Fractal Dimension of Urban form Elements and its Relationships: In the Case of City of Colombo

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ABSTRACT— ***Over the last decade, there has been a rapid increase in the amount of literature on the measurement of urban form simulation. The evolution of urban form studies have been conducted but in existing prediction changes of urban from have not been practiced especially in Sri Lankan spatial planning context. In this background, this research examines the applicability of Fractal Geometry as a method to simulate changes in urban form in Sri Lankan Context and study the relationship between the Fractal Dimension (FD) of Distribution of Transport Network and Buildings. This paper employs mass-radius method to identify the relationship between urban form elements in City of Colombo. Accordingly, study has been conducted urban form studies and calculated FD of road and built-up area has been done using mass-radius method. Findings of the study have revealed a strong correlation and linear relationship between the fractal dimensions of road and built-form. Since there is a strong relationship between FDs of urban form elements and urban form can be simulated. Finally the study concludes Fractal Geometry as a method to simulate urban form changes in Sri Lankan context.***

Keywords— Fractal Geometry, Urban Form, Urban Growth, Road Network, and Built-up area

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1. INTRODUCTION

Believing that a “city or town can be `read' and analyzed via the medium of its physical form” [1]. Subsystems of the city, such as land-use patterns (Batty& Longley , 1989), built-up area distributions (Batty & Xie, 1996), or transportation networks (Chen & Luo, 1998);(Rodin & Rodina, 2000);(Shen, 1997) which are closely related to urban form growth. A city has a very complex transportation network and both city and its transportation networks are complicated systems. As cities grow from smaller to larger their transportation networks generally become more complicated, the urban spaces are filled up more densely by city roads and locations within a city which are more accessible [2].

The evolution process of urban form studies have been conducted in Sri Lankan context. Yet, the applicability of the changes of the urban form using Fractal Geometry is not tested in Sri Lanka. In this background this research is to test the applicability of Fractal Geometry as a method to simulate changes in urban form in Sri Lankan context. This research attempts to predict the urban form elements using Fractal Geometry. Fractal Analysis (i.e. a method developed by Batty et al in 1989) is an effective approach to describe disorder and irregularity of natural or man -made spatial forms. Fractal growth can be regarded as a 2-D space filling process; planar fractal dimension can certainly be used to model2-D urban growth and urban form [3].

The main objective of this study is to test the applicability of Fractal Geometry as a method to simulate changes in urban form. And the sub objective is to study the relationships between the fractal dimension of Distribution of Transport Network and Buildings. This research was designed to identify the relationship between urban form elements in city of Colombo. Understanding urban form changes patterns is important for planners to analyze current phenomena and predict the future development. But in existing prediction changes of urban form has not been practiced in especially in Sri Lankan spatial planning context. It was assumed that there is a definite relationship between fractal dimensions of urban form elements. So, if the technique proves that there is a strong relationship between urban form elements, it will clearly say that simulation of Urban Form Changes can be practiced using Fractal Geometry application.

2. LITERATURE REVIEW

2.1. Urban Form and Urban Growth Simulation

"The word 'form' has many meanings, such as shape, configuration, structure; pattern, organization, and system of relations [4]. Also Individual parcels of urbanized areas can be geometrically planned and designed using Euclidean Geometry. However, the Planar Urban form as a whole system cannot be fully described by Euclidean Geometry. This is because the urban form and its development demonstrate the distinct nature of fractals, namely, irregularity, scale independence, and self-similarity at least for a range of scales. Thus, it is appropriate to regard the urbanized areas as fractals and study their spatial forms in Fractal Geometry [3].

Planar Fractal Growth can be regarded as a 2-D space filling process; Planar Fractal Dimension can certainly be used to model 2-D Urban Growth and Urban Form. Such a Fractal Dimension set may shed more light on the intriguing nature of Fractal Dimension as a spatial dimension measure and its role in Urban Modelling and Spatial Analysis [3]. Fractal theory makes it possible to advance our knowledge about the future of dynamic complex system such as cities and also enables urban systems to be modelled as regards growth process [9]. Models that simulate fractal structures can be calibrated to real situations and used for future predictions based on simple rules of land development [4].

2.2. Fractal Geometry

Geometry is the visual study of shapes, sizes, patterns, and positions also it is expressed by a branch of mathematics that is concerned with the properties of configurations of geometric objects - points, (straight) lines, and circles being the most basic of these. Such objects which show the same kind of irregularity at many scales have been called fractals (from the Latin adjective fractus meaning 'broken') by their inventor and popularizer Mandelbrot (1983, 1990) and the geometry which has emerged in their study is called 'Fractal Geometry'.

Fractal Analysis is an effective approach to describe disorder and irregularity. Fractal, as a term, was coined by Mandelbrot to refer to objects whose forms are essentially irregular, scale invariant, and self-similar. “A fractal is a geometrical figure that consists of an identical motif repeating itself on an ever-reduced scale" [4]. Fractals are irregular shapes with geometries that are scale-invariant. At every scale, the degree of irregularity which characterises the geometry appears to be the same, this being referred to as self-similarity [5].When applied to geographical phenomena, not only can natural geographical phenomena be typical fractal objects but also can artificially designed and planned concrete spatial objects and information infrastructure show fractal properties [2]. Basically Fractal has three following principles. First, fractals are always self-similar, at least in some general senses. On whatever scale, and within a given range you examine a fractal, it will always appear to have the same shape or same degree of irregularity. Fractals can always be described in terms of a hierarchy of self-similar components. Fractals are ordered hierarchically across many scales and the tree is the classic example and other principle is irregularity of Forms [4].

