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Optimal Trajectory Planning and Control of 2 DOF Robotic Manipulator

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ABSTRACT—*Robot manipulators are nonlinear in nature thus requires accurate modeling to present the nonlinear characteristics. This paper proposes kinematic and dynamic model of 2 Degree of Freedom (DOF) robotic manipulator and design of optimal trajectory. Optimal trajectory design is based on the optimization of joint torque. Optimal trajectory design becomes more complicated when considering a complete non linear model. Hence a simplified linearized model is chosen in this work. The design of cubic polynomial trajectory for the comparison of optimal trajectory is also included in this work.*

Keywords—Robotic Manipulator, Trajectory Planning, Optimal Controller

1. INTRODUCTION

Robots are using extensively in manufacturing and service industries. Nowadays the competition among the industries for the development of robots is increased. Hence optimization of trajectory becomes an important factor in robots. In path planning problems, the number of feasible pathsbetween the initial position and final position of a robot is often very large, and the goal is not necessarily todetermine the best solution, but to obtain a good one according to certain requirements and certainconstraints. Classical control system design is generally a trial and error process in which various methods of analysis are used iteratively to determine the design parameters of an 'acceptable' system. Acceptable performance is usually defined in terms of time and frequency domain criteria such as rise time, settling time, peak overshoot, gain and phase margin and bandwidth. Radically different performance criteria must be satisfied such as minimum cost, energy etc by the complex, multi-input,multi-output systems required to meet the demands of modern technology. This paper proposes an optimal trajectory design of 2 DOF robotic manipulator.
Dragan Kostic et al. [1] proposed the problems of modeling and identification for high performance model based control of a robot. An efficient estimation of parameters of the rigid body dynamic model that includes friction effects is also considered. Erkan Zergeroglu et al.[2] considered a kinematically redundant manipulator and designed a model based nonlinear controller that achieves exponential link position and subtask tracking. Mohammad Amin Rashidifar et. al.[3] described Control of Lynx6 robot arm to reach the specified location with minimum error while meeting certain specification without considering the tracking path from the initial position to the final position. A comparison of PID controller result with Fuzzy logic controller (FLC) and Fuzzy supervisory controller (FSC) is also considered in this research work. Vivek Deshpande and P M George[4] derived complete kinematic modeling of 5 DOF Robot arm . Jyoti Ohri et al. [5] described the modeling and control of 2 DOF robotic manipulator and they describe about the robustness comparison of different controllers like proportional Integral and Derivative (PID) and Sliding mode controller (SMC). Optimal trajectory tracking by considering different criteria like minimum energy,time and sum of all rotation using genetic algorithm is discussed [6],[7],[8].Huashan liu et al. discussed about the minimum time optimization problem. Experimental validation on 6 DOF robotic manipulator was also included in this work [9].

Section II discusses about the complete kinematic and dynamic model of 2 DOF robotic manipulator. Section III describes about the trajectory planning which includes the simulation results. Optimal trajectory design and simulation results for torque optimization are included in section IV. Conclusion is given in section V.

2. KINEMATIC AND DYNAMIC MODELING

2.1 Kinematic modeling

Kinematic modeling can be divided into two forward kinematic modeling and inverse kinematic modeling. Forward kinematic modeling is used for representing position and orientation of the end-effector where as inverse kinematic modeling is used for determine the joint angles corresponding to the end-effector position.

2.1.1 Forward Kinematic modeling

A 2 degree of freedom robot arm is considered and by using Denavit-Hartenberg (D-H) notations all the coordinate frames is represented, and is shown in Figure 1. All the joint link parameters are obtained from the coordinate frame and is tabulated in Table I.

Table 1: Joint link parameters for 2 DOF manipulator

i	$\mathbf{a_i}$	$\boldsymbol{\alpha_i}$	$\mathbf{d_i}$	$\boldsymbol{\theta_i}$	$\mathbf{q_i}$
1	0	-90	d_1	θ_1	θ_1
2	l_1	0	0	θ_2	θ_2

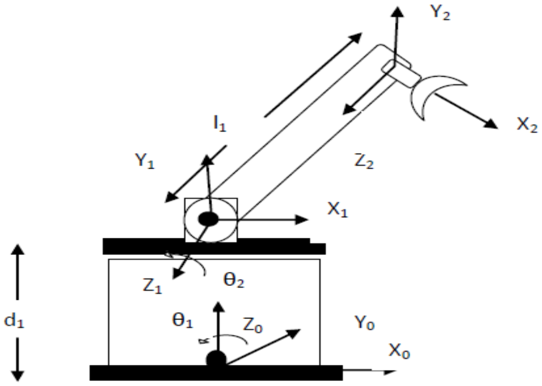

Figure 1: Two DOF Robotic frame assignment

Forward kinematic model of 2 DOF robotic manipulator can be represented as transformation matrix.

$$\begin{bmatrix} C_1C_2 & -C_1S_2 & -S_1 & l_1C_1C_2 \\ S_1C_2 & -S_1S_2 & C_1 & l_1C_2S_1 \\ -S_2 & -C_2 & 0 & \frac{5}{2}-l_1S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where l_1 and d_1 represents the joint-link parameters.

