# Safety of Early Age Loaded Reinforced Concrete Members

Jibrin Mohammed Kaura, Adamu Lawan Ibrahim Aliyu, Mohammed Sulaiman Mahmood

Department of Civil Engineering, Ahmadu Bello University, Zaria

---

**ABSTRACT -** ***This paper examines the effect of construction loads on the safety of reinforced concrete members. A simply supported reinforced concrete slab was considered for the investigation. The probabilistic analysis of time dependent characteristic concrete strength takes into account the slab capacity from casting to maturation. The variation of load was accommodated though a load ratio (dead to live). Adequate review was made on structural reliability analysis using first order reliability method. The focus of the paper was on evaluating the effect of early age loads on the reliability of reinforced concrete members, in which the uncertainties associated with the basic design variables were fully accommodated. First Order Reliability Method was employed though a developed computer program in FORTRAN 77 to accomplish the reliability analysis. It was clearly established that concrete structural members under construction loads has limiting capacity prior to twenty eighth days after casting, and may lead to dramatic lost of stability of the whole system, if shoring and formwork are removed***

**Keywords --** Construction load, reliability, slab

---

## 1. INTRODUCTION

In engineering practice, uncertainty in the quantities of interest is the rule rather than the exception, [4], [3], [11]. Repeated samples of concrete strength specimens from same source yield a set of numbers of which no two are the same. Current British and European structural design codes are based on limit state design approach. An engineering structure represented as being deterministic; to each design variable, it is presumed, that a unique value can be assigned. In turn engineering structures are conventionally modelled to behave in a unique mechanistic way for a given set of the model's parameter values (dimensions etc) and for a given set of input quantities (loads etc). The conclusion is that their responses are predictable with certainty. Yet all of these assumptions of determinism are inconsistent with the uncertainty observed in reality. The need for more appropriate representation and analysis of reality, including the ubiquitous uncertainty, is widely appreciated, [1], [4], [14], [2], [7], [5], [6]. The nature of uncertainties and the manner of dealing with them has been a topic of discussion by statisticians, scientists, engineers and other specialists for a long time. Uncertainties were characterized into two types: aleatory and epistemic. The inherent variability in engineering structures imposes the use of probabilistic model: as such phenomena cannot be dealt with deterministic approaches. This variability is known as "aleatory uncertainty", this uncertainty cannot be reduced. However, both deterministic and probabilistic approaches are built on a number of model assumptions and model parameters that are bases on what is currently known about the physics of the relevant process and the behaviour of systems under given conditions [16]. There is uncertainty associated with this condition, which depends upon state of knowledge, which is referred to as "epistemic uncertainty".

The current use of rapid construction technique, especially in Abuja and other major cities in Nigeria, places pressures on contractors to meet predetermined constructions targets by reducing the time between placements of successive floor, and premature removal of shoring for recycling purposed. This is sometimes in addition to poor workmanship and deliberate omissions from non competent professionals. The major risk is structural collapse during construction. Notable examples are collapse of four store building during construction located on plot 1007 at No. 2 Ikole Street, Garki II Abuja killing 21 persons in August, 2010 [12]. Two hundred and twelve construction workers escaped death, because the incidence occurs in the evening. There is therefore the need to asses the probability of failure of reinforced concrete structures that are subjected to early-age loading prior to the development of their design strength. One of the most influencing time dependent parameter on strength of reinforced concrete structures is the compressive strength, and the characteristic strength of concrete is based on the twenty eight days cube strength. A mathematical model that defined the variation of the concrete strength with age was given in [17] as:

$$f(t) = \frac{t}{(\omega + t\psi)} f(28) \qquad 1$$

where, f(t) is the concrete strength at time t in days, t is the age in days, f(28) is the 28 day strength, $\omega$ and $\psi$ are curing constants [18].

Early-age loading may also have implication on the serviceability of reinforced concrete. Its connection with the long-term deflection of reinforced concrete was established elsewhere [18].