Fractals and fractal dimension can be understood from the viewpoint of geometric measure relation. There is a relationship between different measures. Therefore, for a length, L, an area, A, a volume, V, and a general spatial quantity (any “mass”), M, the measure relation such as

$$L^{1/1} \propto A^{1/2} \propto V^{1/3} \propto M^{1/df}$$

Where df refers to a general dimension, for a Euclidean object, we have df =0 for a point, df =1 for a line, df =2 for a plane, and df =3 for a cube. However, for a fractal object, the df value will vary from 0 to 3 [8].

2.3. Methods of calculating Fractal Geometry

There are various kinds of classifications which are used to estimate the fractal dimension by different authors. In urban and spatial analysis, fractal dimensions are mainly computed using the box-counting method and the mass-radius method. The mass-radius method is used in this study to calculate the Fractal Geometry. The mass dimension defines the relationship between the area located within a certain radius and the size of this radius (or box). This is performed for various radiuses as well as from various points of origin. The mass dimension can be estimated from the log-log plot of the area as a function of the radius [6].

3. STUDY AREA

City of Colombo has been selected as the study area which is the largest city in Sri Lanka. It is located on the West Coast of the island. Colombo is well known as the commercial hub of Sri Lanka. The City of Colombo has historically evolved well connected road network. Such Network was necessary in order to keep the periphery link to the centre in the Capital City of Colombo to facilitate the colonist over the island. Today, roads have become the main arteries of communication and the dominant mode of carrying passengers and goods to and from rural to urban areas of the country.

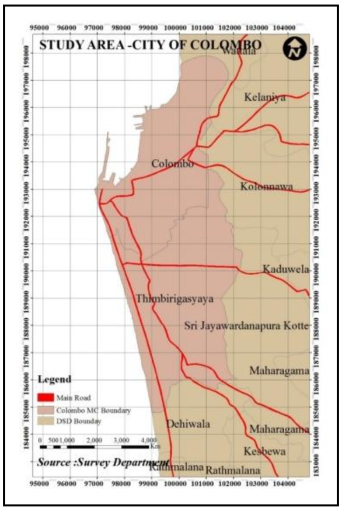

Figure 1: Study Area City of Colombo

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3.1. Evolution process of urban form of Colombo

The city of Colombo has very significant evolution process. The commercial and administrative hub of Sri Lanka was elevated to the status of a city by colonialists. The historical evolution of the spatial form of city of Colombo has been divided into the following Eras.

- Portuguese Period (1505 – 1656)
- Dutch Period (1656 – 1796)
- British Period (1796 – 1948)
- During 1999
- Present (2010)

The City of Colombo was originated from a small seaport. It has been ruled by Portuguese, Dutch and British in different eras. The urban form of City of Colombo was changed at different stages of its growth especially urban form elements of road pattern and built-up pattern. During 8th century, Colombo was used by Arab traders and then in 1505, Portuguese arrived and the settlement was originated from the sea port. The latter period of the Portuguese used Colombo as their capital to control the coastal area. The Portuguese occupation of Colombo ended in 1656 when the Dutch captured the city. The Dutch occupied Colombo and other parts of the coastal Ceylon from 1656 to 1796, a period of 140 years. The British captured Colombo in 1796 but it was not until 1815 that it became the Capital City of the whole island. The primary use of Portuguese and Dutch of Colombo was as a military fort. Unlike Portuguese and Dutch, British started to construct houses and other civilian structures around the fort and construct more roads. During this time they were in the control of the Colombo, they are the people who were responsible for the planning of the city. In some parts of the city tram car tracks and granite flooring laid during the era are still visible even today. British Era of colonization was ended in 1948.And then Urban Development Authority prepared the Colombo Development Plan in 1999.The changes of road network patterns and built patterns are shown in following maps.

