

# Dynamics of Composite Beam with Transverse Non-Propagating Open Crack

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**ABSTRACT**—Cracks or other defects in a structural element influence its dynamic behavior and change its stiffness and damping properties. In this paper, The Critical fracture parameters governing the severity of stress and deformation field ahead of the cracks were evaluated. To ensure the safe, reliable and operational life of structures, it is of high importance to know if their members are free of cracks and, should they be present, to assess their extent. Static, modal, and harmonic analysis were conducted to analyze the dynamic behavior of the cracked composite beam. Also the variation of natural frequencies with different boundary conditions ie, Clamped-Clamped, Clamped-Simply Supported and Cantilever were found out. From the results it is found that the natural frequencies of the beam varied with the depth of the cracks. Also the static stresses tend to increase as the crack depth increases.

**Keywords**—Composite Materials, Crack detection, Damage Diagnosis, Vibration Analysis

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## 1. INTRODUCTION

During operation, all structures are subjected to degenerative effects that may cause the growth of structural defects such as cracks which may lead to the catastrophic failure or breakdown of the structure, as the time progresses. A variety of structural components made of composite materials such as turbine blades, vehicle axles, robot arms, aircraft wings, and helicopter blades can be approximated as composite beams. So it is important to understand the static behavior of cracked composite beams which are extensively used for structural applications for ensuring its safety and health.

Rene [1] reported the effect of fiber orientation on the modes of vibration of a unidirectional fiber reinforced cantilever composite beam. Experimental results were obtained for the natural frequencies and mode shapes of graphite-epoxy and boron-epoxy composite materials having different fiber orientations with respect to cantilever beam axes. Gudmundson [2] presented a first order perturbation method which predicts changes in natural frequencies of a structure resulting from cracks, notches or other geometrical changes. The method was applied to an edge-cracked rectangular beam. Rizos [3] presented a method of measuring the amplitude of a steel beam at two points during forced vibration at one of its natural frequencies. The identification method was based on the assumption of a transverse crack, extending uniformly along the width of the structure. Tita [4] observed the influence of the fibers orientations as well as the stacking sequences on the dynamic behavior of the components with the numerical and experimental analysis, Changes in the stacking sequence yield to different dynamic behavior of the component . Wang [5] investigated the coupled bending and torsional vibration of a fiber-reinforced composite cantilever with an edge surface crack. The coupling of bending and torsion can result from either the material properties or the surface crack. Gaith [6] took a model of a fiber-reinforced composite simply supported beam with a transverse one-edge non propagating open crack. Using this model, the influence of the depth and crack location, anisotropic properties such as orientation angle, and fraction of fibers on the bending natural frequencies of the simply supported composite beam has been investigated.

In the present paper an attempt is made to work out the Finite element model of a composite beam with a transverse one edge non propagating open crack. It is assumed that the crack changes only the stiffness of the beam whereas the mass of the beam remains unchanged and the having uniform width [7]. By using the models of the cantilever cracked composite beam the influence of (i) depth of a crack in a cracked composite beam on stress (ii) the changes of the natural frequencies of the cracked composite beam as a function of the angle of the fibers. (iii) The effect on cracks due to harmonic loads, (iv) Effect of Boundary conditions.

## 2. NATURAL FREQUENCIES OF COMPOSITE BEAM

The natural vibration equation of a mid-plane symmetrical composite beam with bending stretching coupling and transverse shear deformation neglected is given by, (Vinson & Sierakowski (1991) [8])

$$IS_{11} \frac{\partial^2}{\partial w^4} w(x, t) + \rho F \partial^2 w(x, t) / \partial x^2 = 0 \quad (1)$$

It can easily be shown that under these conditions if the beam involves only a one layer, isotropic material, then,

$$IS_{11} = EI = \frac{Ebh^3}{12} \quad (2)$$

and for a beam of rectangular cross-section Poisson's ratio effects are ignored in beam theory.

