A Survey on Applications of Select Mathematical Operators in Other Disciplines

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ABSTRACT---- ***The exponential boom in information technology in the past few decades has altogether revolutionized the way courses belonging to different disciplines are imparted, learnt and employed in the practical fields. Instead of having to understand the true purpose and applicability of pertinent mathematical concepts and operators, the students directly resort to the simulation and modeling software programs pertaining to various disciplines. This leaves the learners with the dilemma of not grasping the true essence and spirit of mathematical concepts and operators working in the background of these software programs. To partly address this issue, this paper presents a survey of various mathematical operators that are frequently employed in other branches of learning in diverse ways.***

Keywords--- Differentia l Operator (D), Integration Operator ($\int$), Nabla Operator (∇), Delta Operator (Δ)

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1. INTRODUCTION

Mathematics is commonly stated as the mother of all sciences. But the application of mathematics is not only limited to the core scientific disciplines, and a vast array of respective sub-disciplines, rather it extends to nearly every possible subject in some way. The exponential boom of IT and the software industry has swiftly replaced the conventional methods that were used to impart mathematical concepts to the students of other disciplines. With simulation and modeling software programs available for so many fields including, but not limited to, physics, chemistry, engineering, economics, biology, space sciences, fluid dynamics, computer sciences, graphics, networks etc.[13], the actual mathematical concepts used to design these simulation programs vanish in the background. This leaves the students and practitioners of these programs unaware of the significance, contribution and understanding of the actual mathematical concepts and operators and their mutual coordination.

Another relevant problem in this regard is that mathematical subjects are sometimes taught by the teachers who themselves have no clue of the true application of the concepts being imparted to the students in other fields of study. This sometimes results in students of other disciplines wondering of why are they even being taught these mathematics courses and what is the actual application of mathematical concepts in the practical field.

The idea of this survey is to initiate a series of publications to address the aforementioned issues and facilitate the learners in understanding the application of various conventional and unconventional mathematical operators employed in other disciplines.

2. APPLICATION OF MATHEMATICAL OPERATORS IN OTHER FIELDS OF STUDY

This section presents various mathematical operators and their applications in other fields, along with examples and mathematical models intermittently, in a brief systematic way.

2.1 $+, -, \pm, \mp$: These symbols are mostly used in arithmetic and trigonometry.

2.2 $\times, \cdot$: These are product symbols, basically used in arithmetic, linear algebra, functional analysis, vector algebra, vector calculus and different areas in physics, chemistry etc.

2.3 $\div$, $/$ **:** These symbols are called division or obelus[1], commonly used in arithmetic. Another important application of these operators is in quotient group.

2.4 √ : This is the square root symbol, generally used in arithmetic and some other areas. $\sqrt{a}$ means a non-negative number whose square is ''a''.

Example: if $z = r\,e^{i\theta}$ in polar form, then $\sqrt{z} = \sqrt{r}\ e^{i\frac{\theta}{2}}$, where $-\pi \angle \theta \leq \pi$ and $\sqrt{-1} = i$.

2.5 ∑ : This symbol is known for summation, mostly used in arithmetic.

Example: $\sum_{n=1}^{k} a_n = a_1 + a_2 + a_3 + \text{..........} a_k$ and $\sum_{n=1}^{5} n^2 = 1^1 + 2^2 + 3^2 + \text{.....} 5^2$ [2]

2.6 ∂ : This symbol or operator is called "Tho", or sometimes "Del", or "Partial Derivative" operator. This is commonly used for partial derivatives in vector calculus and in differential geometry. This is one of first known symbol in mathematics, used by Marquis Decondorcet in 1770, by Adrian Marie Legendre in 1786 and lastly by Carl Gustav Jacob in 1841.[3] The applications of this symbol are as follows.

a. In geometry:
Example: If V be the volume of a cone, ''h'' be the height and ''r '' is radius, then $V = \frac{1}{3}\pi r^2 h \ \ then\ \frac{\partial y}{\partial r} = \frac{2\pi rh}{3}$

b. In economics:

Used in optimization problems with more than one choice of variable in economics.

Example: Marginal propensity to consume (MPC) is given by the model MPC= $\frac{\Delta C}{\Delta Y}$ [4]

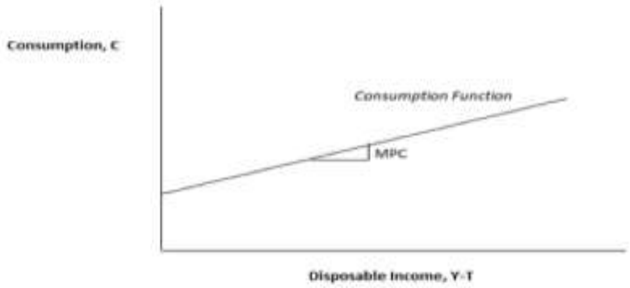
Where Δc is the change in consumption and Δy is the change in disposable income that produced the consumption.