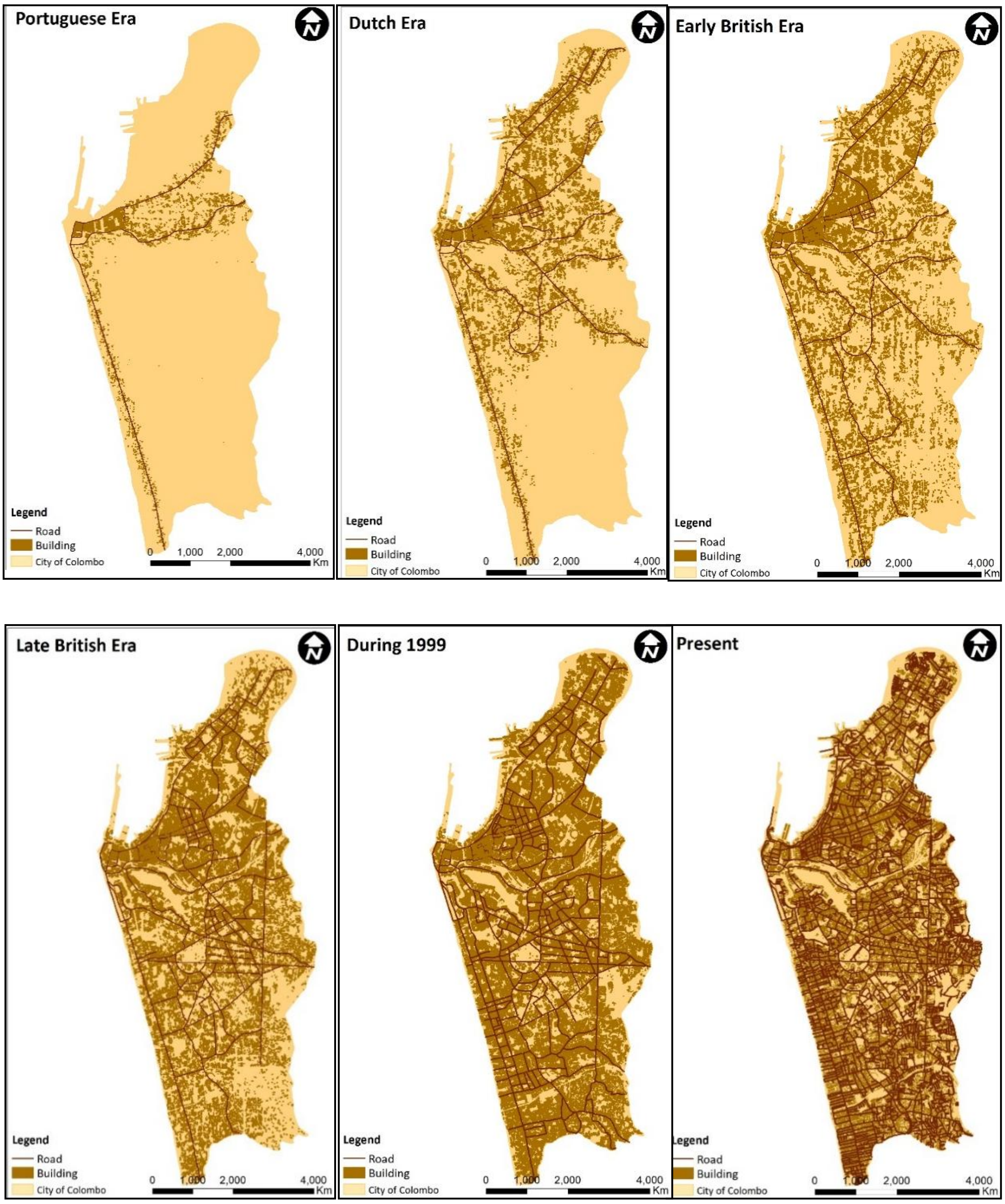

Figure 2 : The changes of road network pattern and built pattern in different eras **[7]**

4. METHOD

The methods of fractal analysis can be used to study the spatial organization of human activities across scales (Frankhauser, 1998). In fact, all fractal geographical entities exhibit the following dimensional relationship:

$$L^{1/1} \propto S^{1/2} \propto V^{1/3} \propto M^{1/3} \quad \textbf{\textit{(1)}}$$

Where L is the length of a geographical entity,
S is the area, V is the volume,
M is any mass measurement, and
D is the fractal geometry of M

4.1. Preparation of Road Pattern and Built-form Pattern maps

Mass Radius method is used to calculate the fractuality of Road Network and Built-up area in different era such as Portuguese, Dutch, Early British, Late British, during 1999 and present. This method was used to calculate the Fractal Geometry of road and built-up area by increasing the 100m radius of circle and calculating the length of road and built-up area in different radius. Both Road Network data and Building data were obtained from AutoCAD data provided by Survey Department (2002) which is the latest updated data. Fort area is considered as Central Business District and the circles are originated from fort area and both road network map and built-up area clipped into 100m interval different circle areas by increasing radius of buffer.