It is handy to know the natural frequencies of beams for various practical boundary conditions in order to insure that no recurring forcing functions are close to any of the natural frequencies, because that would result almost certainly in a structural failure. The natural frequency in radians/unit time are given as

$$\omega_n = \alpha^2 \sqrt{\frac{IS_{11}}{\rho AL^4}} \quad (3)$$

Where  $\alpha^2$  is the co-efficient which varies with the boundary condition.

## 3. NUMERICAL INVESTIGATIONS

Structures are weakened by cracks. When the crack size increases in course of time, the structure becomes weaker than its previous condition. Finally, the structure may breakdown due to a minute crack. The basic problem investigated here is a cracked cantilever composite beam, which has application in aerospace structures and high-speed turbine machinery. The composite beam made of unidirectional graphite fiber-reinforced polyamide having transverse open non propagating crack is used in the analysis. The model chosen for the analysis is shown in the figure 1. It is of uniform cross-section A, having an open-edge transverse crack of depth 'a' at position 'L1'. The width, length and height of the beam are B, L and H respectively. The material property of composite beam is presented in Table 1.

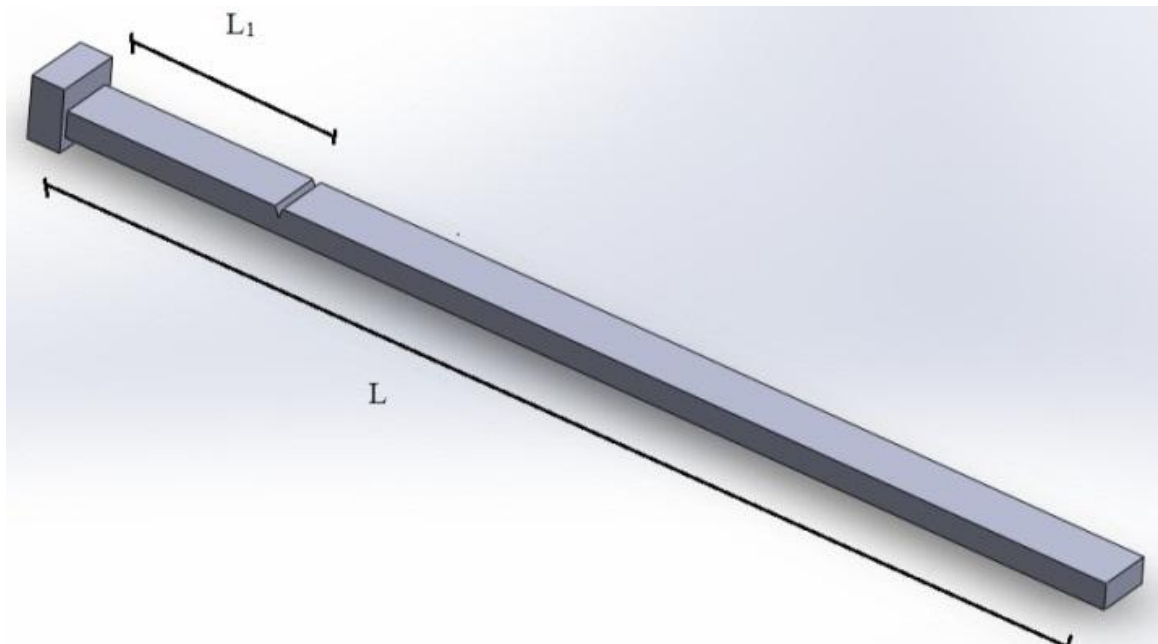


Figure 1: Schematic diagram of a composite cantilever beam with a crack

Table1. Material Properties of Composite Beam

Modulus of Elasticity	$E_m$	2.756 GPa
	$E_f$	275.6 GPa
Modulus of Rigidity	$G_m$	1.036 GPa
	$G_f$	114.8GPa
Poisson's Ratio	$V_m$	0.33
	$V_f$	0.2
Mass density	$\rho_m$	1600 kg/m <sup>3</sup>
	$\rho_f$	1900 kg/m <sup>3</sup>

The finite element solver namely ANSYS 12.1 is used to perform all the necessary computations. In the initialization phase, geometry and material parameters are specified. For the composite beam model with localized crack, material parameters like modulus of elasticity, the modulus of rigidity, the Poisson ratio and the mass density of the composite beam material along with geometric parameters like dimensions of the composite beam, also the specifications of the damage like size of the crack[9], location of the crack and extent of crack are supplied as input data into the preprocessor of the ANSYS 12.1 software. The beam is discretized with Solid 186 element. The model is then solved to obtain the static and Harmonic load for non-cracked and cracked composite beams.

