Conversion of Low Speed Mechanical Energy into Electric Energy - New Electrostatic Generator

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ABSTRACT— ***In power generation from natural energy, especially such as wave and tidal energies, the velocity of the moving body or medium is very slow. In the traditional energy conversion, we are obliged to convert the slow speed motion into high speed motion, and this causes a big energy loss. Accordingly, an energy conversion suited for the low speed motion is required. Namely, the energy conversion of not velocity but displacement type is important in these cases. In the present discussion, we propose a new electrostatic generator based not on electromagnetic but on electrostatic phenomena. We use the charge storage effect of a capacitor. In the present discussion, we discuss not only on the principle but also on the application to wave and tidal generators. If we use electrical double-layer capacitors, the capacitance increase dramatically. This suggest us the possibility of an electrostatic generation using capacitors.***

Keywords— Energy transformation, Displacement-type, Electric double-layer capacitor, Electrostatic generator, Wave/Tidal generation

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1. INTRODUCTION

The traditional converter of kinetic energy into electric energy mostly utilizes the electromagnetic phenomenon and assumes a high speed displacement or rotation of a body or medium. However, in case of energy conversion from natural energies, especially in case of wave and tidal generation, the motion is reciprocal and the velocity is very slow. In case of traditional conversion of energy, the low speed reciprocal motion must be transformed from the low speed reciprocal motion into the high speed rotational motion. This invites an energy loss. Hence, we must develop a new energy conversion method for low speed motion. Namely, a not velocity-type but displacement-type energy converter is required.

In the present paper, we propose a new energy converter using not electromagnetic but electrostatic phenomenon. Electrostatic generators such as the Van de Graff generator is well known [1]. Kelvin water dropper is also well known [2-5]. However, they may not be utilized for large scale power generation. In short, we utilize the electricity storage effect of a capacitor, and this convert displacement into electricity. Recently, a electric double layer capacitor has appeared, and the storage performance of the capacitor is drastically increased [6]. Since a capacitor with capacitance 100F has been developed, this suggests a possibility of high-performance electrostatic generator. In the present paper, we discuss not only the principle of the electrostatic generator but also the application to the wave and/or tidal generator.

Although the output of the wave and wind power generation is very irregular, that of the tidal power is constant. Furthermore, if we install the tidal power generator into a container connected to sea, we can avoid the damage due to strong wind and big wave. The reliability of the tidal power generation is high, though the output is small.

Table 1: List of symbols.
(If there is plurality of capacitors, we discriminate using subscripts. For example, V_i refers to the voltage between capacitor plates of capacitor i .)

Symbol	Definition	Unit	Symbol	Definition	Unit
S	Area of capacitor plate	m^2	ε	permittivity	F/m
Δ	Distance between capacitor	m	t	Time	s

	plates				
$\overline{\Delta}$	Average of Δ	m	r	Resistance	Ω
Q	Charge	C	i	Current	A
V	Voltage	V	V_B	Base voltage of capacitor	V
C	Capacitance	F	EP	Electric output power	W
E	Electric field	V/m	MP	Mechanical input power	W
F	Force acting on electrode	N	$avg(EP)$	Average of EP	W
U	Saved electric energy	J	$avg(MP)$	Average of MP	W
W	Work done by external force	J			

2. PRINCIPLE OF ELECTROSTATIC GENERATOR

2.1. Parallel plate capacitor

Figure 1 illustrates a parallel plate capacitor. If we apply voltage on the facing electrodes, the electric charge is stored in the capacitor.

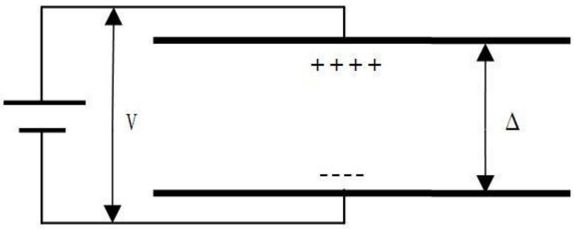

Figure 1: A parallel plate capacitor

The relationship between the voltage V and charge Q is given by

$$Q = CV, \tag{1}$$

where the capacitance C is given by the permittivity ε, area S of the electrode and distance Δ between electrodes by

$$C = \frac{\varepsilon_0 S}{\Delta}. \tag{2}$$

The stored electric energy U is obtained as

$$U = \frac{1}{2}VQ = \frac{1}{2}CV^2. \tag{3}$$

Rewriting, we have

$$U = \frac{1}{2}VQ = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{\varepsilon S}\Delta. \tag{4}$$

The strength E of the electric field and force F acting on the electrode are given by

$$E = \frac{V}{\Delta} = \frac{Q}{C\Delta} = \frac{Q}{\varepsilon S}, \tag{5}$$

$$F = \frac{1}{2}QE = \frac{1}{2}\frac{QV}{\Delta} = \frac{1}{2}Q\frac{Q}{\varepsilon S} = \frac{1}{2}\frac{Q^2}{\varepsilon S} = \frac{1}{2}\frac{C^2(\Delta)V^2}{\varepsilon S}. \tag{6}$$

It is necessary to pay attention to $F \neq QE$. Since the charge on the same electrode does not contribute on F, and the strength of the electric field due to the other electrode is a half of the strength of the electric field due to the both electrodes.

2.2. Principle of a new electrostatic generation using capacitor

Let S be constant. The state of a capacitor can be described by (Δ, E). The initial state is described by (Δ_0, E_0). Q, E, V and F are functions of Δ and E. Since C is a function of Δ, we express C as $C(\Delta)$. Namely, we have

$$Q = C(\Delta)\Delta E = \frac{\varepsilon S}{\Delta}\Delta E = \varepsilon SE, \tag{7}$$

$$V = E\Delta, \tag{8}$$

$$F = \frac{1}{2}QE = \frac{1}{2}\varepsilon SE^2, \tag{9}$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon S}{\Delta}(\Delta E)^2 = \frac{1}{2}\varepsilon S\Delta E^2, \tag{10}$$

$$\delta W = F\delta\Delta = \frac{1}{2}\varepsilon SE^2\delta\Delta. \tag{11}$$

From Eq. (10), the contour plot of the stored energy U is shown in Figure 2. As shown by an arrow in the figure, the gradient of the contour becomes steep, as U increases. We consider the transition of states (Δ, E) as shown in Figure 3.