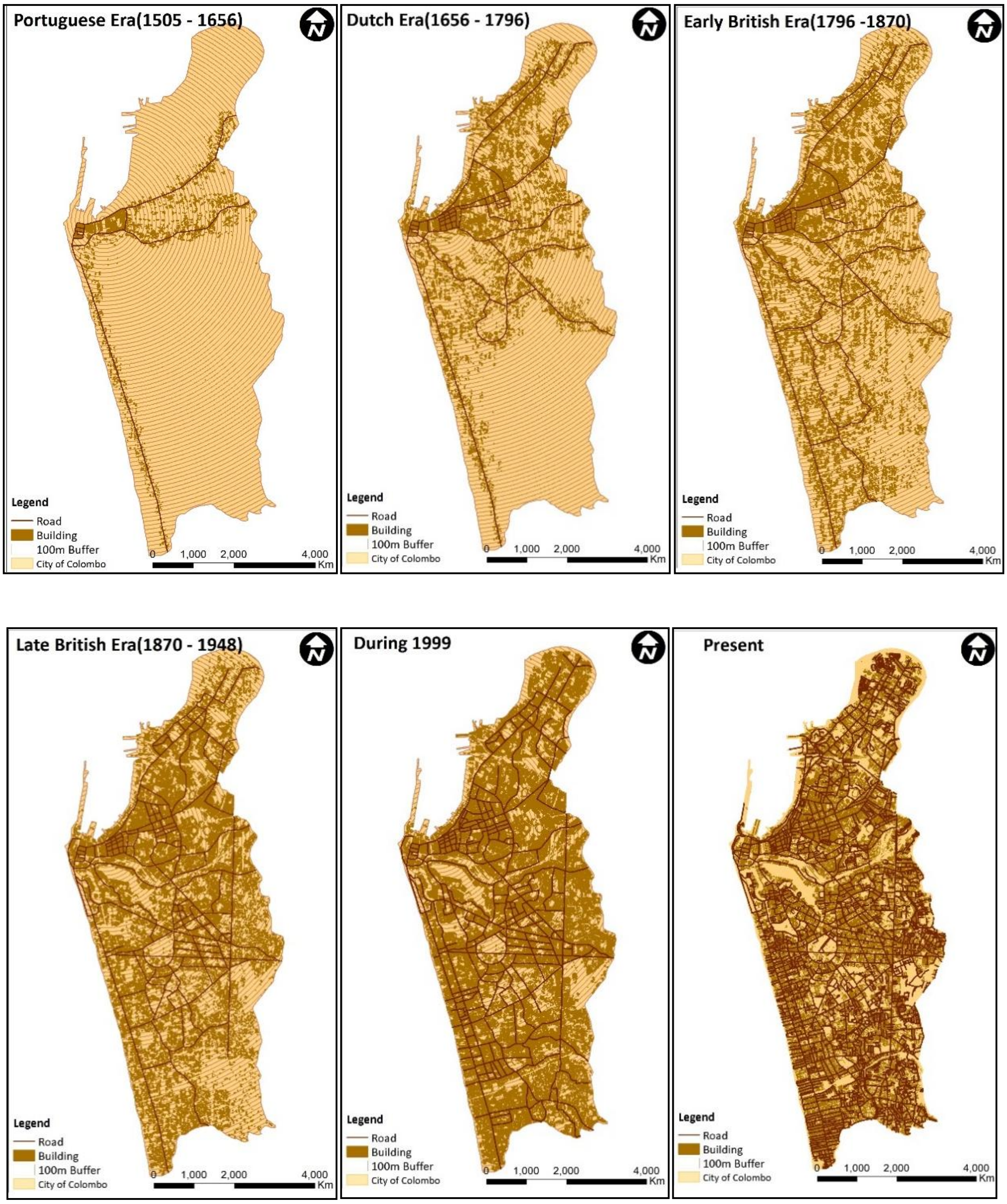

Figure 3 : Buffer Analysis is used to calculate fractal dimension of Road Length & Built-up area within different radius

4.2. Road Pattern Fractal Analysis

Mass –radius is used to calculate the fractal geometry of road by increasing the radius of circle and calculating the length of road in different radius. When the radius of the circle (R) increases, the length of road L(R) also increases correspondingly.

Fractal Geometry of Road Length, the equation can be written as;

$$D(L_i) = log[L(L_i)/L(R_{i-1})]/Log\ [R_i / L_{i-1}] \quad (2)$$

Where,
Ri is the radius at scale i,
L (Li) is the length of road at scale i,
Ri-1 is the cell length of scale i-1, and
L (Li-1) is the length of road at scale of i-1.

4.3. Built-form Pattern Fractal Analysis

Fractal Geometry of Built-up area, the equation can be written as;

$$D(L_i) = log[L(A_i)/L(R_{i-1})]/Log\ [R_i / A_{i-1}] \quad (3)$$

Where,
Ri is the radius at scale i,
L (Ai) is the area of built-up at scale i,
Ri-1 is the cell length of scale i-1, and
L (Ai-1) is the area of built-up at scale of i-1.

4.4. Correlation Analysis of Road Pattern and Built-form Pattern

Then correlation analysis was employed to find out the nature and the strength of relationship fractal geometry of Road length and Built-up area.

5. ANALYSIS AND RESULTS

The relational analysis between the fractal dimensions of road length and the built-up area by using Mass-Radius Method with 100m interval buffer radius in different eras is obtained by plotting the graph. The estimation of fractal dimension of road length for the entire study area during Portuguese period (1505 -1656) ranges from -1.03 to 2.67 (Figure 4) whereas the fractal dimension of built-up area varies from 0.0 to 1.62. The true fractal dimension can be thus arrived at 2.0 for road length, built up area which indicates that the built up area and roads are filled in a two dimensional space. Central Business District (CBD) area has higher fractal dimension of road. In general, the bigger the radius of circle is, the smaller the fractal dimensions of road and built-up area are. Center of the city is having higher fractal dimension of roads and it means that the first few buffers are mostly filled with the road network. When going away from Centre, the dimension has been reduced. And then slowly road fractal dimension is increasing. Then the periphery of the city road fractal dimension has been reduced that means the periphery of the city is not filled by road. Interesting observation is as the dimension of length of road increases dimension of built up area also increases and vice versa which shows that there is proportionality between them. The highest dimension of built-up area is 1.62 (equal to R=0.3km) and road length is 2.67 (equal to R=0.4 km) which means that building have filled nearly centre area greatly. The correlation between FD of Built-up area and Road Length is 0.53.It indicates the highest strong relationship between two variables.