2.1.2 Inverse Kinematic modeling

Inverse kinematic problem is used for determine all possible and feasible set of joint variables, which would achieve the specified position and orientation of the manipulator's end-effector with respect to the base frame. There are two approaches for the solution of inverse problem: closed form solutions and numerical solutions. The closed form solution of 2 dof robot arm is given below.

$$\theta_1 = tan^{-1}(\frac{-a_x}{a_y}) \tag{1}$$

$$\theta_2 = tan^{-1}(\frac{n_y - o_x}{n_x + o_y} \tag{2}$$

$$\theta_1 = \theta_{12} - \theta_2 \tag{3}$$

where [n,o,a] describe the orientation of end-effector.

2.2 Dynamic Modeling

The robotic manipulator complex nature can be described by equations of motion. Lagrangian Euler(LE) and Newton mechanism are used for dynamic modeling where LE is based on energy. Assumptions are considered during modeling such as no backlash, no friction and neglect the effect of control component dynamics. The dynamic model obtained by solving EOM is second order coupled non linear differential equations.The torque equation for a manipulator can be written as

$$\tau_i = \sum_{j=1}^{2} M_{ij}(\theta)\ddot{\theta}_j + \sum_{j=1,k=1}^{2} h_{ijk}\,\dot{\theta}_j\dot{\theta}_k + G_i \quad \text{for i=1,2} \tag{4}$$

where M(θ) is the effective inertia matrix, h(θ,$\dot{\theta}$) is the centrifugal and coriolis acceleration forces and G(θ) is the gravitational loading forces. $\theta, \dot{\theta}, \ddot{\theta}$ are the joint position,velocity and acceleration respectively. The Dynamic modeling equation can be represented in state variable form as

$$X_1 = [\theta_1, \theta_2] \tag{5}$$

$$X_2 = [\dot{\theta}_1, \dot{\theta}_2] \tag{6}$$

$$\dot{X}_1 = X_2 \tag{7}$$

$$\dot{X}_2 = f(X_1, X_2) + g(X_1)\tau \tag{8}$$

$$f(X_1, X_2) = -M^{-1}(X_1)H(X_1, X_2) - M^{-1}G(X_1) \tag{9}$$

$$g(X_1) = M^{-1}(X_1) \tag{10}$$

Optimal control design becomes very complicated for robotic manipulator because of its complex nonlinear model. Hence for simplicity complicated non linear model is converted into a simplified linear model. The linear model of 2 DOF robotic manipulator is given below.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2.5226 & 0 & 0 & 0 \\ 0 & -3.9240 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.6857 & 0 \\ 0 & 1.0667 \end{bmatrix}$$

3. TRAJECTORY PLANNING

Trajectory planning is very important in robotic manipulator for requisite motion of manipulator. Trajectory planning can be done by with or without via points. Initial and final locations are mainly required for trajectory planning which can be obtained by inverse kinematics.

3.1.1 Point to point motion without via point

In this scheme the goal point and travel time are specified. The set of joint variable values for the given goal point are obtained from the inverse kinematics. Two constraints each on the joint position and velocity functions are

$$\theta(0) = \theta^s \tag{11}$$

$$\theta(t_g) = \theta^g \tag{12}$$

$$\dot{\theta}(0) = 0 \tag{13}$$

$$\dot{\theta}(t_g) = 0 \tag{14}$$

The joint position can be generally represented as a polynomial

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{15}$$

where θ^s *and* θ^g are the starting and goal points and t_g is the time to reach goal point. To describe the joint motion the cubic polynomial obtained by considering the constraints is

$$\theta(t) = \theta^s + (\theta^g - \theta^s) * (3t^2/4) - (\theta^g - \theta^s) * (\frac{t^3}{4}) \tag{16}$$

3.1.2 Simulation Results

Point to point motion trajectory without via point is shown in Figure 2 and the torque curve for the corresponding trajectory is shown in Figure 3.

