an anti fuzzy interior ideal of  $S$ .

Conversely, assume that  $f$  is an anti fuzzy interior ideal of  $S$ . By theorem 3.11 ([15]),  $A$  is a ternary subsemigroup of  $S$ . Let  $a \in A$  and  $x, y, w, z \in S$ , then  $f(xyawz) \leq f(a) = r$ . Thus  $xyawz \in A$  and hence  $A$  is an interior ideal of  $S$ .  $\square$

**Corollary 3.9.** A non-empty subset  $A$  of a semigroup  $S$  is an interior ideal of  $S$  if and only if the characteristic function of the complement of  $A$ , that is,  $C_{A^c}$  is an anti fuzzy interior ideal of  $S$ .

**Proposition 3.10.** Let  $f$  be an anti fuzzy ideal of  $S$ . Then  $f$  is an anti fuzzy interior ideal of  $S$ .

*Proof.* Since  $f$  is an anti fuzzy ideal of  $S$ ,  $f$  is an anti fuzzy ternary subsemigroup of  $S$  (c.f [15]), and for any  $a, w, z \in S$ ,  $f(awz) \leq f(a)$ ,  $f(awz) \leq f(w)$  and  $f(awz) \leq f(z)$  which implies that  $f(awz) \leq \max\{f(a), f(w), f(z)\} \leq f(a)$ . Hence  $f$  is an anti fuzzy ternary subsemigroup of  $S$ . Now let  $x, y, a, w, z \in S$ , then  $f(xyawz) = f(xy(awz)) \leq f(awz) \leq f(a)$ . Consequently,  $f$  is an anti fuzzy interior ideal of  $S$ .  $\square$

Regarding the converse, we have the following result.

**Proposition 3.11.** Let  $S$  be a regular ternary semigroup and  $f$  be an anti fuzzy interior ideal of  $S$ . Then  $f$  is an anti fuzzy ideal of  $S$ .

*Proof.* Let  $x, y, z \in S$ . Since  $S$  is regular, for any  $x \in S$  there exists  $a \in S$  such that  $x = xax$ . Then  $f(xyz) = f(xaxyz) \leq f(x)$ . So  $f$  is an anti fuzzy right ideal of  $S$ . Similarly, we can prove that  $f$  is an anti fuzzy left ideal of  $S$ . It remains to show that  $f$  is an anti fuzzy lateral ideal of  $S$ . For this purpose, assume that  $y, a \in S$  such that  $y = yay$  (since  $S$  is regular). Since  $f$  is an anti fuzzy ternary subsemigroup of  $S$ ,  $f(y) = f(yay) \leq f(y) \vee f(a) \vee f(y)$  which implies that  $f(a) \leq f(y)$ . Hence  $f(xyz) = f(xyayz) \leq f(a) \leq f(y)$ . Therefore,  $f$  is an anti fuzzy lateral ideal of  $S$ .  $\square$

**Proposition 3.12.** If  $\{f_i: i \in I\}$  is a family of anti fuzzy interior ideals of  $S$ , then so is  $\bigcup_{i \in I} f_i = \sup\{f_i(x): i \in I, x \in S\}$ , provided it is non-empty.

*Proof.* Let  $x, y, z \in S$ . Then we have

$$\begin{aligned} (\bigcup_{i \in I} f_i)(xyz) &= \sup\{f_i(xyz): i \in I\} \\ &\leq \sup\{\max\{f_i(x), f_i(y), f_i(z)\}: i \in I\} \\ &= \max\left\{\sup\{f_i(x): i \in I\}, \sup\{f_i(y): i \in I\}, \sup\{f_i(z): i \in I\}\right\} \\ &= \max\left\{\left(\bigcup_{i \in I} f_i\right)(x), \left(\bigcup_{i \in I} f_i\right)(y), \left(\bigcup_{i \in I} f_i\right)(z)\right\}. \end{aligned}$$

Hence  $\bigcup_{i \in I} f_i$  is an anti fuzzy ternary subsemigroup of  $S$ .