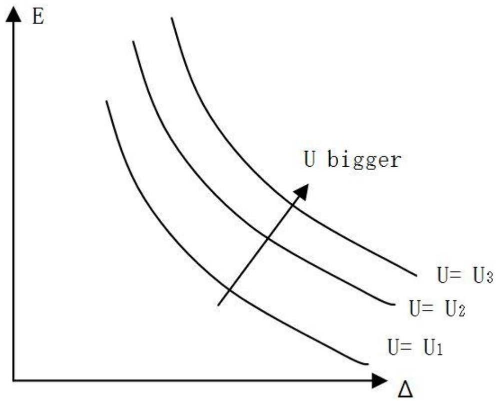

Figure 2: Contour of energy U of a capacitor

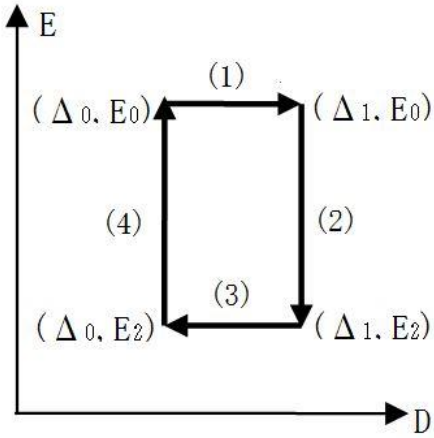

Figure 3: Transition of states

Step (1): We cut off the external electric circuit. Namely, applying the external force $F = (1/2)\varepsilon SE_0^2$, we increase the distance between the electrodes until $\Delta = \Delta_1$ under $Q = Q_0$. The stored electric energy U becomes large by an amount corresponding to the work done by the external force: $W = F(\Delta_1 - \Delta_0) = (1/2)\varepsilon SE_0^2(\Delta_1 - \Delta_0)$. The voltage increases from $V_0 = E_0\Delta_0$ to $V_1 = E_0\Delta_1$. Namely

$$\Delta_0 \to \Delta_1 \text{ under } Q = Q_0 \text{ then } (\Delta_0, E_0) \to (\Delta_1, E_0) \tag{12}$$

$$U_1 = U_0 + W = \frac{1}{2}\varepsilon S\Delta_0 E_0^2 + \frac{1}{2}\varepsilon SE_0^2(\Delta_1 - \Delta_0) = \frac{1}{2}\varepsilon S\Delta_1 E_0^2 \tag{13}$$

Step (2): We connect the external electric circuit. Under $\Delta = \Delta_1$, the electricity is discharged to the external electric circuit until E decreases to E_2. The discharged electric energy is $(1/2)\varepsilon S\Delta_1 E_0^2 - (1/2)\varepsilon S\Delta_1 E_2^2$. The stored electric energy is $U_2 = (1/2)\varepsilon S\Delta_1 E_2^2$. The voltage drops to $V_2 = E_2\Delta_1$. Namely

$$Q_0 \to Q_2 = C(\Delta_1)V_2 = \varepsilon SE_2 \text{ under } \Delta = \Delta_1 \text{ then } (\Delta_1, E_0) \to (\Delta_1, E_2) \tag{14}$$

Step (3): We cut off the external electric circuit. Under $Q = Q_2$, applying the external force $F_2 = Q_2^2/\varepsilon = (1/2)\varepsilon S E_2^2$ to the inverse direction as in step (1), the distance between electrodes is reduced to $\Delta = \Delta_0$. The work done by the external force is $W_2 = F_2(\Delta_1 - \Delta_0) = (1/2)\varepsilon S E_2^2(\Delta_1 - \Delta_0)$. The voltage drops to $V_3 = E_2\Delta_0$. Namely

$$\Delta_1 \to \Delta_0 \text{ under } Q = Q_1 \text{ then } (\Delta_1, E_2) \to (\Delta_0, E_2) \tag{15}$$

Step (4): We connect the external electric circuit. Under $\Delta = \Delta_0$, we supply the electricity from the external electric circuit until $Q = Q_0$. The supplied electric energy is $(1/2)\varepsilon S\Delta_0 E_0^2 - (1/2)\varepsilon S\Delta_0 E_2^2 = (1/2)\varepsilon S\Delta_0\left(E_0^2 - E_2^2\right)$. The voltage returns to $V_0 = E_0\Delta_0$ from $V_3 = E_2\Delta_0$. Namely

$$Q_1 \to Q_0 \text{ under } \Delta = \Delta_0 \text{ then } (\Delta_0, E_2) \to (\Delta_0, E_0) \tag{16}$$

We show below that the net electric energy supplied to the external electric circuit is equal to the net work done by the external force:

Net electric energy supplied to the external electric circuit
= Electric energy discharged to the external circuit in Step (2)
- Electric energy supplied from the external circuit in Step (4)

$$= \frac{1}{2}\varepsilon S\Delta_1 E_0^2 - \frac{1}{2}\varepsilon S\Delta_1 E_2^2 - \frac{1}{2}\varepsilon S\Delta_0\left(E_0^2 - E_2^2\right)$$

$$= \frac{1}{2}\varepsilon S(\Delta_1 - \Delta_0)E_0^2 - \frac{1}{2}\varepsilon S(\Delta_1 - \Delta_0)E_2^2 = \frac{1}{2}\varepsilon S(\Delta_1 - \Delta_0)(E_0^2 - E_2^2), \tag{17}$$

Net work supplied from the external force
= Work supplied from the external force in Step (1)
- Work done to the external force in Step (3)

$$= \frac{1}{2}\varepsilon S E_0^2(\Delta_1 - \Delta_0) - \frac{1}{2}\varepsilon S E_2^2(\Delta_1 - \Delta_0) = \frac{1}{2}\varepsilon S(\Delta_1 - \Delta_0)(E_0^2 - E_2^2). \tag{18}$$

However, if the work done to the external force is thrown away, the efficiency η becomes

$$\eta = \frac{\textit{Net electric energy}}{\textit{Work done by external force}} = \frac{(U_1 - U_2) - (U_0 - U_3)}{U_1 - U_0} = 1 - \frac{U_2 - U_3}{U_1 - U_0}$$

$$= 1 - \frac{(1/2)\varepsilon_0 S\Delta_1 E_2^2 - (1/2)\varepsilon_0 S\Delta_0 E_2^2}{(1/2)\varepsilon_0 S\Delta_1 E_0^2 - (1/2)\varepsilon_0 S\Delta_0 E_0^2} = 1 - \frac{(\Delta_1 - \Delta_0)E_2^2}{(\Delta_1 - \Delta_0)E_0^2} = 1 - \frac{E_2^2}{E_0^2} \tag{19}$$

3. VERIFICATION OF PRINCIPLE OF ELECTROSTATIC GENERATOR BY NUMERICAL SIMULATION

3.1. Verification of principle 1

As shown in Figure 4, we consider a device having a pair of capacitors charged initially. If we change the distance of the electrodes of one of the capacitors, the voltage between the electrodes changes, and an alternating current is generated. We need work done from the outside to change the distance of the electrodes. Then, electricity corresponding to the mechanical input energy is generated and consumed by a resister in the circuit. We conduct simulation calculations and verify the principle of the electrostatic generator.