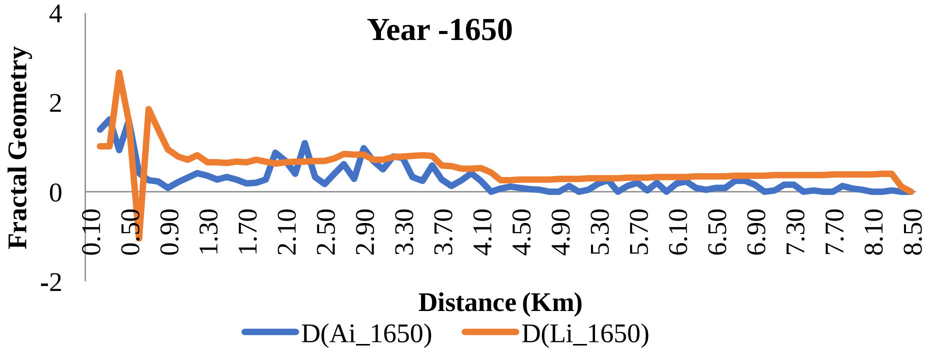

Figure 4 : Fractal Dimension of Built-up Area & Road Length and Radius distance in 1650

The estimation of fractal dimension of road length for the entire study area during Dutch (1656 -1796) ranges from -2.62 to 1.68 whereas the fractal dimension of built-up area varies from to 0.00 to 1.88) (Figure 5). The curve of dimensions of road and built-up area show some rebounds through the general declining process. This can indicate the values of the dimension around the edge of the city may show more instability. The correlation between FD of Built-up area and Road Length is 0.68 during 1750.It indicates the increasing the highest strong relationship between two variable when compared to 1650.

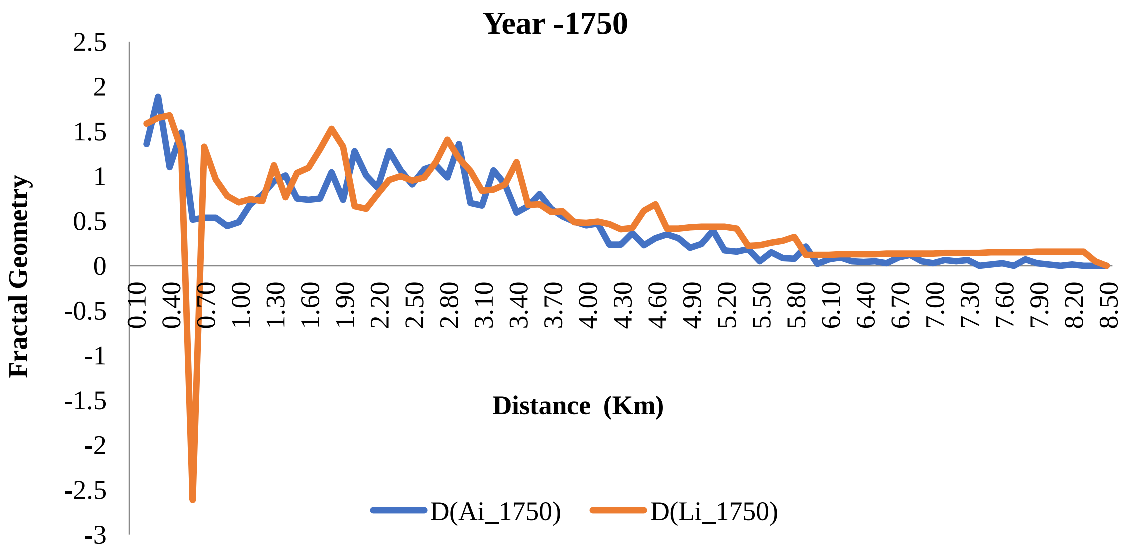

Figure 5 : Fractal Dimension of Built-up Area & Road Length and Radius Distance in 1750

The estimation of fractal dimension of road length for the entire study area during Early British period (1796 - 1870) ranges from -2.62 to 2.19 whereas the fractal dimension of built-up area varies from 0.0 to 1.99 (Figure 6). It is generally understood that higher accessibility greater potential for the development. During this period more roads constructed along with that settlement also emerged. The correlation between FD of Built-up area and Road Length is 0.63 during 1850.It indicates the highest strong relationship between two variable when compared to 1750.

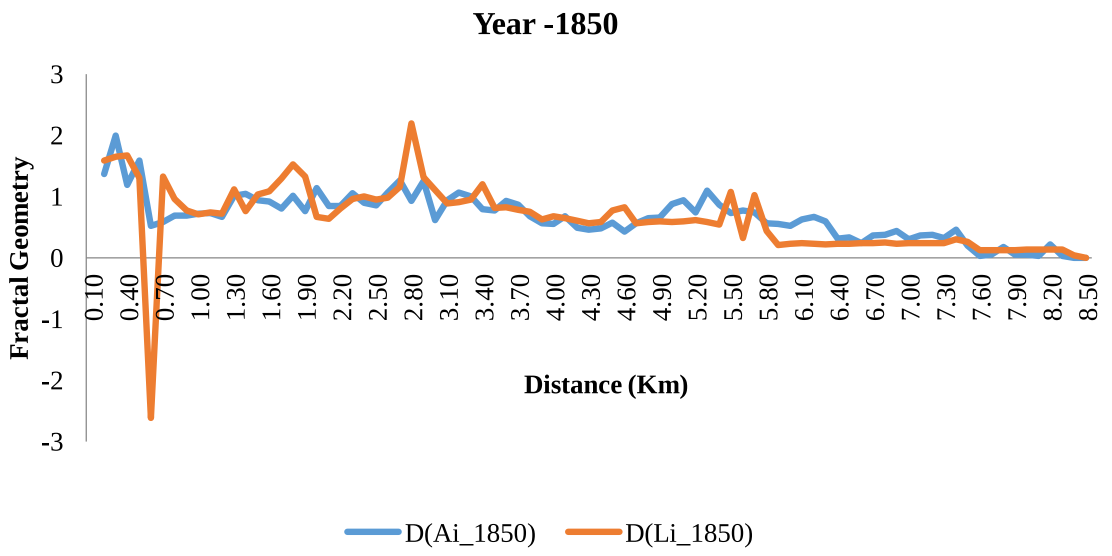

Figure 6 : Fractal Dimension of Built-up Area & Road Length and Radius distance in 1850

