

Evaluation Methodology on Vibration Serviceability of Bridge by using Non-Contact Vibration Measurement Method

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ABSTRACT— *In order to evaluate vibration serviceability by means of non-contact vibration meter, serviceability of existing bridge were experimentally evaluated by using laser vibration meter, and the methodology for the serviceability were also proposed. Test results were analyzed and compared with evaluation results acquired by using accelerometer and LVDT. To testify accuracy of natural frequency by vibration data through laser vibration meter, it was compared with that acquired by using accelerometer. According to test and comparison results, it is showed that serviceability can be evaluated properly enough from the tolerance curve of Reiher-Meister for the estimated acceleration that can be calculated by the numerical differentiation of measured velocity.*

Keywords— Dynamics, Evaluation, Bridges, Field Testing

1. INTRODUCTION

The recent technological advances realized in various fields have also influenced bridge structures resulting gradually in longer span lengths and lighter structures. These advances stressed the importance of the serviceability of the structure as a major topic in the design, and triggered research on the establishment of quantitative criteria enabling to identify the magnitude of the vibrations perceived by the pedestrians or vehicles' passengers crossing the bridge. In general, accelerometers or deflectometers are mounted to evaluate the vibrations of the bridge. However, measurement for the experimental assessment of the vibrational serviceability becomes extremely troublesome when the bridge is elevated at high altitude and is crossing a river or the sea as it is often the case.

This paper analyzes the dynamic characteristics of the bridge using a device enabling to measure vibrations without being mounted on the structure differently from the accelerometers or deflectometers, which are bonded to the structure to assess the vibrational serviceability of the bridge. Thereafter, this paper intends to investigate experimentally the applicability of non-contact device's results for the assessment of the vibrational serviceability. To that goal, the natural frequency of an actual bridge is calculated by using vibration data which are acquired by laser vibration measurement device and the measurements are compared to those obtained from accelerometers in order to examine its applicability.

The laser vibration measurement device overcomes the limitations encountered by the conventional contact-type accelerometer. For example, the device considered in this study offers a very broad range of measurable frequencies running between 0Hz and 30MHz. The available range of measured speed is 0.5 μ m/s ~ 30m/s, which allows a resolution of pm order when referring to the displacement. As a matter of fact, the use of laser annihilates the error of the measuring device caused by mass loading, and the absence of cables eliminates practically the limitations related to the measurement distance. Moreover, the use of laser for measurement remains possible even for very remote distance since its frequency characteristics does not change with the distance. Another major advantage is the high independence from ambient noise influence. Furthermore, the use of a very tiny laser spot enables the measurement of the dynamic characteristics at locations in the structure which were inaccessible for the mounting of sensors. Table 1 compares the performances of accelerometer and laser vibration meter.

Table 1: Comparison of the performances of accelerometer and laser vibration meter

Performance	Accelerometer	Laser vibration meter
Bandwidth	0.5~10,000Hz	0~30MHz
Life cycle cost	High (damage)	High (investment)
Ambient influence	Low	Very low
Application	When absolute vibration is required	Always if optical access is available

2. EVALUATION METHOD OF VIBRATIONAL SERVICEABILITY BY NON-CONTACT VIBRATION MEASUREMENT

2.1 Principles of vibration measurement

The use of the laser Doppler vibrometer (LDV) stands as one of the methods using non-contact devices for the measurement of vibrations in a structure.

The laser beam with a constant period from the LDV is directed at the surface of interest, and the vibration amplitude and frequency are extracted from the Doppler shift of the laser beam frequency due to the motion of the surface (Figure 1). For example, the speedometer used to detect vehicles running beyond the speed limits is also applying the Doppler effect. In other words, when the ultra-short wave with frequency f is shot by the radar on the approaching vehicle, the reflected ultra-short wave has a higher frequency f' due to the motion of the vehicle relative to the radar. Then, the radar converts the difference between these two frequencies f and f' into the vehicle speed so as to display the speed to the measurer.

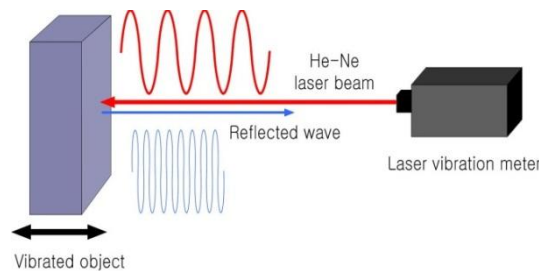


Figure 1: Frequency shift of laser beam by Doppler effect

2.2 Evaluation methods of vibrational serviceability

Even if various methods are applied to assess the vibrational serviceability of structures, the ISO sensitivity curve (ISO 2631, 1997), the formula proposed by Postlethwaite (1944), and the vibration tolerance curves of Reiher-Meister have been adopted to assess the vibrational serviceability of bridges (Kim, 2006). The comparative studies related to the assessment of the vibrational serviceability of in-service bridges applying the ISO standards and the Reiher-Meister curves (Kim et al., 2002; KISTEC, 2000) reported that the assessment criteria of Reiher-Meister include the assessment ranges of the other standards making them more general.