Now  $(\bigcup_{i \in I} f_i)(xyawz) = \sup\{f_i(xyawz): i \in I\} \leq \sup\{f_i(a): i \in I\} = (\bigcup_{i \in I} f_i)(a)$ . Consequently,  $\bigcup_{i \in I} f_i$  is an anti fuzzy interior ideal of  $S$ .  $\square$

**Lemma 3.13.** In a regular ternary semigroup  $S$ , every anti fuzzy ideal is idempotent.

*Proof.* Let  $f$  be an anti fuzzy ideal of a ternary semigroup  $S$ , then obviously  $f * f * f \supseteq f$ . Since  $S$  is regular, so for each element  $a \in S$  there exists an element  $x \in S$  such that  $a = axa = a(xax)a$ . Thus we have

$$\begin{aligned} (f * f * f)(a) &= \bigwedge_{a=pqr} \{f(p) \vee f(q) \vee f(r)\} \\ &\leq \{f(a) \vee f(xax) \vee f(a)\} \\ &\leq \{f(a) \vee f(a) \vee f(a)\} = f(a). \end{aligned}$$

Thus  $f * f * f \subseteq f$ . Hence  $f * f * f = f$ .  $\square$

**Corollary 3.14.** In a regular ternary semigroup  $S$ , every anti fuzzy interior ideal is idempotent.

**Theorem 3.15.** A ternary semigroup  $S$  is regular if and only if every anti fuzzy ideal is idempotent.

*Proof.* Let  $S$  be a regular ternary semigroup and  $f$  be an anti fuzzy ideal of  $S$ . Then lemma 3.13 implies that  $f$  is idempotent.

Conversely, suppose that every anti fuzzy ideal of  $S$  is idempotent. Let  $f, g, h$  be anti fuzzy ideals of  $S$ . Then by theorem 3.18 ([15]),  $f * g * h \supseteq f \cup g \cup h$ . Also,  $(f \cup g \cup h) * (f \cup g \cup h) * (f \cup g \cup h) \supseteq f * g * h$ . Since by lemma 3.14 ([17]),  $f \cup g \cup h$  is an anti fuzzy ideal of  $S$ ,  $(f \cup g \cup h) * (f \cup g \cup h) * (f \cup g \cup h) = f \cup g \cup h$ . Thus  $f \cup g \cup h \supseteq f * g * h$  and hence  $f * g * h = f \cup g \cup h$ . Therefore by theorem 4.1 ([15]),  $S$  is a regular ternary semigroup.  $\square$

**Theorem 3.16.** *Let  $A$  be an interior ideal of a ternary semigroup  $S$ . Then for every  $t \in [0,1]$ , there exists an anti fuzzy interior ideal  $f$  of  $S$  such that  $[f; t] = A^c$ .*

**Proof.** Let  $A$  be an interior ideal of  $S$  and let  $f$  be an anti fuzzy ideal of  $S$  defined by

$$f(x) = \begin{cases} 0 & \text{if } x \in A \\ t & \text{if } x \notin A. \end{cases}$$

Where  $t \in (0,1]$ . Let  $x, y, z \in S$ . If  $x, y, z \in A$  then  $xyz \in A$ . Hence  $f(xyz) = 0 \leq \max \{f(x), f(y), f(z)\}$ . If  $x, y, z \notin A$  then  $f(x) = f(y) = f(z) = t$ , and so  $f(xyz) \leq \max \{f(x), f(y), f(z)\}$ . If exactly one of  $x, y$  and  $z$  does not belong to  $A$  then exactly one of  $f(x), f(y), f(z)$  is equal to  $t$ , so  $f(xyz) \leq \max \{f(x), f(y), f(z)\}$ . Hence  $f$  is an anti fuzzy ternary subsemigroup of  $S$ . Let  $x, y, a, w, z \in S$ . If  $a \in A$  then  $xyawz \in A$ . Thus  $f(xyawz) = 0 = f(a)$ . If  $a \notin A$  then  $f(a) = t$  and so  $f(xyawz) \leq t = f(a)$ . Therefore,  $f$  is an anti fuzzy interior ideal of  $S$ , and it is clear that  $[f; t] = A^c$ .  $\square$