The estimation of fractal dimension of road length for the entire study area during Late British period (1870- 1948) ranges from -2.15 to 2.37 whereas the fractal dimension of built-up area varies from 0.0 to 1.99 (Figure 7). During this period the periphery are has been filled by building when compared to the Dutch period. The correlation between FD of Built-up area and Road Length is 0.76 during 1950.It indicates the increasing the highest strong relationship between two variable when compared to 1850.

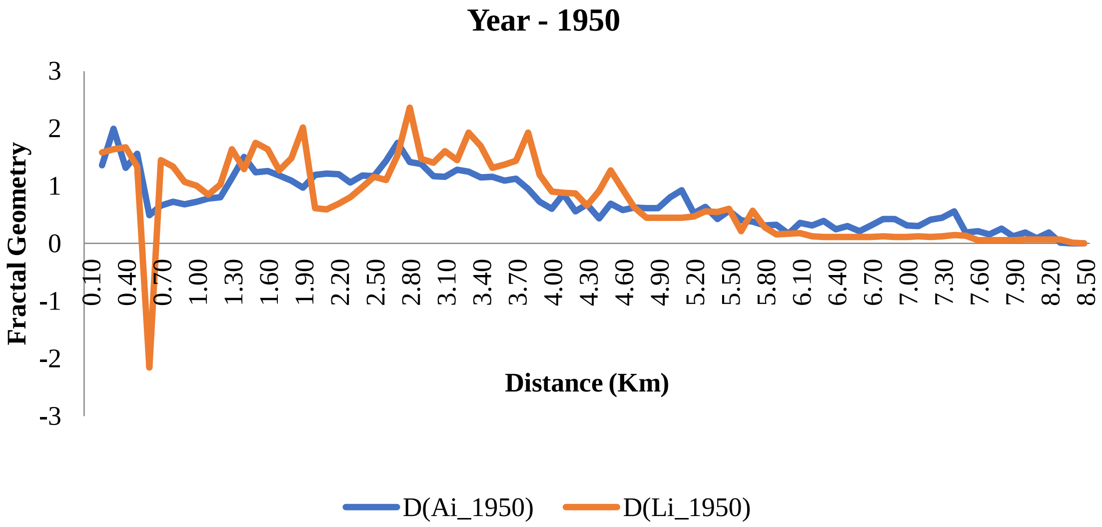

Figure 7 : Fractal Dimension of Built-up Area & Road Length and Radius distance in 1950

The estimation of fractal dimension of road length for the entire study area during 1999 (After Independence period) ranges from -2.15 to 2.11 whereas the fractal dimension of built-up area varies from 0.14 to 1.99 (Figure 8) .The curve of dimensions of road and built-up area show some rebounds through the general growing process. This can indicate the values of the dimension are increasing with the road construction and settlement emergence. The correlation between FD of Built-up area and Road Length is 0.78 during 1999.It indicates the increasing the highest strong relationship between two variable when compared to 1950.

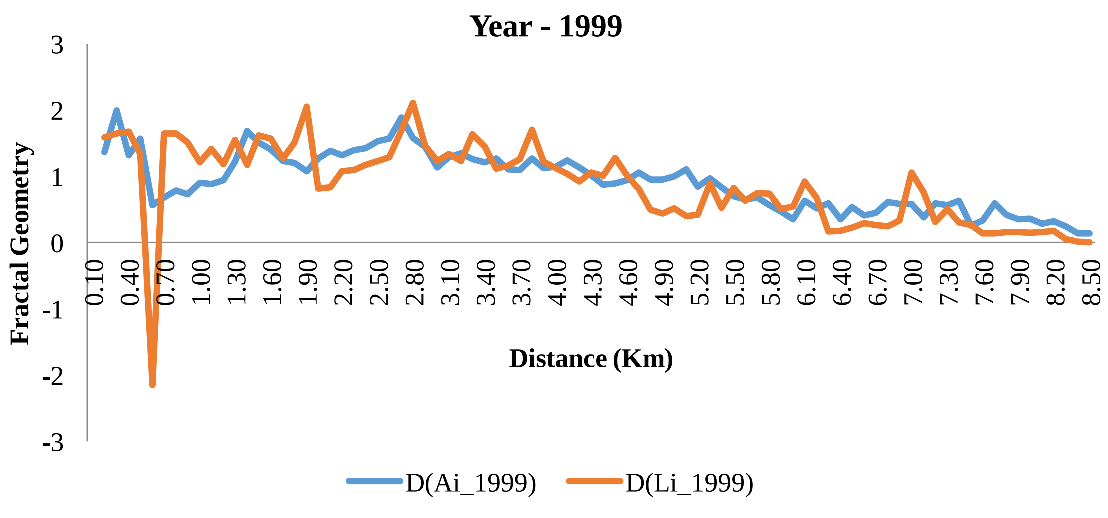

Figure 8 : Fractal Dimension of Built-up Area & Road Length and Radius distance in 1999

The estimation of fractal dimension of road length for the entire study area during 2010 (present) ranges from -2.27 to 2.18 whereas the fractal dimension of built-up area varies from 0.46 to 1.99 (Figure 9). The curve of dimensions of road and built-up area show some rebounds through the general increasing process with the similar pattern.
The correlation between FD of Built-up area and Road Length is 0.79 during 2010.It indicates the increasing the highest strong relationship between two variable when compared to 1999.