Accordingly, this paper adopts the Reiher-Meister curves for the assessment of the vibrational serviceability of bridges. Figure 2 shows the Reiher-Meister's tolerance curves with respect to the amplitudes of displacement (Figure 2(a)) and acceleration (Figure 2(b)). The horizontal axis represents the frequency and the vertical axis stands for the amplitude of displacement and acceleration. These curves are the results of large scale tests and assess the vibrational serviceability by five degrees of perception from "just perceptible (E)" to "painful (A)". It can be observed that the sensitivity to the amplitude of displacement increases with higher frequencies whereas the sensitivity to the amplitude of acceleration weakens at higher frequencies (Kim, 2006; Kim et al., 2002).

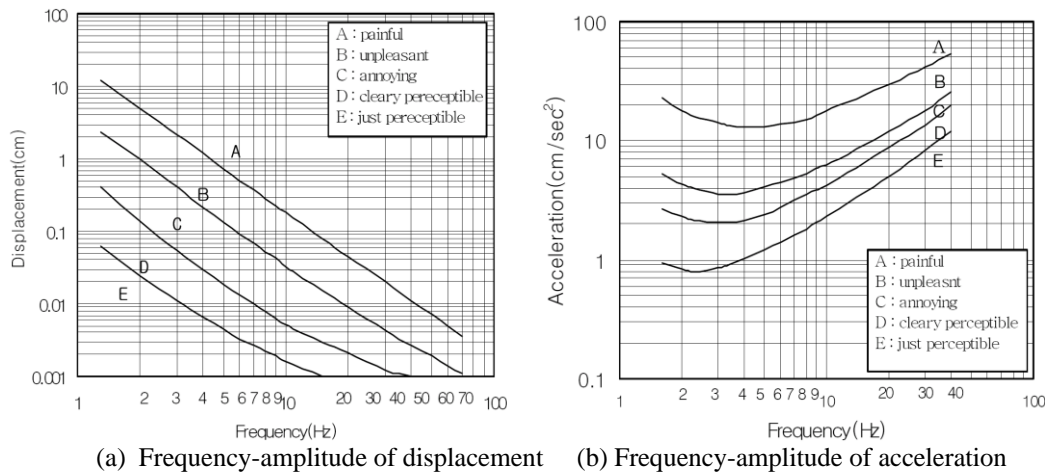


Figure 2: Tolerance curves of Reiher-Meister

The procedure for the assessment of the vibrational serviceability proceeds as follows. First, the natural frequency of the bridge of interest is calculated through FFT analysis using the signals measured by the sensors installed on the structure. The maximum amplitudes of the measured displacement and acceleration are used for the analysis. Here, the maximum amplitude corresponds to the half of the peak to peak value of the input signals in the time history. The so-obtained natural frequency and the maximum amplitudes are then compared to the Reiher-Meister tolerance curves to assess the vibrational serviceability of the bridge (Kim, 2006; Kim et al., 2002).

3. EXAMINATION OF THE APPLICABILITY OF NON-CONTACT VIBROMETER

Prior to assessing the vibrational serviceability of a bridge using a non-contact vibrometer, the applicability of the non-contact vibrometer was verified by direct evaluation by means of tests on the accuracy of the measurements produced by the vibration measurement method according to the site conditions. A portable non-contact LDV was selected and used to calculate the natural frequency of the actual bridge. The results were then compared to the calculated results obtained from accelerometers installed at identical locations. Here, the vibration measurement method according to the site conditions means appropriate installation spots and setting methods considering the cases where the measurement spot is easily accessible and the cases where access is impossible due to topographical features or due to the height of the bridge.

3.1 Test bridge

The bridge selected for the verification of the performance of the vibrometer is Sab-Kyo Bridge, located on national road 34 in Korea (Figures. 3 and 4). Sab-Kyo Bridge is a steel box girder bridge with length of 560 m and width of 20.1 m. It is a ten-span bridge with two central spans of 80 m and four spans of 50 m at both sides. The measurement spot is located at the bottom of the steel box of the left-hand side 50 m-span (A1-P1) and the height from the ground to the bottom of the steel box runs around 440 cm.