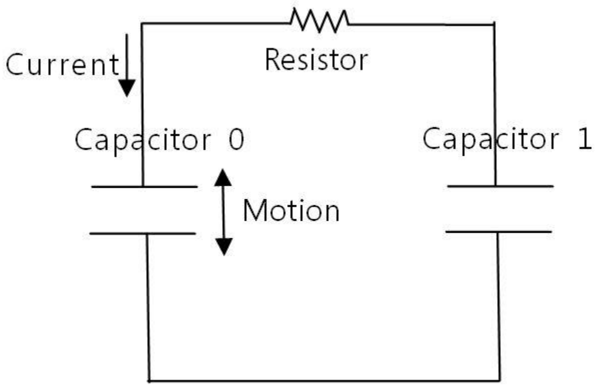

Figure 4: A system for verifying the principle of an electrostatic generator

In this case, the distance Δ_0 of capacitor 0 is driven, and the distance Δ_1 of capacitor 1 is held constant. The basic equations are given by

$$V_0 + ri = V_1 \tag{20}$$

$$\frac{dQ_0}{dt} = i\,,\ \frac{dQ_1}{dt} = -i\,, \tag{21}$$

$$Q_0 = C_0(\Delta_0)V_0\,,\ Q_1 = C_1(\Delta_1)V_1\,, \tag{22a, b}$$

$$\Delta_0 = \overline{\Delta} + A\sin\left(\frac{2\pi}{T}t\right),\ \Delta_1 = \overline{\Delta}\,. \tag{23a, b}$$

From Eq. (20), we have

$$i = \frac{1}{r}(V_1 - V_0)\,. \tag{24}$$

Substituting Eqs. (22) and (24) into (21), we obtain

$$\frac{dC_0(\Delta_0)V_0}{dt} = \frac{1}{r}(V_1 - V_0)\,,\ \frac{dC_1(\Delta_1)V_1}{dt} = -\frac{1}{r}(V_1 - V_0)\,. \tag{25}$$

Then, differential equations for the valteges V_0 and V_1 are derived as

$$\frac{dV_0}{dt} + \left(\frac{dC_0(\Delta_0)}{dt}\bigg/C_0(\Delta_0)\right)V_0 = \frac{1}{rC_0(\Delta_0)}(V_1 - V_0)\,, \tag{26a}$$

$$\frac{dV_1}{dt} + \left(\frac{dC_1(\Delta_1)}{dt}\bigg/C_1(\Delta_1)\right)V_1 = -\frac{1}{rC_1(\Delta_1)}(V_1 - V_0)\,. \tag{26b}$$

Since Δ_0 and Δ_1 are given by Eq. (23), we are required to solve Eq. (26) under the initial condition:

$$\Delta_0 = \Delta_1 = \overline{\Delta}\,,\ V_0 = V_1 = V_B\,,\ Q_0 = Q_1 = C(\overline{\Delta})V_B\,,\ i = 0 \text{ at } t = 0\,. \tag{27}$$

The electric force F_0 and F_1 acting on electrodes 1 and 2 are given by

$$F_0 = \frac{1}{2}Q_0E_0 = \frac{1}{2}\frac{C_0^2(\Delta_0)V_0^2}{\varepsilon S_0}\,,\ F_1 = \frac{1}{2}Q_1E_1 = \frac{1}{2}\frac{C_1^2(\Delta_1)V_1^2}{\varepsilon S_1} \tag{28}$$

The average electric output power $avg(EP)$ and average mechanical input power $avg(MP)$ are defined as

$$avg(EP) = \lim_{\tau\to\infty}\frac{1}{\tau}\int_0^\tau ri^2dt\,,\ avg(EP) = \lim_{\tau\to\infty}\frac{1}{\tau}\int_0^\tau F_0\frac{d\Delta_0}{dt}dt\,. \tag{29}$$

As proved in Appendix A, the conservation of energy gives

$$avg(EP) = avg(MP)\,. \tag{30}$$

Numerical results are shown in Figure 5. Parameters used in the simulation are shown in Table 2. In this case, $avg(EP) = 199$ and $avg(MP) = 246$, the difference between them is not small.