#### 4. ANALYSIS AND DISCUSSIONS

##### 4.1 Static Analysis

A linear static analysis was carried out on cantilever beams with and without cracks with a point load of 1 KN applied at the free end of cantilevered beam and the results so obtained is shown in Table 2 and 3. As the depth of crack increases the intensity of stress also increases as expected. Stress for crack up to 0.2 depth is about three times and for crack up to at 0.6 depth is sixty times that of the intact beam.

Table 2: Stress at various depth of beam without crack at  $L_1 = 0.1L$  &  $V = 0.1$

Angle of fiber (Ply Angle)	Volume fraction of fiber	Stress (N/mm <sup>2</sup> ) Without crack		
		at 0.2 Depth	at 0.4 Depth	at 0.6 Depth
0	0.1	103.67	34.56	34.56
15		107.27	35.96	35.96
30		103.48	34.48	34.48
45		103.68	34.55	34.554
60		103.68	34.56	34.56
75		103.68	34.56	34.56
90		103.68	34.56	34.56

Table 3: Stress at various depth of beam with crack  
at  $L_1=0.1L$  &  $V=0.1$

Angle of fiber (Ply Angle)	Volume fraction of fiber	Stress (N/mm <sup>2</sup> ) With crack		
		Crack at 0.2 Depth	Crack at 0.4 Depth	Crack at 0.6 Depth
0	0.1	341.0	438.79	1997.7
15		338.3	447.79	2018.3
30		293.3	476.52	2039.1
45		326.2	486.6	2140.6
60		334.1	498.56	2160.4
75		289.7	424.09	2406.1
90		285.19	417.92	2426.4

#### 4.2 Modal Analysis

The beam assumed to be made of unidirectional graphite fiber-reinforced polyamide containing transverse open crack having a cantilever boundary condition. The geometrical characteristics and material properties of the beam are taken as the same as those used in Krawczuk&Ostachowicz [10] in order to check the accuracy of the present analysis. The geometrical characteristics, the length (L), height (H) and width (B) of the composite beam were chosen as 1.0m, 0.025 and 0.050 m respectively.

The non-dimensional natural frequencies are normalized according to the following relation;

$$\omega_n(\alpha) = L \sqrt{\frac{w(\alpha)}{I S_{11} \rho F}} \quad (4)$$

Where  $\omega(\alpha)$  is the natural frequency of the beam computed for each value of the Ply angle ( $\alpha$ ). Natural frequencies of uncracked composite beam and beam with crack depth upto 0.2 depth is presented in Table 4 and Table 5 respectively.

Table 4: Natural frequencies of Uncracked Composite Beam with a fibre volume fraction of 0.1

Angle of Fibres (Ply Angle, $\theta$ )	Natural frequencies of Uncracked Composite Beam with a fibre volume fraction of 0.1		
	1st Natural frequency (Rad/s)	2nd Natural frequency ( Rad/s )	3rd Natural frequency ( Rad/s )
0	106.09	686.5	1924.3
15	103.15	596.66	1865.9
30	93.205	537.89	1381.9
45	75.424	405	1121.4
60	36.213	282.84	792.04
75	35.648	234.34	600.74
90	35.403	229.03	593.92

Table 5: Natural frequencies of a damaged beam having a crack of 5mm depth at location 0.1 times the length of the beam

Angle of fiber (Ply Angle)	Natural frequencies of a damaged beam having a crack of 5mm depth at location 0.1 times the length of the beam	
	1st Natural frequency ( Rad/s )	2nd Natural frequency ( Rad/s )
0	91.478	617.88
15	88.139	605.91
30	74.655	504.45
45	59.938	384.88
60	43.336	276.8
75	35.639	218.8
90	35.381	210.74

Results of Modal analysis of un-cracked beam are shown in figure 2 and composite beams with crack having depth 0.2, 0.4 and 0.6 times total depth of the beam are presented in Figure 3, 4 & 5 respectively.