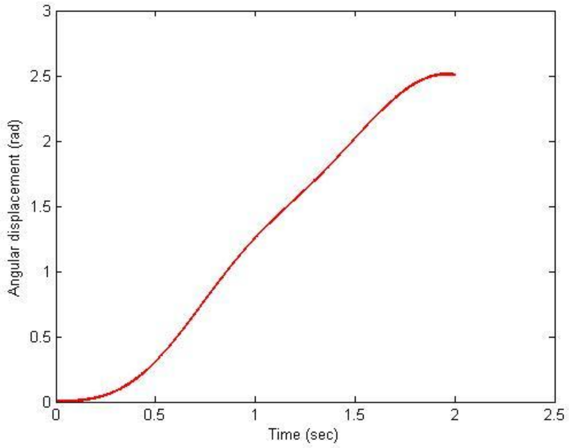

Figure 2. Cubic Trajectory

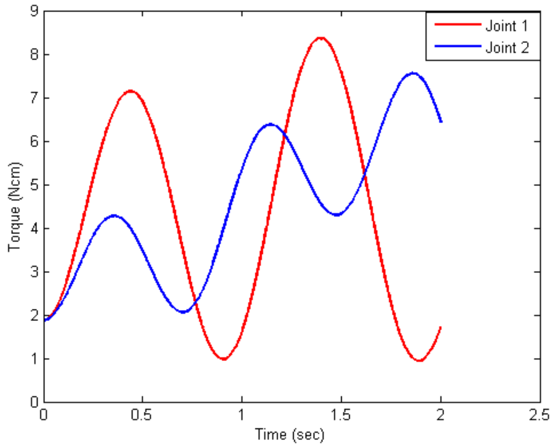

Figure 3. Joint torques of robotic manipulator

4. OPTIMAL TRAJECTORY DESIGN

Optimal control law is used in multi disciplinary applications such as biological systems, communication networks and socio-economic systems etc.Optimal control is used to determine the control signals that will cause a process to satisfy the physical constraints and at the same time minimizes some performance criterion.The performance measure is

$$J = \frac{1}{2}\int_{t_0}^{t_f} (T_1^2 + T_2^2)dt \tag{17}$$

The Hamiltonian can be written as

$$H = \frac{1}{2}(T_1^2 + T_2^2) + P_1 dx_1 + P_2 dx_2 + P_3 dx_3 + P_4 dx_4 \tag{18}$$

where P_1,P_2,P_3, and P_4 are Lagrangian multipliers. The costate equations are

$$\dot{P} = -\frac{\partial H}{\partial x} \tag{19}$$

The control effort can be obtained by solving the equation given below.

$$\frac{\partial H}{\partial T} = 0 \tag{20}$$

4.1 Simulation Results

Position and velocity constraints given in (11),(12),(13), and (14) are used for solving optimal control equations. The optimal trajectory and control effort obtained using the above method is shown in Figure 4 and Figure 5. Large number of trajectories can be created between two points; one type of trajectory (Cubic) design is added in this paper and is shown in Figure 2. Finally the optimal trajectory is shown in Figure 4; by applying optimal trajectory the torque can be optimized. It is clear from Fig 5, that the joint torques is minimized up to 30 percentages.

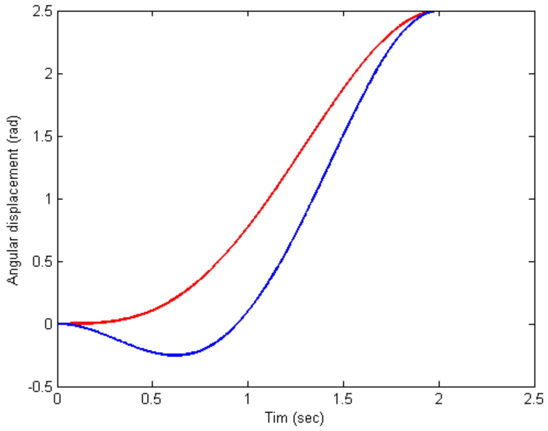

Figure 4. Optimal Trajectory

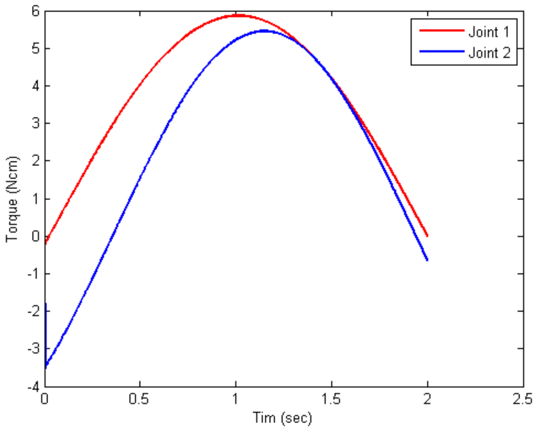

Figure 5. Joint torque of manipulator

5. CONCLUSION

Optimal trajectory design and control is presented in this paper. Conventional optimal control approach is used for the optimization of each joint torque. Optimal control design is very difficult for complicated non linear model. Hence this paper considered a simple linearized model for the analysis. A cubic polynomial trajectory design is also included for the comparison. The analysis results shows that the optimal control gives better torque reduction and control.

This work considered a simple linearized model for avoiding the complexity. Hence this work can be extended in future by considering complete non linear model. Numerical methods can be adopted for avoid the complexity, and optimization of parameter other than torque such as energy, time can also be considered.

6. ACKNOWLEDGMENT

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