Solved Example: For a person whose income and consumption are given below, what is the MPC of that person.

Income	consumption
USD 150	USD 150
200	180

$\Delta C = 30$, $\Delta Y = 50$ so that MPC = $\frac{30}{50} = .6 \ \ or\ 60\%$

Also, suppose you receive a bonus of USD 1000 with your salary and you decide to spend USD 900 of this marginal increase in the income on a new car, then your MPC will be $\frac{900}{1000} = .9 \ \ or\ 90\%$



The MPC Graph [4]

c. In Physics, Engineering and Medical Sciences:

In physics it is used in fluid flows and for measuring force in a spring. The most important use of the ∂ operator in medical sciences is to calculate Nerve Conduction.

A nerve conduction study (NCS), also called a nerve conduction velocity (NCV) test--is a measurement of the speed of conduction of an electrical impulse through a nerve. NCS can determine nerve damage and destruction.

During the test, the nerve is stimulated, usually with surface electrode patches attached to the skin. Two electrodes are placed on the skin over the nerve. One electrode stimulates the nerve with a very mild electrical impulse and the other electrode records it. The resulting electrical activity is recorded by another electrode. This is repeated for each nerve being tested. The nerve conduction velocity (speed) is then calculated by measuring the distance between electrodes and the time it takes for electrical impulses to travel between electrodes. A related procedure that may be performed is electromyography (EMG). An EMG measures the electrical activity in muscles and is often performed at the same time as NCS. Both procedures help to detect the presence, location, and extent of diseases that damage the nerves and muscles.[5] The model for calculating it is given by the following second order partial differential equation

$\frac{1}{r_l}\frac{\partial^2 V}{\partial x^2} = c_m \frac{\partial V}{\partial t} + \frac{V}{r_m}$ Where r_l is longitudinal resistance, r_m is membrane resistance, c_m is capacitance due to electrostatic force and V IS voltage [$V = iR$]

2.7 Δ : The Del operator was introduced by Laplace (1749-1827), and in mathematics, this operator Del (Δ) is called Laplace operator or Laplacian. This operator has several applications, for instance

a. In Physics:
For gravitational potential and its model is given by $\Delta u = mg\Delta h$ [6]
Where *u* represents potential energy, *m* represents mass of the object, *g* represents gravitational acceleration and *h* represents height.

b. Harmonic function

$\nabla^2 f = 0 \; or \; \Delta f = 0$, where ∇ is the Nabla operator (which we will discuss in the next section)

c. Blob and Edge Detection

A blob is a group of connected pixels in an image that share some common property. The mathematical model is given by $\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$

The mathematical model of edge detection model is given by $c = \begin{pmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{pmatrix}$, where $\sum I_x^2$ is the sum over a small region around the hypothetical corner, $\sum I_x I_y$ is the gradient with respect to x times gradient with respect to y and the matrix is symmetric.[7]

Other uses of this operator are in Electric diffusion equation for heat fluid flow, in wave Propagation and in quantum mechanics.

2.8 ∇ : The Nabla operator is commonly used to find gradient, divergence, and curl but some of the most important applications of this operator are in

a. Differential geometry

b. Calculus of finite differences as backward difference operator

c. Discrete fractional calculus

d. Computer science field of abstract interpretation and the mathematical model is given by $X'_{n+1} = X'_n \ \nabla Y'_n$ where X'_n any sequence and Y'_n also sequence

e. Naval engineering to designate the volume displacement of a ship or any other water-borne vessel or to designate weight displacement. The mathematical model is given by $\nabla = \frac{\Delta}{density\, of\ sea\, water}$

2.9 $\left(\int\right)$: This is the Integral operator. "Integration is a way of adding cut pieces, whose thickness is very small compared to its length, width or its diameter".

Example: Filling a tank from tap integration the flow (adding up all the little bits of water gives the volume of water in the tank)

Generally there are two types of integration (i) Indefinite, and (ii) Definite integral.

Indefinite integral: it is used to find displacement from velocity and velocity from acceleration.

Example: $\int \frac{ds}{dt}\,dt \ or \int v\,dt = s$ and $\int \frac{d^2s}{dt^2}\,dt \ or \int a\,dt = v$

Also I can use this indefinite integral for proving the equations of motion also in an easy way.