Figure 3: Side view of Sab-Kyo bridge

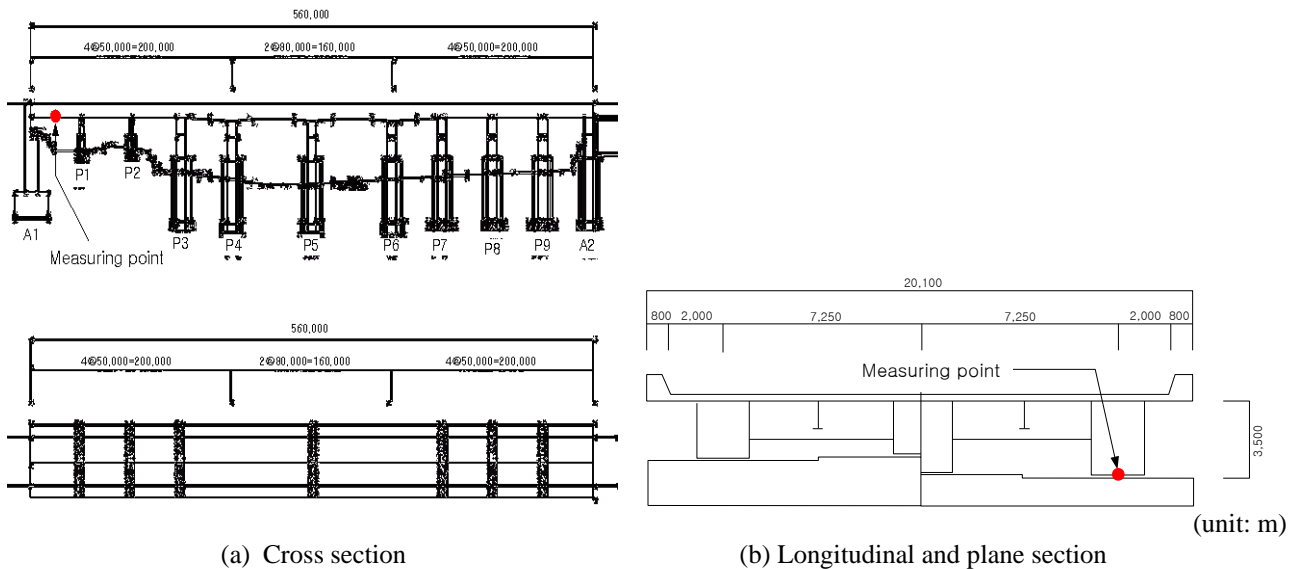


Figure 4: Location of vibration measurement point

3.2 Test method

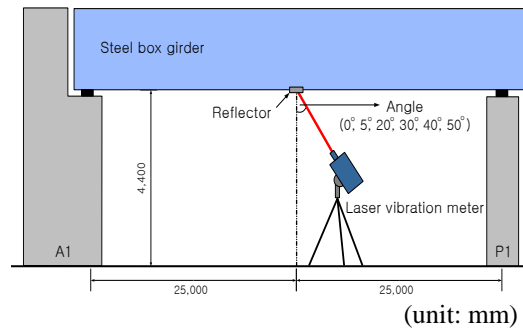
The methods generally adopted to excite the bridge for the calculation of its natural frequency are the forced vibration method using an exciter, the impact test, the method using a testing vehicle, and the ambient vibration test applying operation loads like traffic load or wind load. The ambient vibration test measures the vibrations induced by the loads (traffic, wind, etc.) acting the operation of the bridge. These vibrations are then processed to obtain the natural frequency of the bridge. This method is widely used since it easy to apply and does not require special devices to excite the structure. Accordingly, this method is selected for the test. As shown in Figure 5, the measurement spot is located at the bottom of the steel box of the left-hand side 50 m-span (A1-P1) and the height from the ground to the bottom of the steel box is about 4.4 m.

As listed in Table 2, the test variables are the duration and angle of measurement, and the eventual attachment of a reflector. A total of 10 cases are assumed. Here, the selection of the measurement duration as test variable has been governed by the consideration of the passage of an arbitrary vehicle with unspecified weight during the execution of the ambient vibration test. The angle of measurement stands for the discharge angle of the laser beam with reference to the straight line perpendicular to the bottom of the steel box as shown in the test setting of Figure 5.

The results of the research dedicated to ambient vibration measurement recommend to set the duration of measurement to 20 minutes to obtain stable and accurate measurements. This recommendation aims to afford sufficiently long time for the measurement so as to enhance the readability of the response spectrum (Ahn, 2000). Considering this recommendation, 4 durations of measurement with a longest time of 30 minutes are adopted in our tests. Furthermore, the selection of the angle of measurement as test variable has been dictated by the will to consider the case where the laser beam cannot be directed perpendicularly to the measurement surface as it may occur for inaccessible locations due to the site conditions like the height or topography of the bridge. In addition, the natural frequency of the bridge is also measured by means of accelerometers attached separately in order to verify the feasibility of the measurement results produced by the non-contact vibrometer.

Table 2: List of test variables

Type	Time duration(min)	Angle	Reflector	Number of tests
Case 1	30	0°	Attached	4
Case 2	20			2
Case 3	15			2
Case 4	5			2
Case 5	20		not attached	2
Case 6	15	5°	Attached	1
Case 7		20°		1
Case 8		30°		1
Case 9		40°		1
Case 10		50°		1



(a) Conceptual drawing of measurement



(b) Scaffold set-up and accelerometer installation



(c) Vibration meter set-up



(d) PC program setting



(e) Vibration measurement

Figure 5: Test set-up

3.3 Test results and discussion

Natural frequency according to the duration of measurement

Even if the duration of measurement was varied from 5 minutes to a maximum of 30 minutes in order to consider the passage of unspecified vehicles, all the measurements resulted in an identical natural frequency of 2.12 Hz as 1st natural frequency (Figure 6). The identical natural frequency obtained through the varying durations of measurement can be explained by the following two reasons. The traffic was relatively poor during the period of the test and, in view of the measured signals, the absence of vehicles inducing relatively large amplitude of vibration could not provoke interaction of interest. In addition, the longest span Sab-Kyo Bridge reaches 80 m and its side spans are 50 m long, which classify it as a short-to-medium span bridge. Accordingly, the effect of wind loads on the dynamic characteristics of the bridge remains insignificant compared to long-span bridges like cable-stayed bridges. In the ambient vibration test, the effects of the white noise-like wind and traffic loads remained relatively small and led to identical natural frequency despite the change of the duration of measurement.

Natural frequency in the case of not attached reflector

In general, a reflective film specially fabricated to have a high reflexivity is attached on the measurement spot to obtain a stable signal and prevent the dispersion of the laser beam. However, this reflector could not be attached in Case 5 due to the inaccessibility of the measurement spot and this case was adopted for trial test considering the case of not attached reflector.