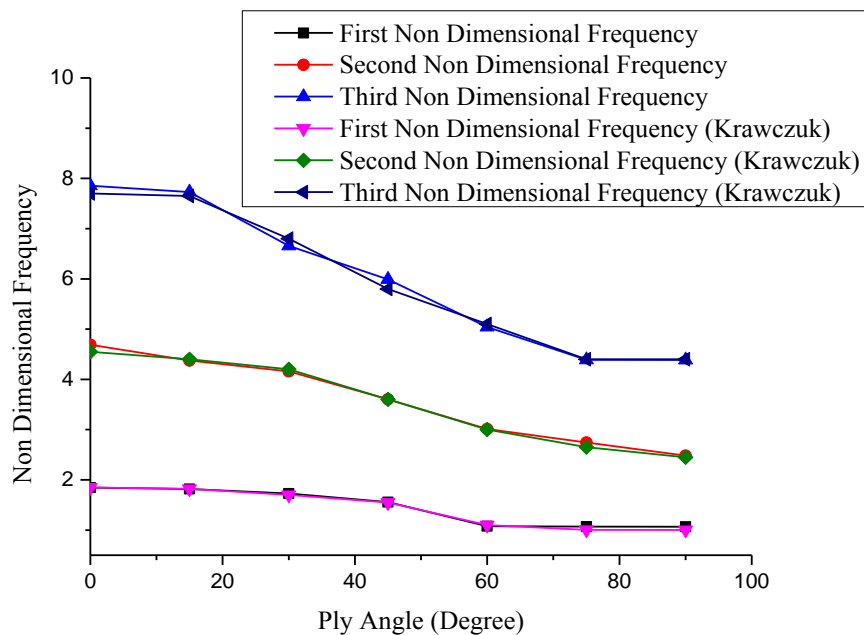


Figure 2: Comparison of first two Non Dimensional Natural Frequencies with present analysis and Krawczuk&Ostachowicz (1995) for un-cracked composite beam.

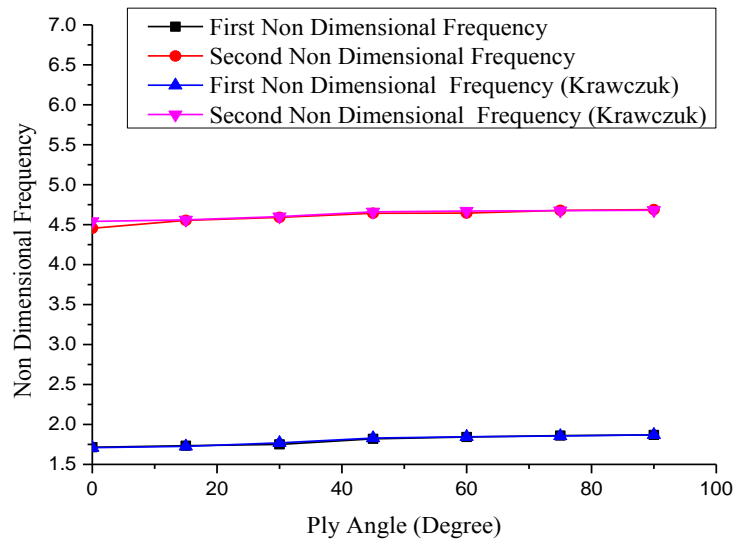


Figure 3: Comparison of first two Non Dimensional Natural Frequencies of the cracked composite beam with RCD=0.2 and RCL = 0.1