Table 2: Parameters used in the simulation

A	1	S_0	10	V_B	100
T	20	S_1	10	R	4
$\overline{\Delta}$	2	ε	10	dt	0.02

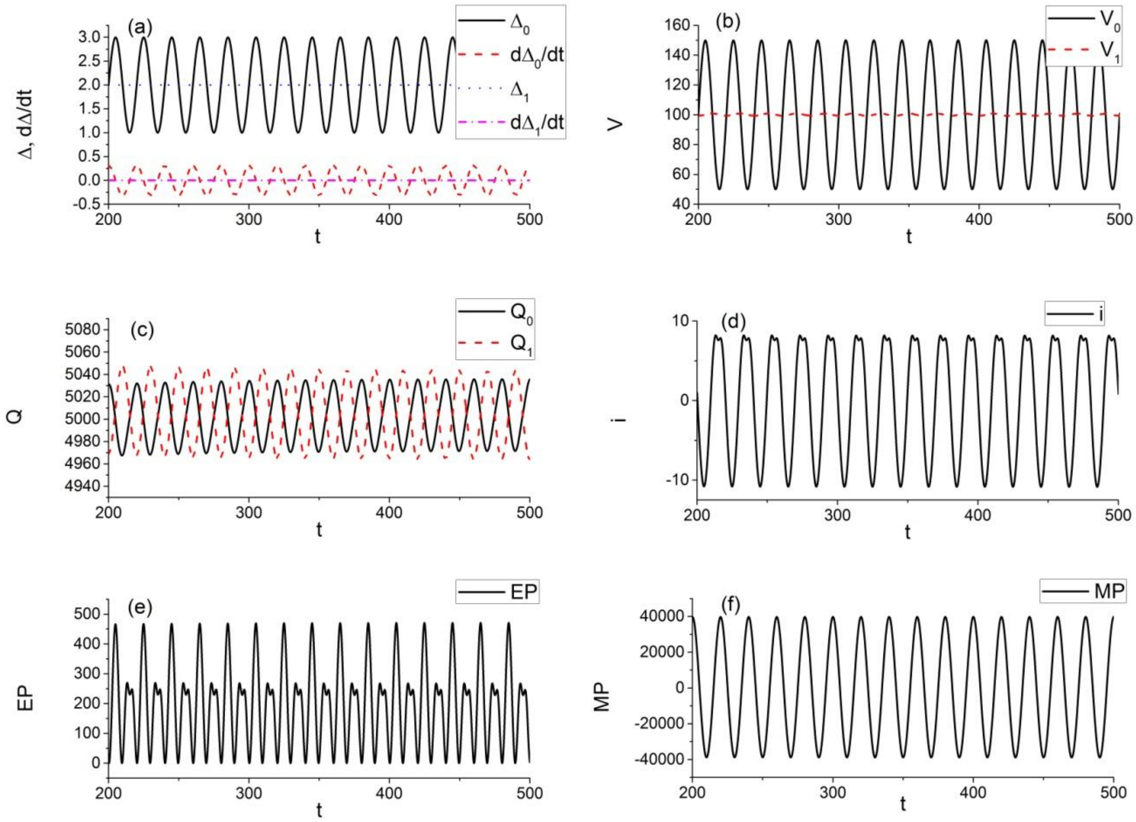

Figure 5: One pair of electrodes is fixed and the other pair of electrodes is driven sinusoidally. ($r = 0.4$; (a) Distance Δ between electrodes and the time rate of change $d\Delta/dt$; (b) Voltages V_0 and V_1 between electrodes; (c) Stored charges Q_0 and Q_1 ; (d) Current i ; (e) Electric input power EP ; (f) Mechanical output power MP)

3.2. Verification of principle 2

We consider a different system from 3.1. As shown in Figure 6, we consider a device having a pair of capacitors charged initially. If we change the distance between electrodes of one capacitor sinusoidaly and that of the other capacitor in opposite phase, an alternating current is generated in the circuit, since the voltage between electrodes of one capacitor goes up and down with that of the other capacitor in opposite phase. Then, electricity corresponding to the mechanical input energy is generated and consumed by a resister in the circuit. We conduct simulation calculations and verify the principle of the electrostatic generator.

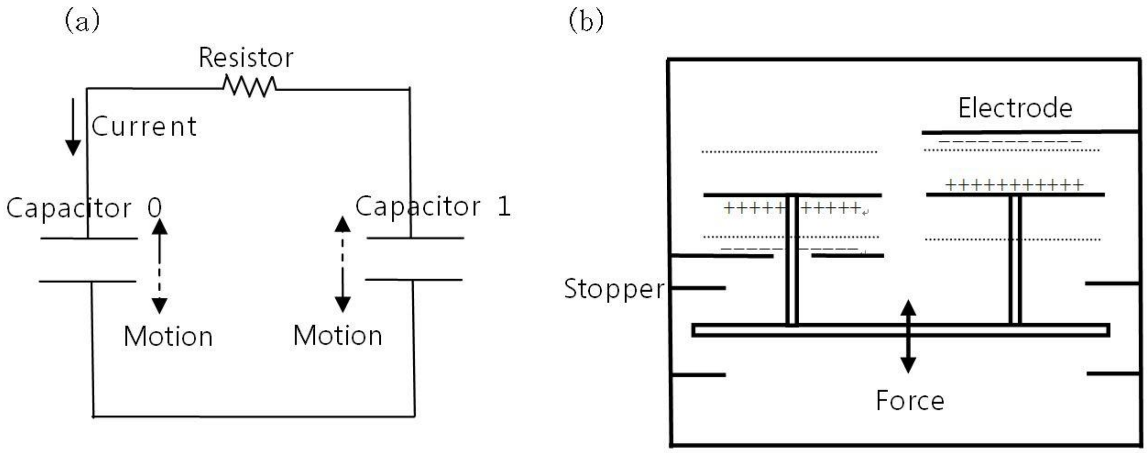

Figure 6: A system for verifying the principle of an electrostatic generator. ((a) Electric circuit; (b) Mechanisml) The distance Δ_0 between electrodes of capacitor 0 and the distance Δ_1 between electrodes of capacitor 1 are driven with reverse phase

3.3.1. A case when capacitors 1 and 2 are connected always

In Eqs. (20~26), we replace Eq. (23) by

$$\Delta_0 = \overline{\Delta} + A\sin\left(\frac{2\pi}{T}t\right),\ \Delta_1 = \overline{\Delta} - A\sin\left(\frac{2\pi}{T}t\right). \tag{31a, b}$$

We are required to solve Eq. (26) under the initial condition (27)

Parameters used in the simulation are given in Table 3. The numerical results are shown in Figure 7. In this case, $avg(EP) = 1003$ and $avg(MP) = 1116$, the difference between them is small.