In view of the wave of the measured signal, the first test was polluted by slight noise before and after the passage of the vehicle. In the second cycle of test, the input wave was polluted by noise at the whole but no particular feature could

be observed in its overall shape. A natural frequency of 2.12 Hz was derived from the input signal identically to Case 1 to Case 4. The absence of change in the natural frequency can be explained by the satisfactory state of the surface of the steel box at the measurement spot and the relatively short measurement distance shorter than 5 m.

Natural frequency according to the angle of measurement

The angle formed by the line perpendicular to the measurement surface and the discharge line of the laser beam was varied as 5°, 20°, 30°, 40°, 50° for a total of 5 cases. The natural frequency was measured during 15 minutes for each of these cases. In most cases, no problem occurred for the location of the laser focus and the sensitivity adjustment. However, for the angle of 50°, it was difficult to capture visually the location of the focus, which impeded to place exactly the focus on the 1 cm rectangular reflector and, by the way, induced relative loss of the measurement sensitivity.

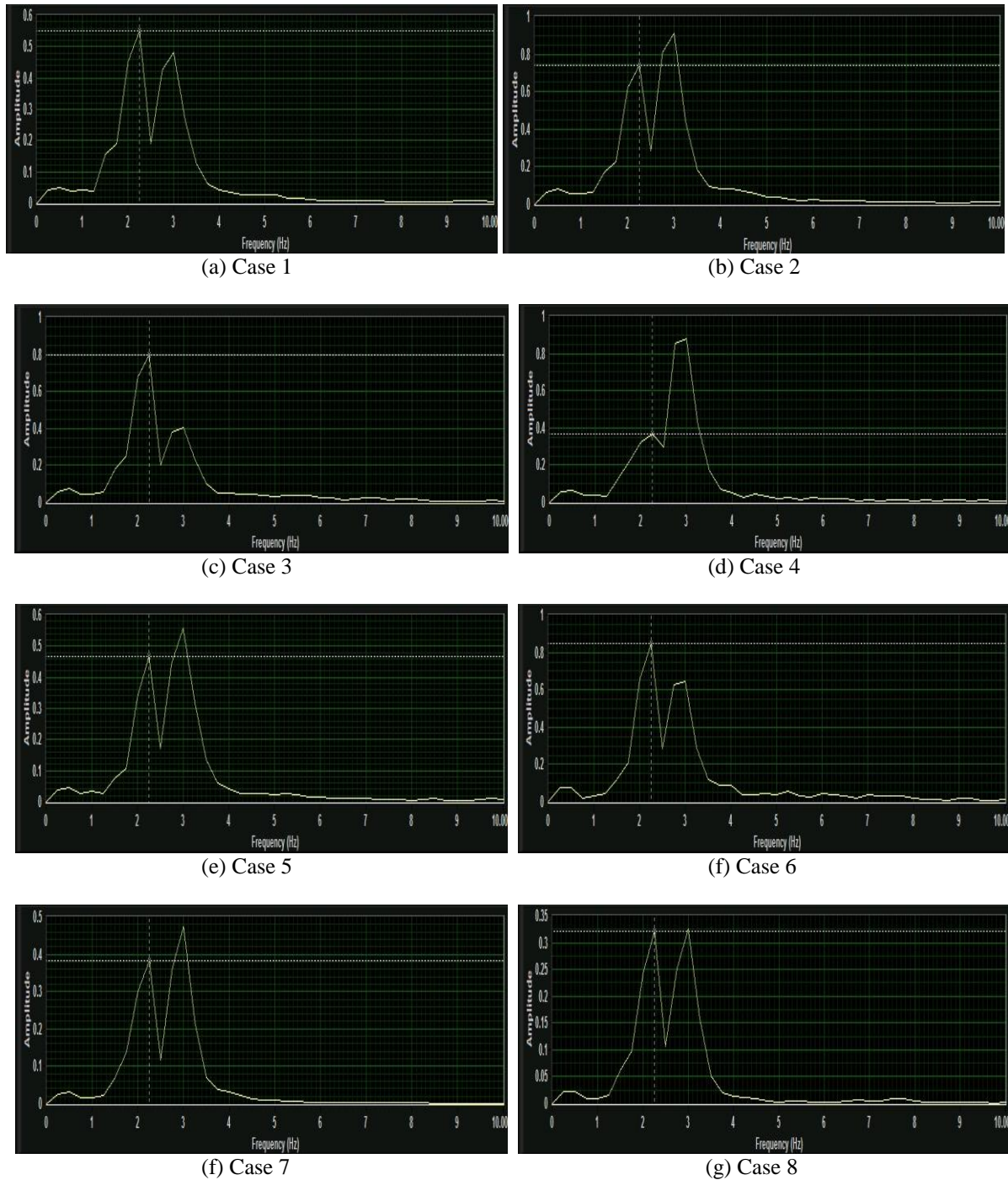


Figure 6: Frequency analysis results(Continued)