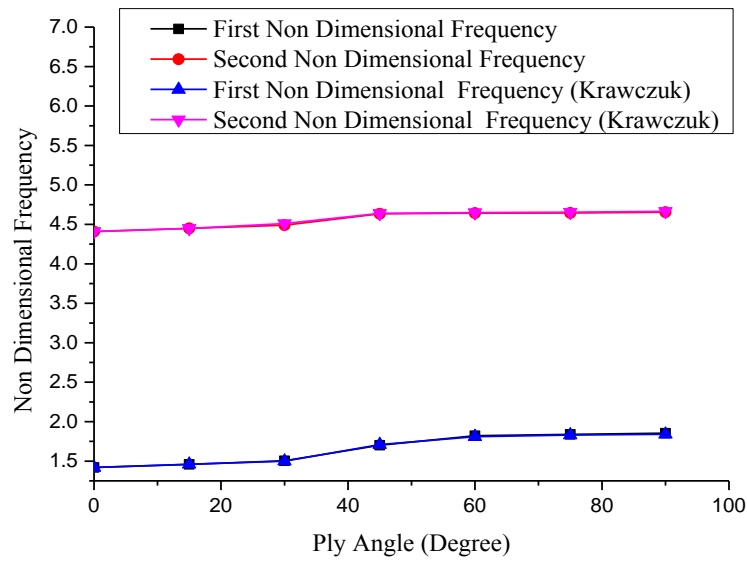


Figure 4: Comparison of first two Non Dimensional Natural Frequencies of the cracked composite beam with RCD=0.4 and RCL = 0.1

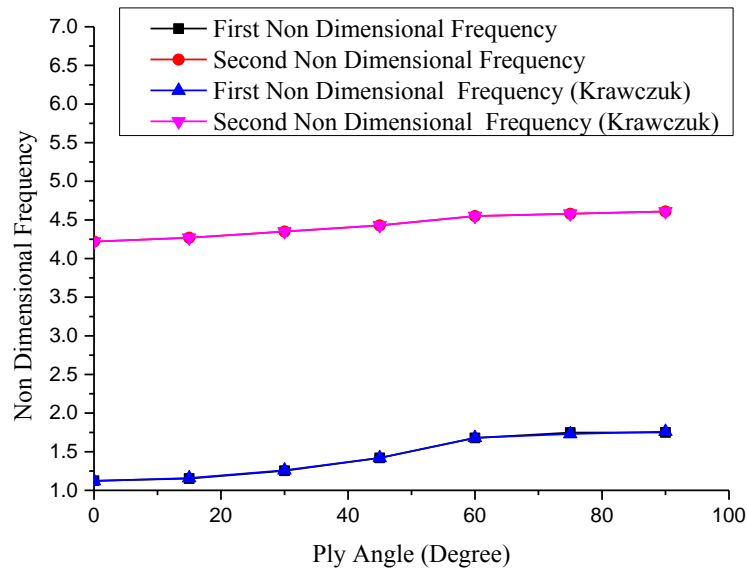


Figure 5: Comparison of first two Non Dimensional Natural Frequencies of the cracked composite beam with RCD=0.6 and RCL = 0.1

Also Non dimensional frequencies of intact composite beam were found out for different boundary condition such as Clamped-Clamped condition and Clamped-Supported. And the results are shown in figure 6 and 7 respectively.

From the figures it can be noticed that non dimensional frequencies is maximum at Clamped-Clamped condition. Non dimensional frequencies decrease in the order of Clamped - Clamped condition, Clamped-Simply Supported and cantilever condition.