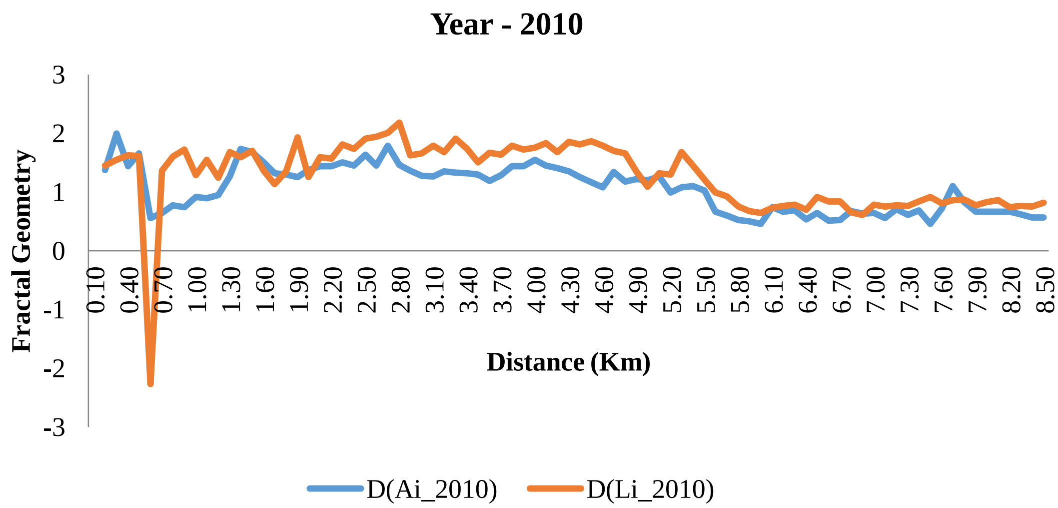

Figure 9 : Fractal Dimension of Built-up Area & Road Length and Radius distance in 2010

5.1. Correlation Analysis

The correlation between Road Length and Built-Up Area with respect to radius with the time changes is shown in Table 4.3.The correlation co-efficient value falls the range between road length and built-up area 0.349-0.796.Since the correlation value is positive throughout the evolution process and the value is increasing. There is a high positive relationship between road length and built-up area at 2010.This trend shows that there is a possibility to have higher relationship between built-up area and road in future.

Table 1 : Correlation between Fractal dimension of Built-up area and Road Length in different Eras

	1650	1750	1850	1950	1999	2010
Correlation between Fractal dimension of Built-up area and Road Length	0.538	0.686	0.635	0.767	0.786	0.796
	Correlation is significant at the 0.01 level (2-tailed)					

The following graph shows the fractal dimension of built-up area in different eras .When the radius is increased correspondingly, Fractuality of Built-up area increases. Since the fractal dimension of built-up area change is seen regular similar pattern in different radius, it indicates that there is similarity in the growth of building pattern in the city of Colombo area.