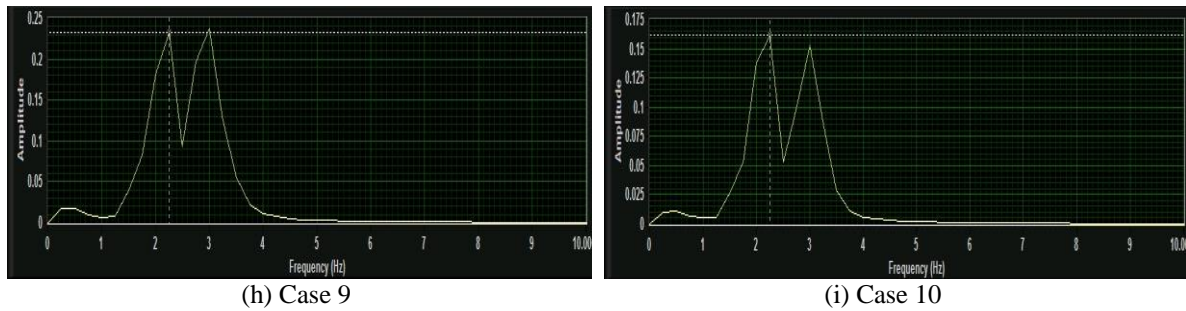


Figure 6: Frequency analysis results

The natural frequency measured with respect to the given tilt angles was 2.12 Hz for all the cases from Case 6 to Case 10, and corresponded to the natural frequency measured when the laser beam was perpendicular to the measurement surface. Accordingly, the amount and path of the reflected wave generated when the laser beam is discharged obliquely to the measurement surface are seen to have insignificant effect on the measurement of the natural frequency.

For Case 10 with a tilt angle of 50° , the location of the focus deviated slightly from the exact position and degraded the measurement sensitivity. However, such conditions did not affect the frequency analysis. This can be explained by the very satisfactory state of the surface of the steel box of Sab-Kyo Bridge. However, in general, the measurement angle, a , is determined with respect to the geographical features of the site. If δ is the maximum expected deflection, the minimum length, l , of the reflector is determined by Equation 1. Since the laser focus should be located within the reflector, the measurement sensitivity is maintained in the optimal state (Figure 7). For example, in the case of Sab-Kyo Bridge for which a maximum dynamic deflection of 10 mm was measured, the measurement height, H , is 4.4 m and the horizontal distance, D , from the reflector to the measurement location is 5.24 m leading to a measurement angle, a , of 50° (the distance of 5.24 m is assumed considering the topographical conditions). In this case, the minimum length, l , of the reflector should be approximately 24 mm to maintain optimal measurement sensitivity.

$$l = 2\delta \tan a = 2\delta \frac{D}{H} \quad (\text{Equation 1})$$

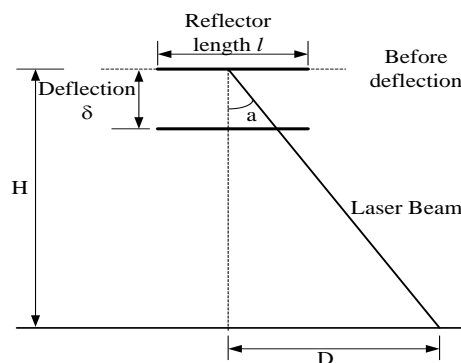


Figure 7: Determination of the length of the reflector for optimal measurement sensitivity

Comparison of the measurements and the values obtained by the simplified formula

The ambient vibration test of Sab-Kyo Bridge using a LVD resulted in a natural frequency of 2.12 Hz. The feasibility of this result is verified by comparing it with that obtained from the simplified formula for the calculation of the natural frequency and that derived from the measurement of the accelerometers.

The factors affecting the natural frequency of a bridge are diversified including the type of the bridge and piers, the span length, the type of bearings, the weight and speed of the vehicle, and the state of the road surface. However, research reported that the natural frequency is highly correlated to the span length (Bachmann, 1995; Lee, 1997), and can be expressed by the following simplified Equation 2 and Equation 3.

$$f = \frac{100}{L} \quad (\text{Equation 2})$$

$$f = 80L^{-0.87} \quad (\text{Equation 3})$$

Substituting the span length of 50 m of Sab-Kyo Bridge in these equations leads to respective values of 2.00 Hz and 2.66 Hz for the natural frequency. The difference between the experimentally obtained value of 2.12 Hz and the value calculated by Equation 2 falls thus within 6%. Moreover, the natural frequency derived from the measurement of the accelerometers being 2.075 Hz, this difference drops down to about 2%. Besides, the long-term monitoring of Sab-Kyo Bridge gave a natural frequency of 2.14 Hz (MOCT, 2003), reducing the difference to below 1%. Accordingly, no particular problem could be observed regard to the measurement of the natural frequency of the bridge.

4. ASSESSMENT OF VIBRATIONAL SERVICEABILITY

The feasibility of the non-contact LDV has been verified considering the duration of measurement, the tilt angle of measurement, and the eventual attachment of reflector as test variables. The next step is to investigate experimentally the applicability of the non-contact vibrometer for the assessment of the vibrational serviceability by comparing the results with those provided by accelerometers and displacement gauges on an in-service bridge.

The velocity response according to the traffic was measured using the non-contact vibrometer. In addition, the acceleration and displacement were measured by installing accelerometers and LVDTs. The measurement of these responses is necessary if the Reiher-Meister curves shown in Figure 2 are used to assess the vibrational serviceability of the structure. However, the use of the LDV selected as non-contact vibrometer measures the velocity of the structure, which means that the displacement should be obtained by numerical integration of the velocity and the acceleration should be estimated by numerical derivation of the velocity. Accordingly, it appears that the accuracy of the so-estimated acceleration and displacement should be verified. Therefore, experimental verification will be conducted by exploiting the data measured on the bridge. To that goal, the acceleration and displacement measured by the accelerometers and LVDTs will be compared to the values obtained by numerical derivation and numerical integration of the measurements provided by the non-contact LDV. Here, the PC program developed for the establishment of the non-contact vibration measurement system is used for the data acquisition and frequency analysis. The numerical integration and derivation are performed in the time domain.