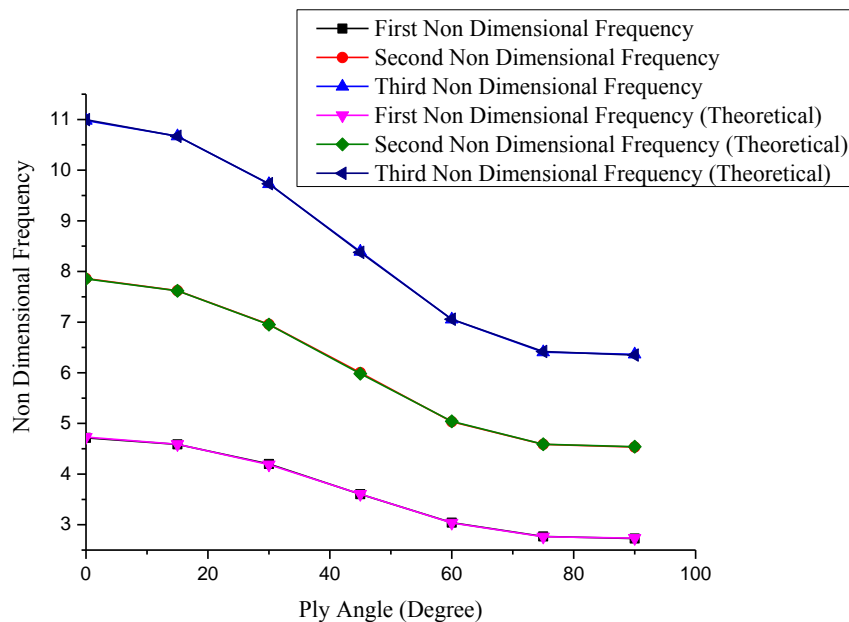


Figure 6: Comparison of First Three Non Dimensional Natural Frequencies of Present Analysis with Theoretical Results for clamped-clamped condition.

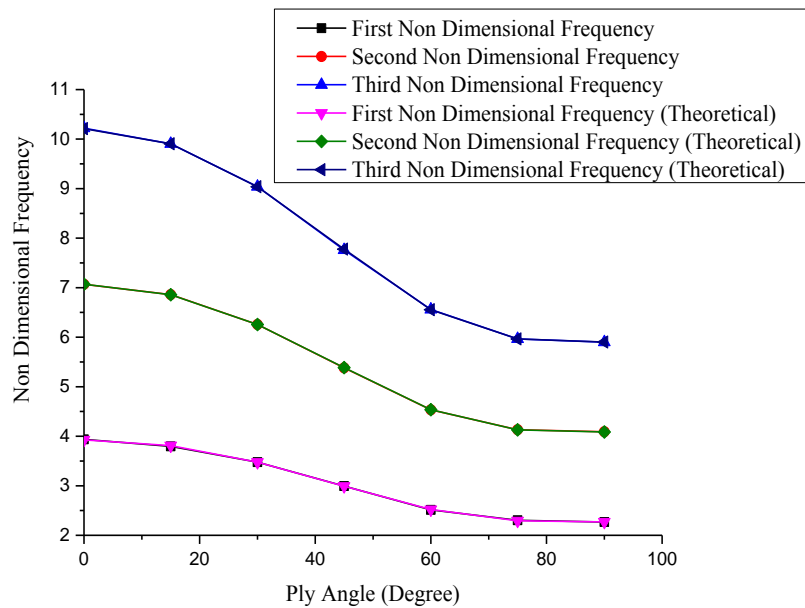


Figure 7: Comparison of First Three non-dimensional natural frequencies of present analysis with theoretical results for clamped-simply supported condition.

### 4.3 Harmonic Analysis

Any sustained cyclic load will produce a sustained cyclic response (a harmonic response) in a structural system. Harmonic response analysis gives the ability to predict the sustained dynamic behavior of structures, thus enabling to verify whether or not designs will successfully overcome resonance, fatigue, and other harmful effects of forced vibrations. Harmonic response analysis is a technique used to determine the steady state response of a linear structure to loads that vary sinusoidally (harmonically) with time. This analysis technique calculates only the steady-state, forced vibrations of a structure. The transient vibrations, which occur at the beginning of the excitation, are not accounted for in a harmonic response analysis [11]. Out of the three different methods available ie, Full, Reduced and Modal Superposition methods, the Full method is adopted here since it makes use of the full stiffness and mass matrices.

Harmonic analysis of cracked cantilever composite beam with crack depth 0.2 times the total depth of the beam is done and displacement at crack tip is found out for harmonic load of 1KN with ply angle 0 to 90 degrees and the results for 0, 45 and 90 degrees are shown in figure 8, 9 and 10 respectively. Displacement due to harmonic load varies sinusoidally from 0 to 90 degree ply angle.