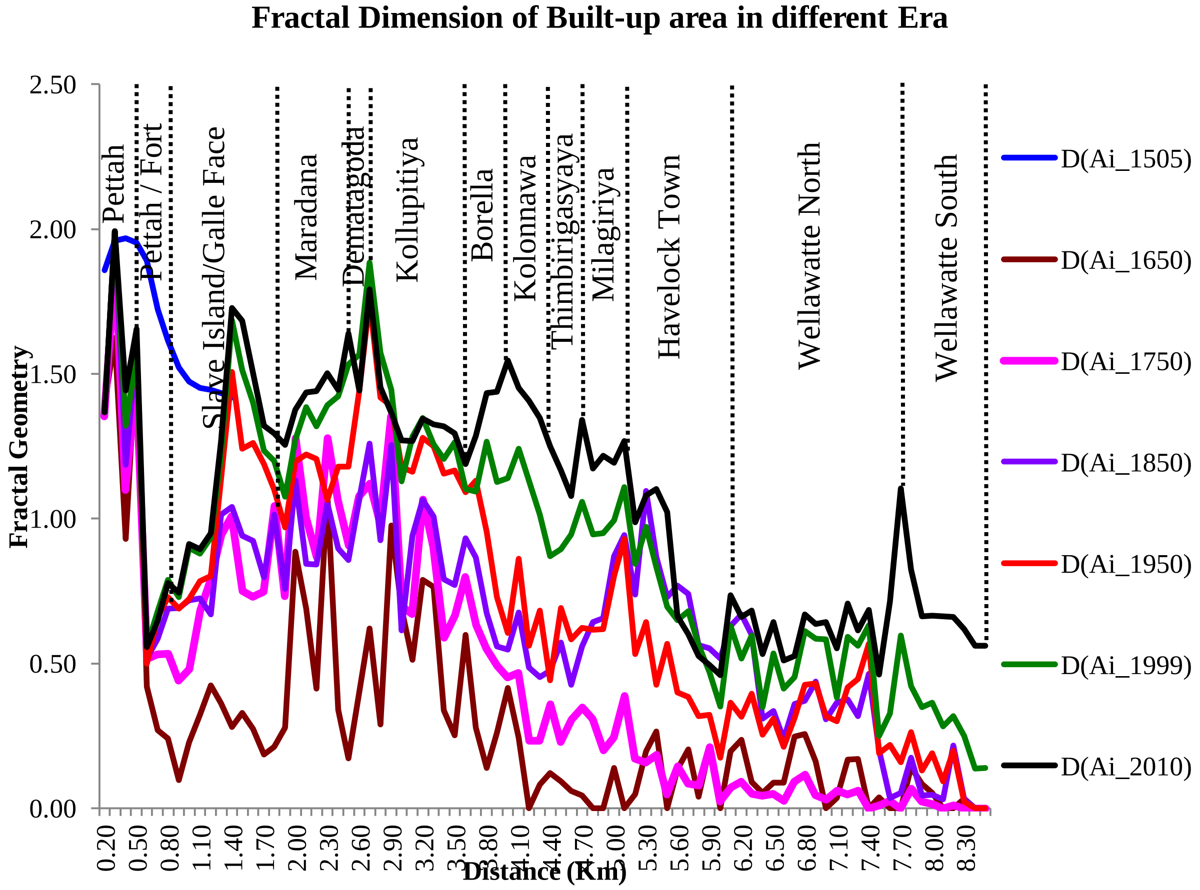

Figure 10 : Fractal Dimension of Built-up area in different Era

Since there is strong correlation and linear relationship between Fractal Dimension of road and Built-Up Area, the Urban Form Pattern can be simulated.

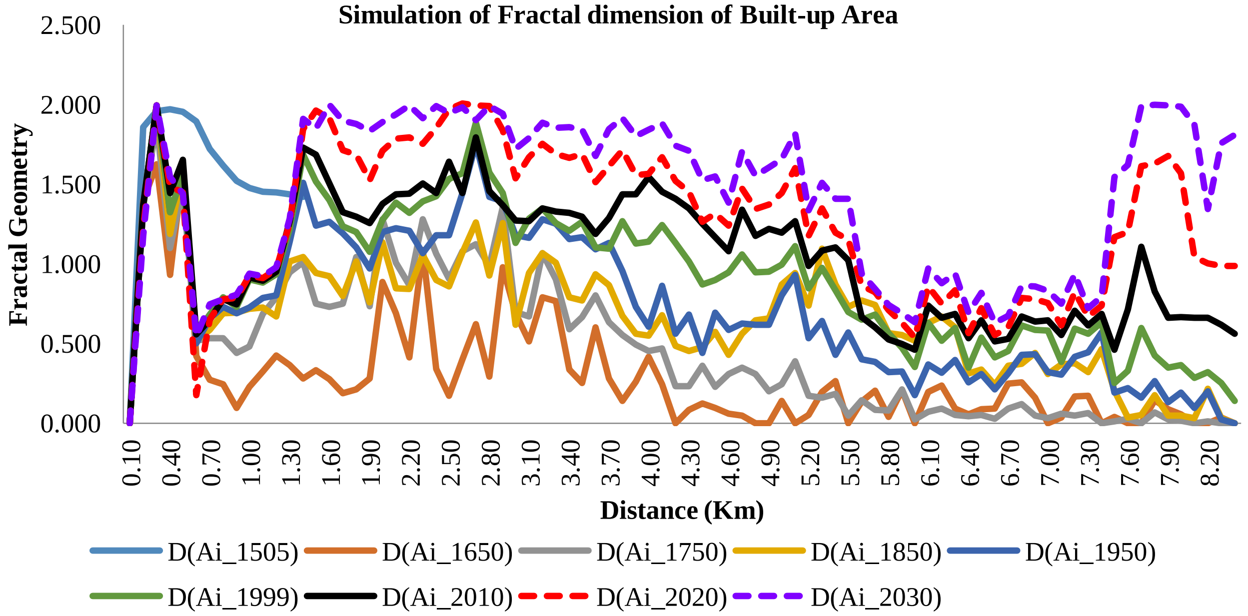

Figure 11 : Simulation of Fractal dimension of Built-up Area

6. CONCLUSION

Understanding urban form changes patterns is important for planners to analyze the current phenomena and predict the future development. The main finding of investigation of relationship between Fractal Geometry of urban form elements and the results showed that the correlation values is high between road length and built-up area with respect to radius with the time changes and the values falls the range between road length and built-up area 0.349-0.796 positive (** Correlation is significant at the 0.01 level (2-tailed). This trend shows that there is a possibility to have higher relationship between built-up area and road in future. Since there is a strong relationship between fractal dimensions of urban form elements which show the patch to simulate the urban form changes.

This study suggests that this tool might be a tool for prediction of urban form changes and relational patterns have been used to make policy planning with respect to road development and settlement development. Accordingly, study showed the applicability of Fractal Geometry as an urban form simulation tool through one case study application; further the research can be developed into used to study other city of the country.

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