4.1 Test method

The bridge selected for the assessment of the vibrational serviceability is Sab-Kyo Bridge, the bridge used for the ambient vibration test conducted for the verification of the feasibility of the non-contact vibrometer. Here also, the measurement spot for the accelerometer, LVDT and LDV is located at the bottom of the steel box of the left-hand side 50 m-span (A1-P1).

Since the height from the ground to the measurement spot runs around 4.4 m, a scaffold was installed to attach the accelerometer and LVDT. After their installation, the sensors were connected to a dynamic data logger and the setting was completed to measure the dynamic displacement and acceleration by setting the initial values of the program (Figure 8). The setting process of the LDV was identical to that applied for the feasibility test. Figure 8(d) illustrates the setting completed for the assessment of the vibrational serviceability of Sab-Kyo Bridge by adjusting the optimal laser focus. As shown in Figure 8(d), the measurement spot is identical for the accelerometer, LVDT and LDV to enable simultaneous measurement. The velocity is measured using the LDV and the results are compared with those obtained by the conventional contact-type vibration measurement method to verify experimentally the applicability of the non-contact vibrometer for the assessment of the vibrational serviceability of the in-service bridge.



(a) Accelerometer and LVDT

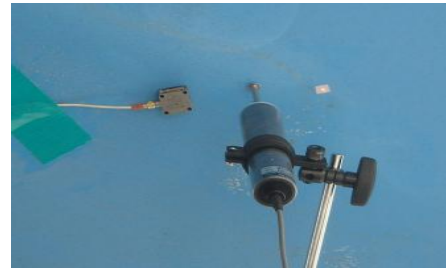


(b) Data logger set-up

Figure 8: Test set-up(Continued)



(c) Vibration meter set-up

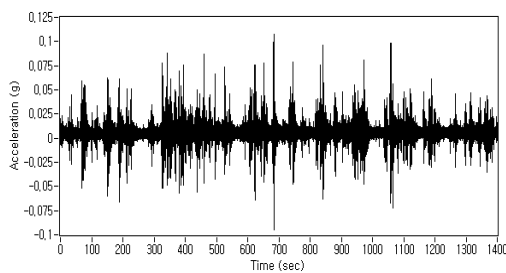


(d) Focus adjustment and measuring

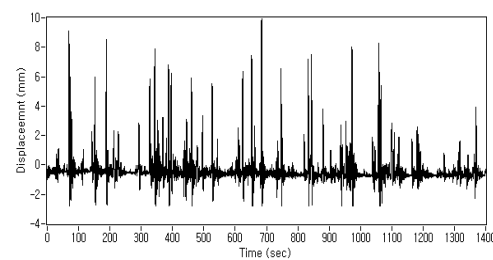
Figure 8: Test set-up

4.2 Vibrational serviceability with respect to the amplitude of acceleration

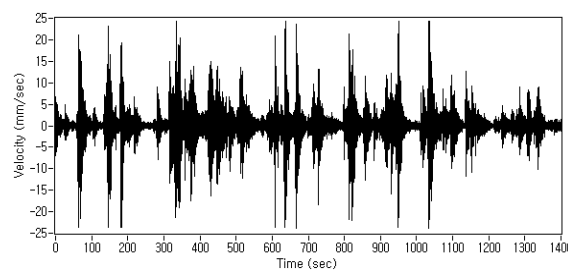
Figure 9 plots the acceleration, displacement and velocity responses measured during 30 minutes at identical location.



(a) Acceleration time history



(b) Displacement time history



(c) Velocity time history

Figure 9: Measured time histories

The representative section among the whole measured velocity responses is selected to estimate the acceleration component by numerical derivation. The estimated acceleration is then compared to the signal measured by the accelerometer. Figure 10(a) plots the acceleration time history measured by the accelerometer during an identical measurement period. Figure 10(b) plots the velocity time history measured by the laser vibration meter in the same section. Figure 10(c) plots the acceleration estimated by numerical derivation of the velocity time history without signal processing. Figure 10(a) and Figure 10(c) should show practically similar trends but are seen to exhibit totally different patterns. Such discrepancy can be attributed to a particular frequency component like the noise, which affects the whole signal due to the numerical derivation. The comparison is done using the numerical derivation after filtering of the measured velocity response to remove the effect of the disturbed signal.

A low pass filter at 4 Hz was applied on the measured velocity response prior to the estimation of the acceleration by numerical derivation shown in Figure 10(d). Figure 11 compares the acceleration measured by the accelerometer and the acceleration estimated from the signal of the LDV.