Imposed load on building is a time variant loading [14]. The current provision in the British Standard Code of practice [9] considered the load as time invariant. The coefficient of variation in the order of 150% [13]. Eurocode 2 [9] recommended a constant value of a partial safety factor equal to 1.5 to account for any uncertainty in the applied loading. Considering the high coefficient of variation of imposed load, coupled with application of such loads to immature concrete member lead to a question on the safety of such members that need urgent answer. To accommodate uncertainties, safety assessment must employ reliability methods of analysis. In this paper, reliability assessment of early-age loaded one-way simply supported slab designed to Eurocode 2 [9] was assessed using First Order Reliability Method (FORM). The slab is assumed to have failed when its bending capacity is exceeded.

## 2. LIMIT-STATE DESIGN PHILOSOPHY

A limit state is a situation where a structure ceases to fulfill one of the specific functions or conditions for which it was originally designed. In practice, the study of structural safety is concerned with the violation of the ultimate limit-state [14].

In the limit state design, the structural inadequacy or failure is expressed through the following equation:

$$\Phi R = \gamma_D S_D + \gamma_L S_L \qquad 1$$

where: R is the member resistance, $\Phi$ is the partial factor of R. $S_D$, $S_L$ are the dead and live load effects respectively. $\gamma_D$ and $\gamma_L$ are the partial factors of $S_D$ and $S_L$ respectively.

Equation 1 was originally developed during the 1960's for reinforced concrete codes. It enables the live load to have greater "partial" factor than the dead load, in view of the formers greater uncertainty and it allowed a measure of workmanship variability and uncertainties bout resistance modeling to be associated with the resistance [15]. Exceedence of limit state condition (failure) is only accepted, if the probability of failure $P_f$ is small. An important limitation of this deterministic approach is that, this "small" is not specified [2].

The inconsistency of the level of safety and unexpected failure of some structures due to uncertainties, coupled with the desire to estimate the level of safety achieved in design justify a code review not only to check and maintain safety level but also to properly accommodate randomness and uncertainties.

### 2.1 Probabilistic Design Method

In a probabilistic design approach, every mechanism is described by its mathematical model. On he basis of such a model, the so called reliability function

$$Z = strength\ (R) – Load\ (S) \qquad 2$$

is defined with regard to the limit state considered. For the purpose of reliability assessment of structures, ultimate limit state is usually considered. The failure function Z = 0, is defined as the boundary between the area associated with failure (negative value of Z) and non-failure(positive value of Z). The probability of failure can be expressed by P{Z<0). The probability of failure is expressed as follows

$$P\{<0\} = \iint_{r<s} f_R(r) f_S(s)\, dr\, ds \qquad 3$$

where: $f_R(r)$ = probability density function of R, $f_S(s)$ = probability density function of S, $f_R(r)f_S(s)\ dr\ ds$ the probability that R is situated between r and r+dr while S is situated between s and s+ds simultaneously. Neither R and S do have to be normally distributed.

Closed-form solution of equation 3 is only possible when R and S are normally distributed [5]. Reliability estimation is performed by representing each random variable by its first two moments, i.e. its mean and standard deviation.

### 2.2 First Order Reliability Method (FORM)

When R and S are independent and both normally distributed (or have been transformed to normally distributed variables), then Z would be normally distributed. This implies that the first two moments of Z (mean and standard deviation) can be calculated from:

$\mu(Z) = \mu(R) – \sigma(S)$ 4

$\sigma^2(Z) = \sigma^2(R) + \sigma^2(S)$ 5

The probability of failure follows from

$$P\{<0\} = \iint_{r<s} f_R(r) f_S(s)\, dr\, ds$$
$$= \Phi(-\beta) \quad 6$$

Where, *Φ(-β) is the* standard normal distribution for the variable β, β is the Hasofer-Lind reliability index is a measure in standard deviation of the distance that the mean μ(Z) is away from the safety-failure interface [14]. Given by:

$$\beta = \frac{\mu(Z)}{\sigma(Z)} \quad 7$$

In general however, Z is not linear [14]. Thus with all the random variables $X_i$ implicitly normally distributed, Z will not be normally distributed. The only way to obtain the first two moments of Z is to linearise it. This can be done by obtaining approximate moments through the expression of Z as a "first-order' Taylor Series expansion about some point $X^*$ (Checking point). Then:

$$Z_{lin} = Z(X_1^*, X_2^*, \ldots\ldots X_n^*) + \sum_{i=1}^{n} \left(\frac{\partial Z}{\partial X_i}\right)(X_i - X_i^*) = 0 \quad 8$$