Table 3: Parameters used in simulation

A	1	S_0	10	V_B	100
T	20	S_1	10	r	4
$\overline{\Delta}$	2	ε	10	dt	0.02

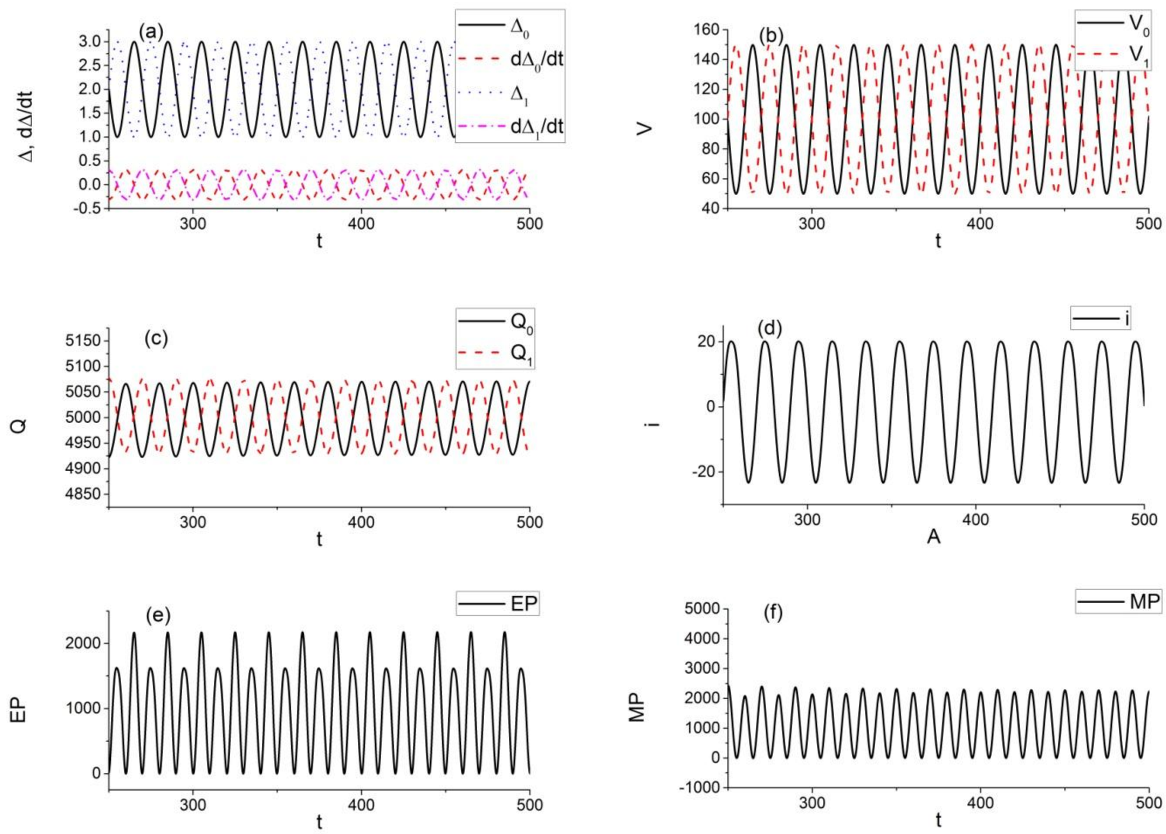

Figure 7: Two pairs of electrodes are driven sinusoidally with opposite phase each other. ($r = 4$; (a) Distance Δ between electrodes and the time rate of change $d\Delta/dt$; (b) Voltages V_0 and V_1 between electrodes; (c) Stored charges Q_0 and Q_1; (d) Current i; (e) Electric input power EP; (f) Mechanical output power MP)

The numerical results in which $r = 4$ is replaced by $r = 0.04$ in Table 3 are shown in Figure 8. In this case, $\overline{EP} = 11151$ and $\overline{MP} = 11149$, they are almost equal.

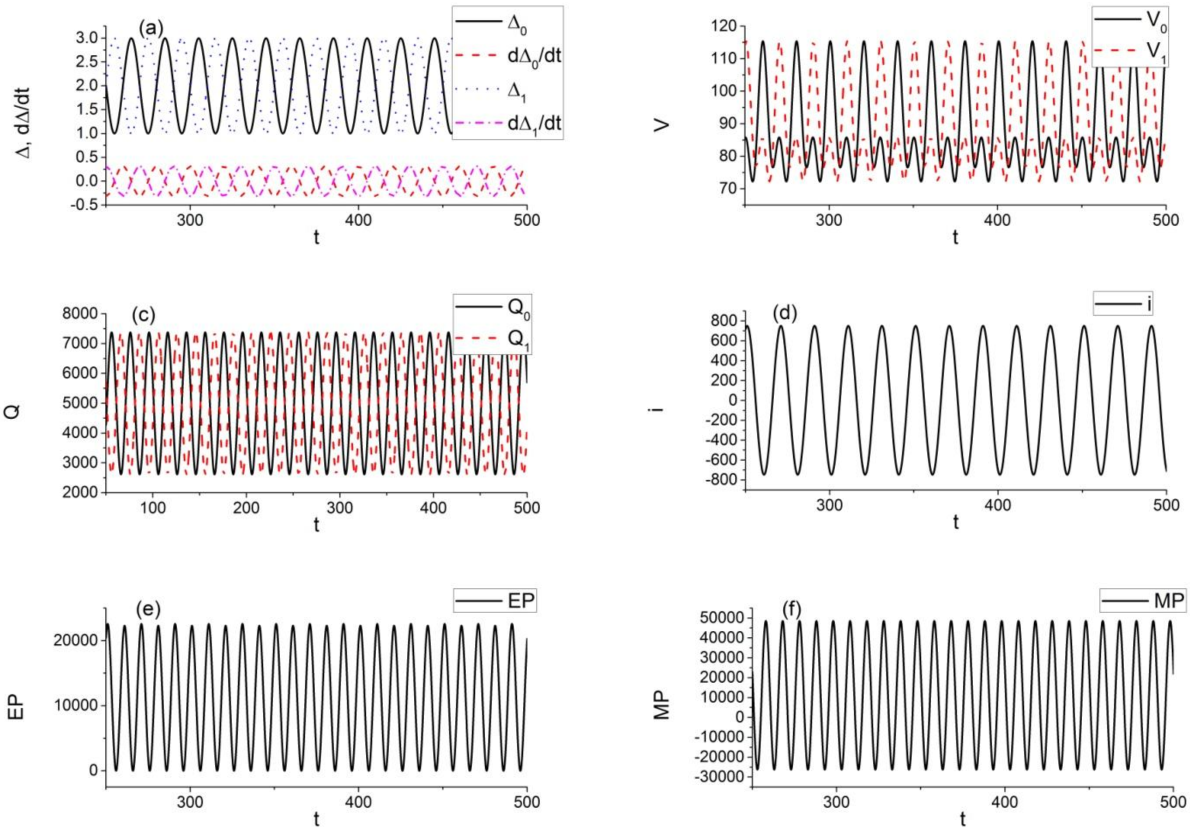

Figure 8: Two pairs of electrodes are driven sinusoidally with opposite phase each other. ($r = 0.04$; (a) Distance Δ between electrodes and the time rate of change $d\Delta/dt$; (b) Voltages V_0 and V_1 between electrodes; (c) Stored charges Q_0 and Q_1 ; (d) Current i ; (e) Electric output power EP ; (f) Mechanical input power MP)

Effects of resistance r and base voltage V_B on the average electric output power $avg(EP)$ and mechanical input power $avg(MP)$ are shown in Figure 9. Effects of wave amplitude A and period T on the average electric output power $avg(EP)$ and mechanical input power $avg(MP)$ are shown in Figure 10.