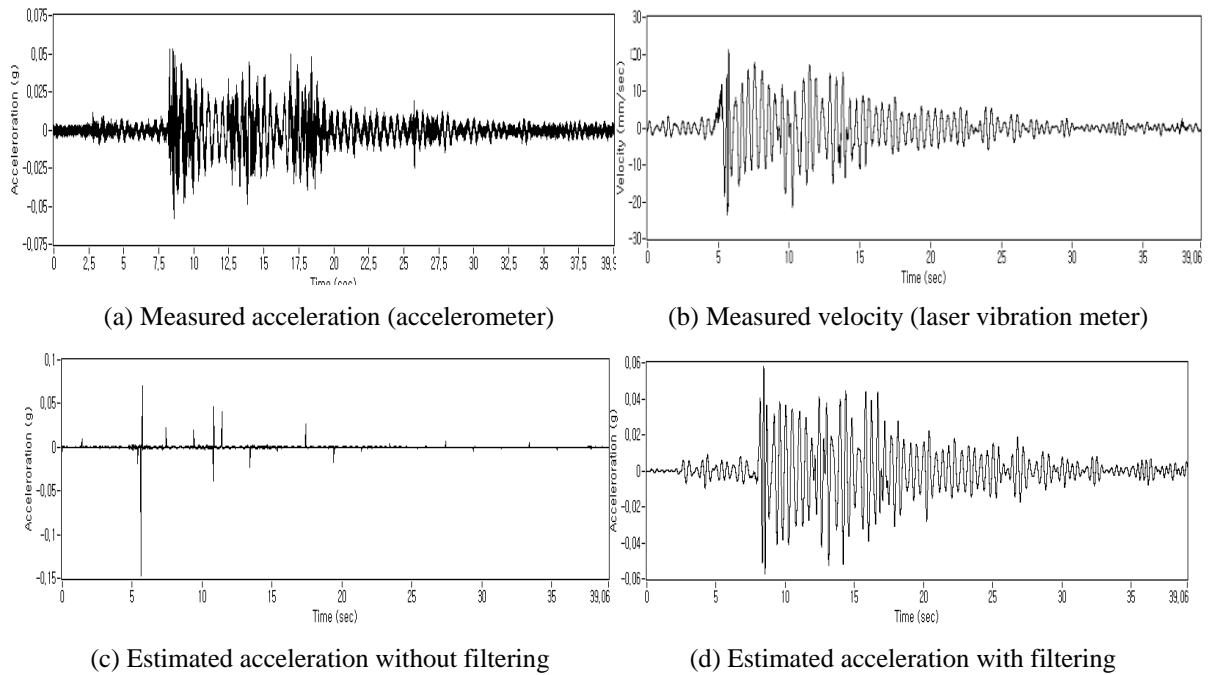


Figure 10: Time history analysis results

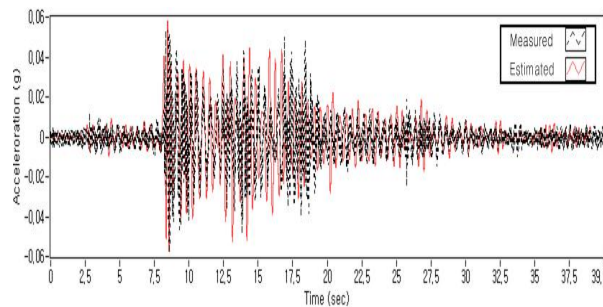


Figure 11: Comparison of measured and estimated accelerations

The maximum amplitude should be used when assessing the vibrational serviceability by the Reiher-Meister tolerance curves. The maximum amplitude of the measured acceleration is 53 cm/sec^2 , and that of the estimated acceleration is 58 cm/sec^2 . And the frequency analysis results of the measured acceleration, the natural frequency is 2.08 Hz while the natural frequency derived from the measured velocity is 2.12 Hz. Using these values to assess the vibrational serviceability by the Reiher-Meister tolerance curves leads to the same degree of perception A as shown in Figure 12.

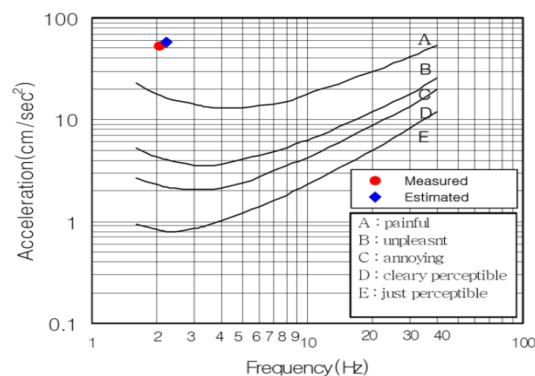


Figure 12: Serviceability evaluation (Frequency-Acceleration)

4.3 Vibrational serviceability with respect to the amplitude of displacement

Figures. 13 and 14 compare the characteristics of the displacement measured by the LVDT to those of the displacement estimated using the signal measured by the LDV. It can be observed that the estimated deflection exhibits an acceptable trend. However, the maximum amplitude of displacement shown in Figure 13 for the assessment of the vibrational serviceability using the Reiher-Meister tolerance curves reaches 0.93 cm for the estimated displacement and 0.55 cm for the measured displacement. This indicates that larger error is generated than when using the acceleration estimated from the measured velocity. This larger error can be attributed by the cumulation of the error during the numerical integration of the measured velocity.