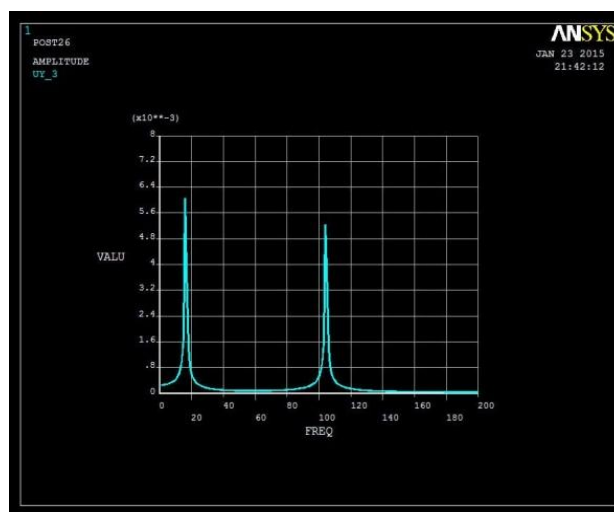


Figure 8: Result of displacement at 0.2 times the crack depth obtained through harmonic analysis for ply angle 0 degree.



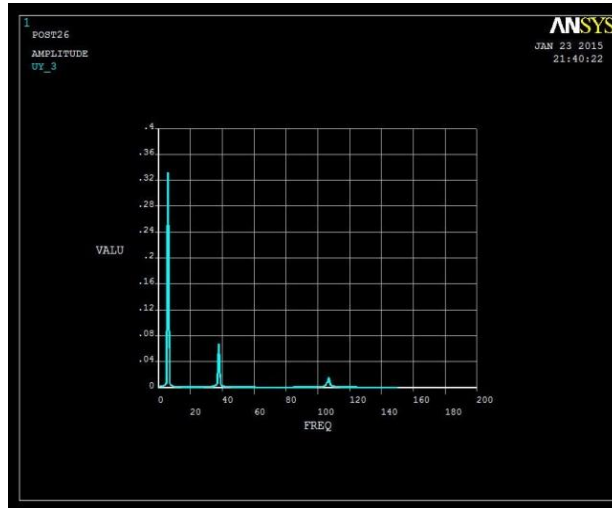


Figure 9: Result of displacement at 0.2 times the crack depth obtained through harmonic analysis for ply angle 45 degree.

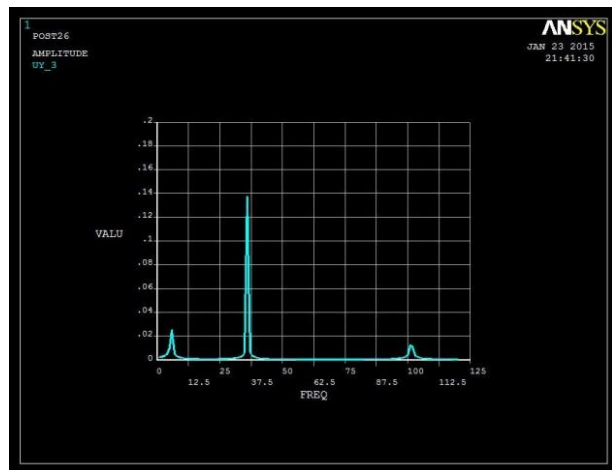


Figure 10: Result of displacement at 0.2 times the crack depth obtained through harmonic analysis for ply angle 90 degree.

## 5. CONCLUSION

The following conclusions can be made from the present investigations of the composite beam finite element having transverse non-propagating one-edge open crack.

- 1) The stress occurring at the crack tip increases as the depth of the crack increases.
- 2) Variations in natural frequencies occur with boundary conditions.
- 3) For an intact composite beam, Natural frequency is maximum at 0 degree and reduces gradually with increasing the fiber angle up to 90 degree.
- 4) In case of composite beam with crack, as the angle of fibers ( $\alpha$ ) increases the value of the natural frequencies also decreases.
- 5) Decrease in the natural frequencies become more intensive with the increase in the depth of crack.
- 6) Non dimensional frequencies vary with respect to the applied boundary conditions.
- 7) Displacement due to harmonic load at the crack tip is minimum at 0 degree ply angle.
- 8) Displacement due to harmonic load tends to have maximum value towards 45 to 90 degree ply angle.

As further studies, the dynamic stability of beam by introducing inclined cracks, delamination etc in place of transverse crack can be employed.

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