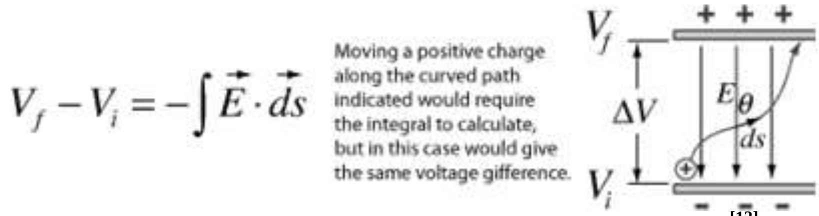
a. Prove by using indefinite integral $v = u + at$
Solution: we knows that $v = \int adt$ integrating it we get $v = at + u$ where u is the initial velocity, v is the final velocity and a is acceleration
And again integrating it by using indefinite integral form we get $s = \int vdt$
$s = \int (u + at)\,dt$ or $s = ut + \frac{at^2}{2} + c$ when t=0 , s=0 then c=0 now the equation of motion is like $s = ut + \frac{at^2}{2}$

b. In Electrical Circuits:

Another use of indefinite integral is in electrical circuit like, if the current i (in amperes) in an electric circuit equal s the rate of change of the charge q (in coulomb) i.e. $i = \frac{dq}{dt}$ or $i\,dt = dq$ or $q = \int i\,dt$ or $v = \frac{1}{c}\int i\,dt$

Here $q = cv$ also we can put this one in differential equation form $i = c\frac{dv}{dt}$

Where $\frac{dv}{dt}$ is instantaneous rate of change (volts per second)
Also this symbol gives a very important relation in between voltage difference and electric field, relation is given below



Voltage Difference and Electric Field Relationship Graph [12]

c. In radioactive decay:
The indefinite integral is also used in the calculation of radioactive decay. Radioactive decay involves discrete events of nuclear disintegration. The number of events is so large that it can be treated like a continuum and the methods of calculus employed to predict the behavior. The result from the decay probability can be put in the differential form as given below

$dN = -\lambda N dt$ Or $\dfrac{dN}{N} = -\lambda dt$ integrating it with respect to t we get $\ln \mathrm{N} = -\lambda t + c$ where c is integration constant or $N = N_0 e^{-\lambda t}$

2.10 $\int_a^b f(x)dx$ **:** When the symbol $(\int)$ is given in the form of $\int_a^b f(x)dx$ then the integration is called definite integral here integration constant is not required because it gives us the definite value.
Most commonly used of the definite integral are as

a. Finding the volume of solid revolution. The best examples are i) Volume of a wine cask, ii) Volume of a watermelon.

b. Finding centroid of an area.

c. Finding centre of mass. The best example is to find the centre of mass of tilt slab construction.

d. Finding moment of inertia

e. Finding work done by a variable force $W = \int_c^b f(x)\,dx$

f. Finding work done by electric charges $W = \int_a^b k\dfrac{q_1 q_2}{r^2}dr$ where q_1 , q_2 charges in coulombs, k is constant and r is distance

g. Average value of a function: if f(x) which is continuous on the closed interval [a, b] then average value of f(x) from x=a to x=b is given by $y_{avrg} = \dfrac{1}{b-a}\int_a^b f(x)\,dx$ [8]
Also average value '' $\bar{a}$ '' of the acceleration a(t) over the time interval $t_1\, to\ t_2$ is given by $\bar{a} = \dfrac{1}{t_2 - t_1}\int_{t_1}^{t_2} a(t)\,dt$ and for the HEAD INJURY CRITERIA (HIC) model is given by

HIC $= \max.(t_1 or\ t_2)\left\{(t_2 - t_1)\left[\dfrac{1}{t_2 - t_1}\int_{t_1}^{t_2} a(t)\,dt\right]^{2.5}\right\}$ [9]

The best example for that is to calculate "head injury criteria" in between 1950 when accident was occurred then there is no one known for safety because there was no safety Belt, no Air Bags etc are available in car's but in 1960 to 1970 companies are trying to provide such thing for safety purpose and the mathematical model is given by

SEVERITY INDEX (SI) $= \int_0^T \{a(t)\}^{2.5} dt$ [10] where T is the duration of the deceleration during the crash and a(t) is the deceleration at time t and the index 2.5 was chosen for the head and other indices were used for other parts of the body

h. Calculate force due to Liquid Pressure by definite integral:

$F = \rho DA$ where F is force, A is area , ρ is density and D is depth in liquid and the total force due to liquid pressure is given by $F = \rho \int_a^b xDdD$ where x is the length in meters

Finally some most important definite integral which are given below [11] they are applicable indifferent areas

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2} \qquad \int_0^\infty xe^{-x^2} dx = \frac{1}{2} \qquad \int_0^\infty x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$$

$$\int_0^\infty x^3 e^{-x^2} dx = \frac{1}{2} \qquad \int_0^\infty x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8} \qquad \int_0^\infty x^5 e^{-x^2} dx = 1$$

3. FUTURE EXTENSIONS

This paper has presented a survey of some mathematical operators and their common applications in different fields of study. A lot of room exists for similar to be conducted in future on numerous other mathematical concepts and operators.

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