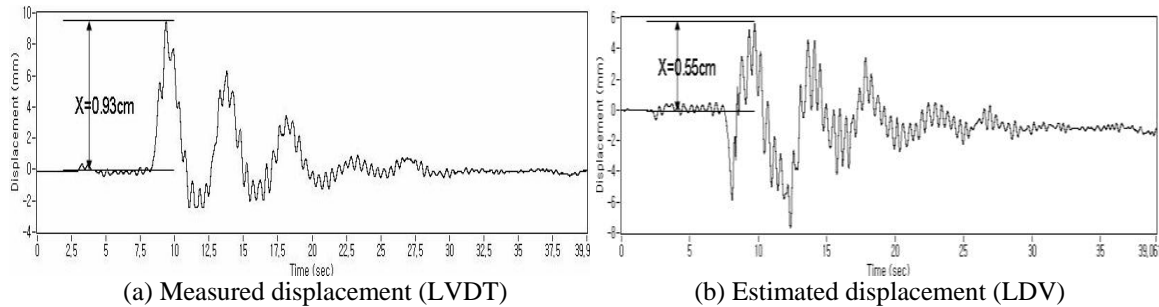


Figure 13: Measured and estimated amplitudes of displacement

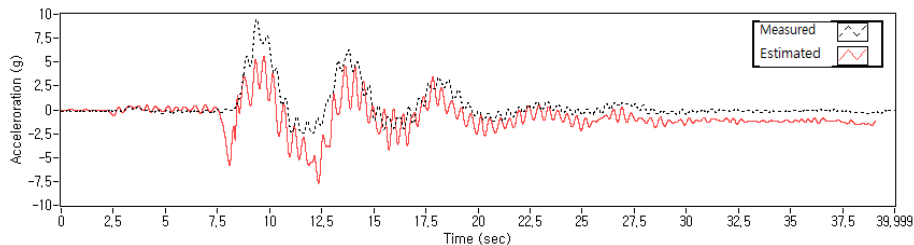


Figure 14: Comparison of measured and estimated displacements

Figure 15 compares the displacement response measured directly by the LVDT and the displacement estimated from the LDV's velocity response. The vibrational serviceability assessed using the measured displacement is B, which is higher by 1 degree compared to the serviceability level of C produced by estimated displacement. The error cumulated through the numerical integration results thus in a serviceability assessment different from that provided by the LVDT. Assuming that the assessment provided by the LVDT is correct, the displacement estimated by the numerical integration of the velocity in the time domain appears to lead to erroneous assessment results.

Moreover, the comparison of the vibrational serviceability related to the amplitude of acceleration and that related to the amplitude of displacement reveals that the degree of perception is A for the amplitude of acceleration and becomes B when using the amplitude of displacement. This difference has already been reported in other experimental studies (Kim, 2006; Kim, 2002). Accordingly, the assessment of the vibrational serviceability using only the amplitude of acceleration or only the amplitude of displacement appears to be inappropriate. Therefore, the vibrational serviceability should be assessed by using both amplitudes and selecting the worst result.

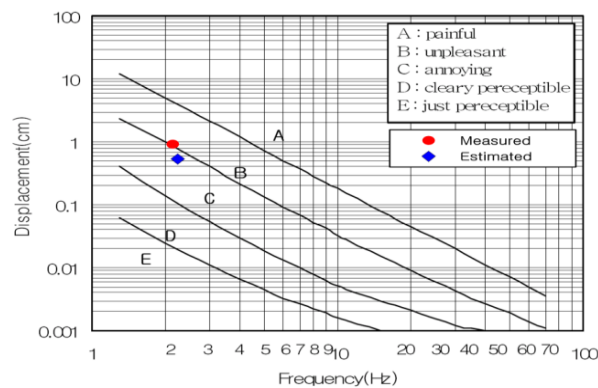


Figure 15: Serviceability evaluation (Frequency-Displacement)

5. CONCLUSIONS

This study implemented first verification tests on the applicability of the non-contact vibration meter for the assessment of the vibrational serviceability and verified its feasibility on an actual bridge. Based on these preliminary results, the vibrational serviceability of an in-service bridge was assessed. The following conclusions can be drawn from the experimental results.

(1) Field test was performed considering a maximum tilt angle of 50° as test variable to examine the accuracy of the measured natural frequency according to the measurement conditions. The corresponding results revealed that, if the measurement sensitivity is maintained to prevent any effect on the measurement, the changes in the amount and path of the reflected wave that may occur when the laser beam is discharged obliquely to the measurement surface have no significant effect on the measured natural frequency.

(2) Compared to the vibrational serviceability assessment results using the acceleration measured by the accelerometer, the vibrational serviceability assessed using the acceleration estimated by numerical derivation of the velocity response that has been filtered was verified to be applicable for the assessment of the vibrational serviceability of real bridges based on the Reiher-Meister tolerance curves.

(3) A minimum measurement duration of 20 minutes is necessary to acquire data in bridges under operation using the portable non-contact vibration meter to consider irregular loadings. A minimum duration of 1 minute is suggested in the case of bridge under construction in which external load is not applied or in the case where static loading is applied.

(4) When the displacement is estimated by numerical integration of the measured velocity in the time domain, the accumulation of the numerical error was seen to result in relatively large discrepancies between the measured and estimated displacements. In view of the tests performed in this study, such discrepancies appeared to lead to different degrees of vibrational serviceability. Attention should be paid when applying this approach.

(5) The comparison of the vibrational serviceability related to the amplitude of acceleration and that related to the amplitude of displacement revealed that different results could be derived in the vibrational serviceability assessment. Accordingly, the assessment of the vibrational serviceability using only the amplitude of acceleration or only the amplitude of displacement appears to be inappropriate.

(6) This study investigated the assessment of the vibrational serviceability of a short-to-medium span steel box girder bridge using a non-contact vibration meter. Vibrational serviceability assessment should be implemented on long-span bridges like cable-stayed bridges as well as further comparative studies for the generalized application of the proposed measurement method.

6. ACKNOWLEDGEMENT

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