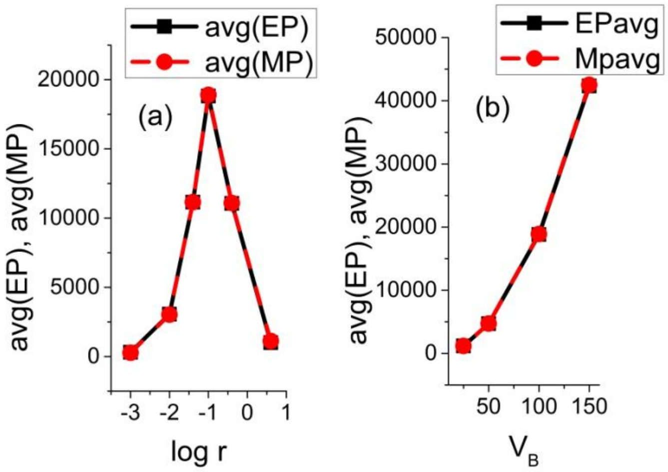

Figure 9: Effects of resistance r and base voltage V_B on average electric output power $avg(EP)$ and mechanical input power $avg(MP)$. ((a) Effects of resistance r ; (b) Effects of base voltage V_B)

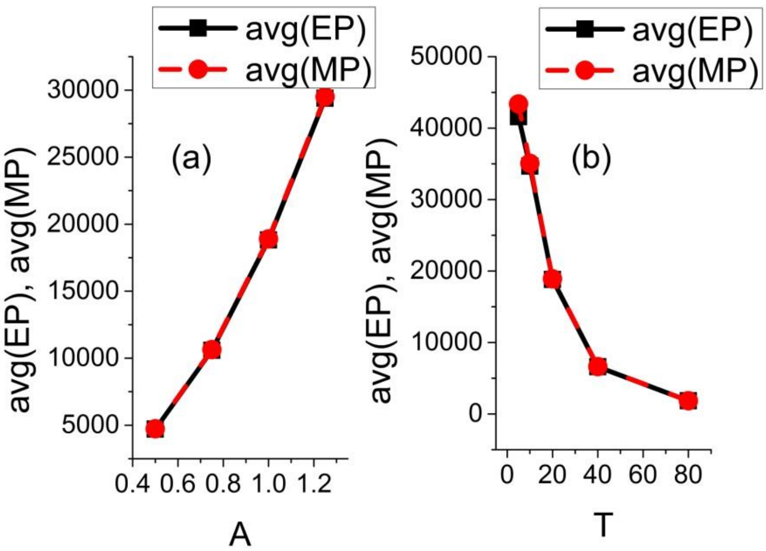

Figure 10: Effects of wave amplitude A and period T on average output electric power $avg(EP)$ and mechanical input power $avg(MP)$. ((a) Effects of amplitude A; (b) Effects of period T)

3.3.2. A case when capacitors 1 and 2 repeatedly connected and disconnected

In case of the tidal generation, since the wave period T becomes very long, we may need the following idea. For example, we connect the electric circuit when $nT + 0.25T \le t < nT + 0.35T$ and $nT + 0.75T \le t < nT + 0.85T$, $n = 0, 1, 2, \cdots$ and disconnect otherwise. Namely, we replace Eqs. (26a) and (26b) with

$$\frac{dV_0}{dt} + \left(\frac{dC_0(\Delta_0)}{dt}\bigg/C_0(\Delta_0)\right)V_0 = \begin{cases} \dfrac{1}{rC_0(\Delta_0)}(V_1 - V_0) & \text{for } nT + \begin{Bmatrix}0.25\\0.75\end{Bmatrix}T \le t < nT + \begin{Bmatrix}0.35\\0.85\end{Bmatrix}T \\ 0 & \text{otherwise} \end{cases}, \tag{32a}$$

$$\frac{dV_1}{dt} + \left(\frac{dC_1(\Delta_1)}{dt}\bigg/C_1(\Delta_1)\right)V_1 = \begin{cases} \dfrac{-1}{rC_0(\Delta_0)}(V_1 - V_0) & \text{for } nT + \begin{Bmatrix}0.25\\0.75\end{Bmatrix}T \le t < nT + \begin{Bmatrix}0.35\\0.85\end{Bmatrix}T \\ 0 & \text{otherwise} \end{cases}. \tag{32b}$$

The parameters used in the numerical calculation are given in table 4. The numerical results are shown in Figure 11. In this case, $avg(EP) = 411934.6$ and $avg(MP) = 421329.8$, the difference between them is small.