#### 4. ANTI FUZZY CHARACTERISTIC INTERIOR IDEALS

In [2], the authors introduced the definition of characteristic interior ideals and fuzzy characteristic interior ideals of a semigroup. In this section we define anti fuzzy characteristic interior ideal of a ternary semigroup  $S$  and study some basic properties of this notion. Let  $S$  and  $T$  be a ternary semigroups. By a homomorphism we mean a mapping  $\alpha: S \rightarrow T$  satisfying  $\alpha(xyz) = \alpha(x)\alpha(y)\alpha(z)$  for all  $x, y, z \in S$ . In what follows,  $Aut(S)$  will denote the set of all automorphisms of the ternary semigroup  $S$ .

**Definition 4.1.** *An interior ideal  $M$  of  $S$  is called a characteristic interior ideal of  $S$  if  $\alpha(M) = M$  for all  $\alpha \in Aut(S)$ .*

**Definition 4.2.** *An anti fuzzy interior ideal  $f$  of  $S$  is called an anti fuzzy characteristic interior ideal of  $S$  if  $f(\alpha(x)) = f(x)$  for all  $\alpha \in Aut(S)$ .*

**Theorem 4.3.** *A non-empty fuzzy subset  $f$  of  $S$  is an anti fuzzy characteristic interior ideal of  $S$  if and only if  $L[f; t] \neq \emptyset$  is a characteristic interior ideal of  $S$ .*

**Proof.** Let  $f$  be an anti fuzzy characteristic interior ideal of  $S$  and  $L[f; t] \neq \emptyset$ . Then by theorem 3.7,  $L[f; t]$  is an interior ideal of  $S$ . Let  $\alpha \in Aut(S)$  and  $x \in L[f; t]$ , then  $f(x) \leq t$  and  $f(\alpha(x)) = f(x) \leq t$ . Hence  $\alpha(x) \in L[f; t]$ . This implies that  $\alpha(L[f; t]) \subseteq L[f; t]$ . For the reverse inclusion, let  $x \in L[f; t]$  and  $y \in S$  such that  $\alpha(y) = x$ . Then  $f(y) = f(\alpha(y)) = f(x) \leq t$  and so  $y \in L[f; t]$ . Consequently,  $\alpha(y) \in \alpha(L[f; t])$  whence  $x \in \alpha(L[f; t])$ . Therefore,  $L[f; t] \subseteq \alpha(L[f; t])$ . Thus we conclude that  $L[f; t] = \alpha(L[f; t])$ .

Conversely, let  $L[f; t]$  be a characteristic interior ideal of  $S$  for all  $t \in [0,1]$ . Let  $\alpha \in Aut(S)$ ,  $x \in S$  and  $f(x) = t$ . Then  $x \in L[f; t]$ . Since  $L[f; t] = \alpha(L[f; t])$ , then  $\alpha(x) \in L[f; t]$  and hence  $f(\alpha(x)) = t \leq t$ . Then  $\alpha(x) \in L[f; t] = \alpha(L[f; t])$ . Since  $\alpha$  is injective, it follows that  $x \in L[f; t]$ . This implies that  $f(x) \leq t$  and hence  $t \leq t$ . Thus we obtain  $f(\alpha(x)) = t = f(x)$ , showing that  $f$  is an anti fuzzy characteristic interior ideal of  $S$ .  $\square$

**Theorem 4.4.** *A non-empty subset  $A$  of a ternary semigroup  $S$  is a characteristic interior ideal of  $S$  if and only if the fuzzy set  $f$  of  $S$  is defined by*

$$f(x) = \begin{cases} t & \text{if } x \notin A \\ r & \text{if } x \in A \end{cases}$$