Table 4: Parameters used in simulation

A	1	S_0	10	V_B	100
T	20	S_1	10	r	0.001
$\overline{\Delta}$	2	ε	100	dt	0.02

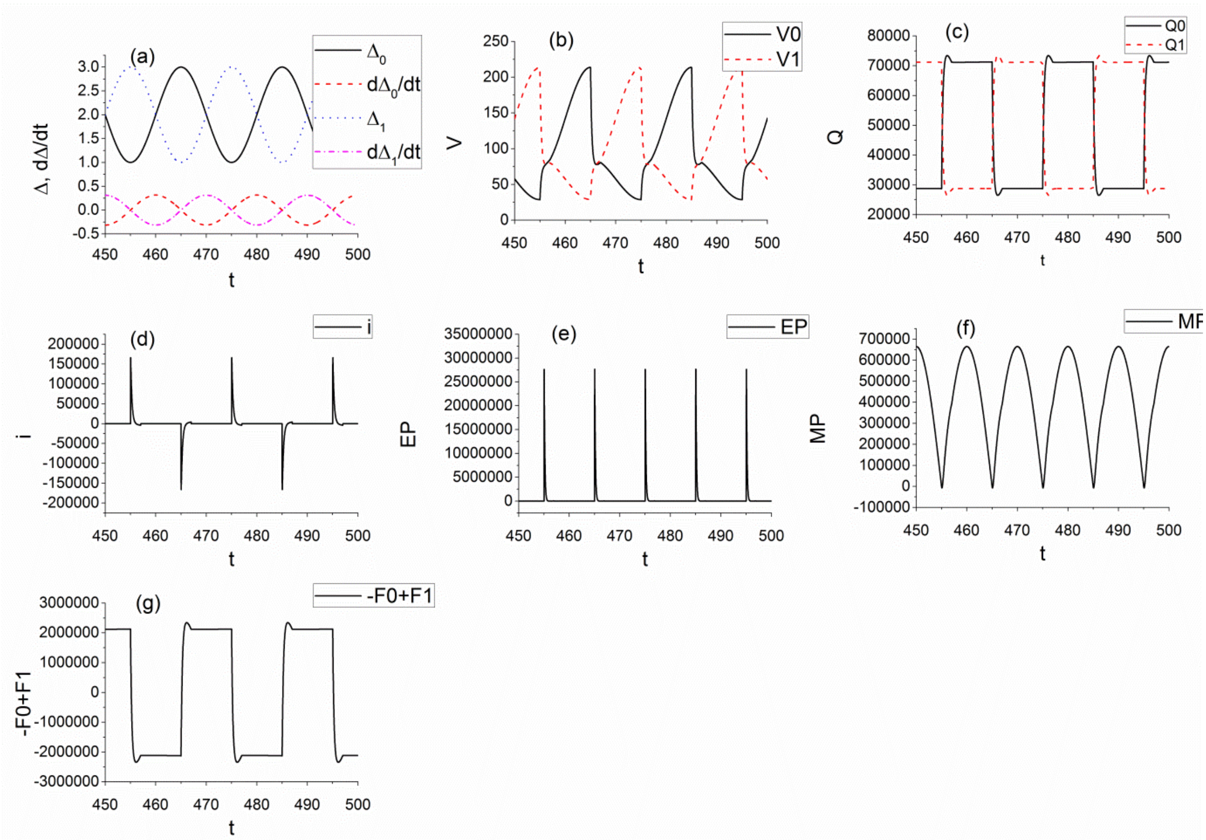

Figure 11: Two pairs of electrodes are driven sinusoidally with opposite phase each other. ($r = 0.001$; (a) Distance Δ between electrodes and the time rate of change $d\Delta/dt$; (b) Voltages V_0 and V_1 between electrodes; (c) Stored charges Q_0 and Q_1; (d) Current i; (e) Electric output power EP; (f) Mechanical input power MP; (g) Force acting on the device $F = -F_0 + F_1$)

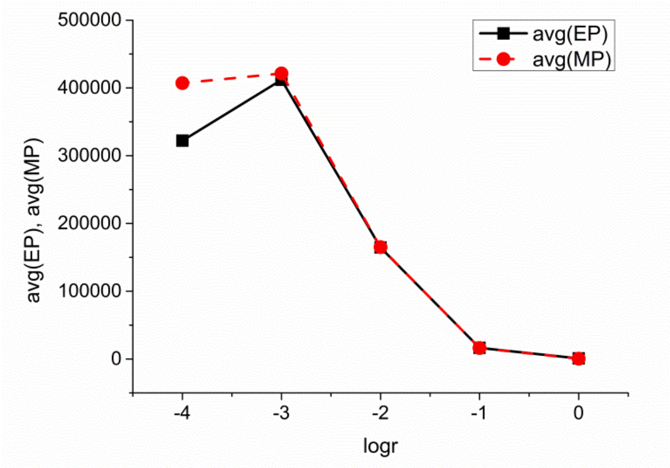

Figure 12: Effects of resistance r on average electric output and mechanical input powers $avg(EP)$ and power $avg(MP)$. (Period $T = 20$)

On the assumption that it is used for the tidal power generation, we show a result when the wave period T is 12hr or 43200sec in Figure 13. The computational conditions are shown in Table 5. In this case the average electric output power $avg(EP)$ and $avg(MP)$ are 172W and 180W, respectively. The difference is small.

Table 5: Parameters used in simulation

A	1	S_0	10	V_B	100
T	43200	S_1	10	r	0.1
$\overline{\Delta}$	2	ε	100	dt	2.16

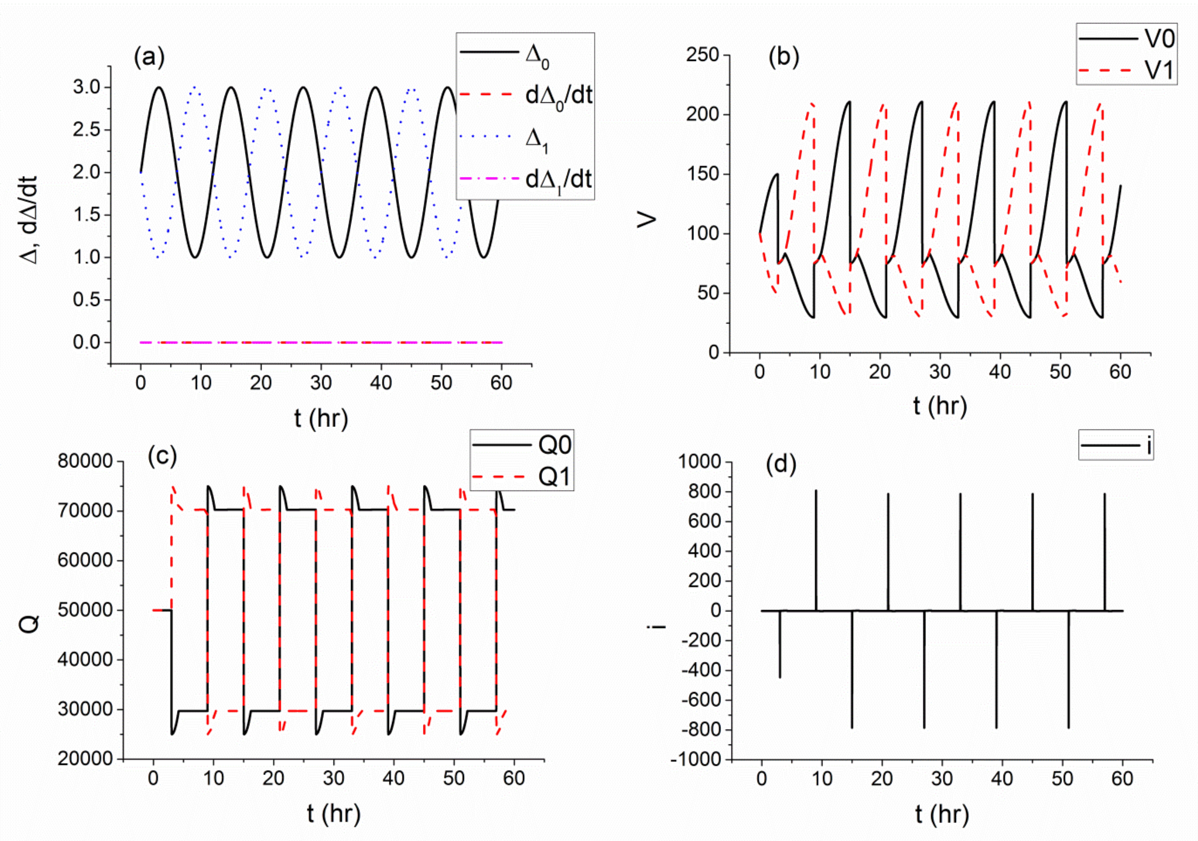

Figure 13: Case when wave period T is 12hr or 43,200sec. ((a) Distance and time derivatives of electrodes; (b) Voltage between electrodes; (c) Charge of capacitor 0 and 1; (d) Electric current in circuit)

4. CONCLUSIONS

Principle of a new electrostatic generator using capacitors and possibility of the application to wave and/or tidal generation are discussed. In case of wave and/or tidal generation, an energy convertor suitable to the slow speed motion of a moving body or media would be required. Not velocity type but displacement type energy conversion would be required. It would reduce the energy loss in the conversion of slow reciprocating motion into high speed rotational motion. It would reduce the energy loss when slowly oscillating fluid motion is transformed into high speed rotational motion of a turbine.