*is an anti fuzzy characteristic interior ideal of  $S$ , where  $t, r \in [0,1]$  such that  $t \geq r$ .*

**Proof.** Suppose  $A$  is a characteristic interior ideal of  $S$  and  $x \in S$ . By theorem 3.8,  $f$  is an anti fuzzy interior ideal of  $S$ . If  $x \in A$ , then  $f(x) = r$ . Let  $\alpha \in Aut(S)$ , then  $\alpha(x) \in \alpha(A) = A$  and hence  $f(\alpha(x)) = r$ . If  $x \notin A$  then  $f(x) = t$  and for all  $\alpha \in Aut(S)$ ,  $\alpha(x) \notin \alpha(A)$ , whence  $f(\alpha(x)) = t$ . Thus we see that  $f(\alpha(x)) = f(x)$  for all  $x \in S$ .

Conversely, assume that  $f$  is an anti fuzzy characteristic interior ideal of  $S$ . Then by theorem 3.8,  $A$  is an interior ideal of  $S$ . Let  $\alpha \in Aut(S)$  and  $a \in A$ . Then  $f(a) = r$  and  $f(\alpha(a)) = f(a) = r$ . It follows that  $\alpha(a) \in A$  and hence  $\alpha(A) \subseteq A$ . Now let  $a \in A$  and  $b \in S$  such that  $\alpha(b) = a$ . Then  $f(b) = f(\alpha(b)) = f(a) = r$ . If possible, suppose  $b \notin A$ ,

then  $f(b) = t$ . Since  $f(\alpha(b)) = f(b) = t$ , it follows that  $\alpha(b) = a \notin A$ , which is a contradiction. Hence  $b \in A$  and consequently  $\alpha(b) \in A$ . Thus we have  $A \subseteq \alpha(A)$ . Therefore,  $A$  is a characteristic interior ideal of  $S$ .  $\square$

**Lemma 4.5.** *If  $f$  is an anti fuzzy interior ideal of  $S$ , then so is  $f^t$  for every real number  $t \geq 0$ , where  $f^t$  is non-empty fuzzy set of  $S$  defined by  $f^t(x) = (f(x))^t$  for all  $x \in S$ .*

**Proof.** Let  $x, y, a, w, z \in S$  and let  $t \geq 0$  be any real number. If  $f(x) \geq f(y) \geq f(z)$ , then  $f(xyz) \leq f(x) \vee f(y) \vee f(z) \leq f(x)$ . This implies that  $f^t(xyz) = (f(xyz))^t \leq (f(x))^t = f^t(x)$  and  $f^t(x) \geq f^t(y) \geq f^t(z)$ . Hence  $f^t(xyz) \leq f^t(x) \vee f^t(y) \vee f^t(z)$ . The argument is similar if  $f(x) \leq f(y) \leq f(z)$ . Therefore,  $f^t$  is an anti fuzzy ternary subsemigroup of  $S$ . Since  $f$  is an anti fuzzy interior ideal of  $S$ , we obtain  $f(xyawz) \leq f(a)$ . It follows that  $f^t(xyawz) = (f(xyawz))^t \leq (f(a))^t = f^t(a)$ . This completes the proof.  $\square$

Let  $f$  be an anti fuzzy characteristic interior ideal of a ternary semigroup  $S$ . Let  $x \in S$  and  $\alpha \in \text{Aut}(S)$ . Then  $f(\alpha(x)) = f(x)$ , which implies that  $f^t(\alpha(x)) = (f(\alpha(x)))^t = (f(x))^t = f^t(x)$  for every real number  $t \geq 0$ . Thus we have the following theorem

**Theorem 4.6.** *Let  $f$  be an anti fuzzy characteristic interior ideal of a ternary semigroup  $S$ , then  $f^t$  is an anti fuzzy characteristic interior ideal of  $S$  for every real number  $t \geq 0$ .*

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