We proposed a new electrostatic generator that does not use the electromagnetic phenomenon. It uses the electrostatic phenomenon. In a word, it uses the charge storage effect of a capacitor. It converts the displacement into the electricity.

We also studied the conversion cycle and efficiency when a capacitor converts the oscillatory displacement into the electric energy. Furtheremore, we discussed the application to the wave and/or tidal generation. We believe that it will improve the conversion efficiency from the natural energy such as wave and tide to the electricity. It can also be applied to the power generation by animal power and human power in developing countries. It will also contribute to ensure the energy in emergency.

Recently, a electric double layer capacitor has appeared, and the storage performance of the capacitor is drastically increased. Since a capacitor with capacitance 100F has been developed, this suggests a possibility of high-performance electrostatic generator.

5. REFERENCES

[1] Wikipedia, Electrostatic generator, http://en.wikipedia.org/wiki/Electrostatic_generator.
[2] On a self-acting apparatus for multiplying and maintaining electric charges, with applications to illustrate the voltaic theory, Phil. Mag. S. 4. Vol. 34. N0. 231. Nov. (1867), pp. 391-396.
[3] Kelvin water dropper, http://web.archive.org/web/20050208210755/http://www.csiro.au/helix/experiments/dhexpkelvin.shtml.

[4] R. A, Schumacher, The Kelvin water dropper also known as Lord Kelvin's thunderstorm, U-tube, http://www.youtube.com/watch?v=8Jx1pvFiaoI, (2011).
[5] A. G. Marin, W. van Hoeve, P. G.Sánchez, L. Shui, Y. Xie, M. A. Fontelos, J. C. T. Eijkel,A. van den Berg, D. Lohse, The microfluidic Kelvin water dropper, (http://arxiv.org/pdf/1309.2866v2.pdf), physics.flu-dyn, Sept. (2013).
[6] Wikipedia, Electric double-layer capacitor, http://en.wikipedia.org/wiki/Electric_double-layer_capacitor

APPENDIX A. CONSERVATION OF ELECTRIC CHARGE AND ENERGY

First, we obtain the conservation law of charge. Rewriting Eqs. (21a) and (21b), we have

$$\frac{dQ_0}{dt}=\frac{1}{r}(V_1-V_0)\,,\quad \frac{dQ_1}{dt}=-\frac{1}{r}(V_1-V_0) \tag{A1.a, b}$$

Adding Eqs. (A1.a) and (A1.b), we obtain the conservation law of charge:

$$\frac{d(Q_0+Q_1)}{dt}=0\,. \tag{A2}$$

Next, we obtain the conservation law of energy. Subtracting Eqs. (A1.a) and (A1.b), we have

$$\frac{d(Q_0-Q_1)}{dt}=\frac{2}{r}(V_1-V_0)\,. \tag{A3}$$

Multiplying V_1-V_0 on both sides, we obtain

$$(V_1-V_0)\frac{d(Q_0-Q_1)}{dt}=\frac{2}{r}(V_1-V_0)^2 \tag{A4}$$

Rewriting the left hand side, we derive

$$\begin{aligned}
(V_1-V_0)\frac{d(Q_0-Q_1)}{dt}&=V_1\frac{dQ_0}{dt}+V_0\frac{dQ_1}{dt}-V_0\frac{dQ_0}{dt}-V_1\frac{dQ_1}{dt}\\
&=(V_1-V_0)\frac{dQ_0}{dt}-V_0\frac{dC_0(\Delta_0)V_0}{dt}-V_1\frac{dC_1(\Delta_1)V_1}{dt}\\
&=(V_1-V_0)\frac{V_1-V_0}{r}-V_0\frac{d\left(\frac{\varepsilon_0 S_0}{\Delta_0}V_0\right)}{dt}-V_1\frac{d\left(\frac{\varepsilon_1 S_1}{\Delta_1}V_1\right)}{dt}\\
&=\frac{(V_1-V_0)^2}{r}-V_0\frac{-\varepsilon_0 S_0}{\Delta_0}\frac{V_0}{\Delta_0}\frac{d\Delta_0}{dt}-V_0\frac{\varepsilon_0 S_0}{\Delta_0}\frac{dV_0}{dt}-V_1\frac{-\varepsilon_0 S_1}{\Delta_1}\frac{V_1}{\Delta_1}\frac{d\Delta_1}{dt}-V_1\frac{\varepsilon_0 S_1}{\Delta_1}\frac{dV_1}{dt}\\
&=\frac{(V_1-V_0)^2}{r}+Q_0E_0\frac{d\Delta_0}{dt}-Q_0\frac{dV_0}{dt}+Q_1E_1\frac{d\Delta_1}{dt}-Q_1\frac{dV_1}{dt}\\
&=\frac{(V_1-V_0)^2}{r}-2(-F_0+F_1)\frac{d\Delta_0}{dt}-\frac{dQ_0V_0}{dt}-\frac{dQ_1V_1}{dt}-\frac{dQ_0}{dt}(V_1-V_0)\\
&=-2(-F_0+F_1)\frac{d\Delta_0}{dt}-2\frac{dU_0}{dt}-2\frac{dU_1}{dt}\,.
\end{aligned} \tag{A5}$$

Substituting Eq. (A5) into Eq. (A4), we have

$$-2(-F_0+F_1)\frac{d\Delta_0}{dt}-2\frac{dU_0}{dt}-2\frac{dU_1}{dt}=\frac{2}{r}(V_1-V_0)^2\,. \tag{A6}$$

Rewriting, we obtain

$$(F_0-F_1)\frac{d\Delta_0}{dt}=\frac{dU_0}{dt}+\frac{dU_1}{dt}+\frac{1}{r}(V_1-V_0)^2\,. \tag{A7}$$

Integrating Eq. (A7) with respect to time, the conservation law of energy:

$$\int_{t_1}^{t_2}(F_0-F_1)d\Delta_0=\left[U_0(t_2)+U_1(t_2)\right]-\left[U_0(t_1)+U_1(t_1)\right]+\int_{t_1}^{t_2}\frac{1}{r}(V_1-V_0)^2dt \tag{A8}$$

is obtained. Taking the average of one period T, we finally derive

$$\frac{1}{T}\int_t^{t+T}(F_0-F_1)d\Delta_0=\frac{1}{T}\int_t^{t+T}\frac{1}{r}(V_1-V_0)^2dt\,. \tag{A9}$$

The left hand side is the average mechanical input power $avg(MP)$ and the right hand side is the average electric output power $avg(EP)$.