Where, $Z_{lin}$ is the linearised reliability function, Z is linearised in ( $X_1^*, X_2^*, \ldots\ldots X_n^*$)., z is the number of stochastic variables in the reliability function, $\left(\frac{\partial Z}{\partial X_i}\right)$ is the partial derivative of Z, with respect to $X_i$, evaluated in $X_1 = X_1^*$. The mean value and the standard deviation of $Z_{lin}$ are:

$$\mu_{Zlin} = Z(X_1^*, X_2^*, \ldots\ldots X_n^*) + \sum_{i=1}^{n} \left(\frac{\partial Z}{\partial X_i}\right)(\mu_{Xi} - X_i^*) \quad 9$$

and

$$\sigma_{Zlin} = \sqrt{\left[\sum_{i=1}^{n} \left(\frac{\partial Z}{\partial X_i}\right)^2 \cdot \sigma_{xi}^2\right]} \quad 10$$

The safety index can be approximated by:

$$\beta = \frac{\mu(Z_{lin})}{\sigma(Z_{lin})} \quad 11$$

For non-normal basic variables, a transformation from the physical (x) space is performed. If the basic variables are assumed to be stochastically independent, the transformation is defined by

$Ui = \Phi^{-1}(F_i(X_i))$ 12

with standard normal variables $U_i$ and its inverse by:

$X_i = F^{-1}(F_i(U_i))$ 13

Application of the inverse transformation allows the failure function in the physical (x) space to be evaluated. In this research, computation of safety indices was achieved through a user defined FORTRAN-based computer program, that call FORM5 [10] as a subroutine.

## 3. SETUP OF NUMERICAL EXPERIMENT

The performance function of the structural member under investigation is given by:

$G(X) = (M_R - M_S)$ 14

The safety index is given by:

$$\beta = \frac{G(x)}{\sqrt{(A_1 + A_2 + \dots\dots\dots\dots + A_n)}} \quad 15$$

$$\text{where: } A_i = \left(\frac{\partial G(X_i)}{\partial X_i}\right)^2 (\sigma_{Xi})^2 \quad 16$$

The following algorithm is used in the development of the FORM5, [10] in conjunction with a transformation method that transformed non normal variables to standard normal variables.

**Step one:** Assign starting value of the random variables $X_i = X_i^*$ for all i. (Mainly $X_i^* = \mu_i$ (the mean of $X_i$) are taken as the starting values. The statistics of the random variables are presented in Table 1.

**Table 1:** Statistics of the Random Variables

| Random Variable | Distribution Model Type | Coefficient of Variation | Mean | Standard Deviation |
|---|---|---|---|---|
| Characteristic strength of concrete, $f_{cu}(t)$ | Lognormal | 0.17 | $30.0\frac{t}{(\omega+t\psi)}$ | ---- |
| Reinforcement ratio, $\rho$ | Normal | 0.05 | Default | ----- |
| Area of reinforcing steel, $A_s$ | Normal | 0.01 | Default | ----- |
| Depth of slab, H | Normal | 0.025 | Default | ----- |
| Concrete cover to reinforcing steel, C | Gamma | 0.17 | 20.0 mm | 3.4 mm |
| Imposed load, $Q_K$ | Exponential | 1.5 | 1.5 kN/m$^2$ | 2.25 mm |
| Uncertainty in resistance model, $\theta_R$ | Lognormal | 0.07 | 1.1 | 0.077 |
| Uncertainty in load model, $\theta_S$ | Lognormal | 0.2 | 1.0 | 0.2 |
| Diameter of reinforcing steel, $\Theta$ | Normal | 0.05 | 12.0 mm | 0.6 |

*** ω** = 4.0, ψ = 0.85, t is the age of concrete in days
(Sources: [13], [17], [18])

**Step two:** Calculate $\mu(Z_{lin})$ and $\sigma(Z_{lin})$

$$\mu(Z_{lin}) = G(X) = [0.167\theta_R f_{cu}\frac{A_s}{\rho}(h\text{-}c\text{-}\frac{\Theta}{2})] - [\frac{(1.35\alpha+1.5)\,\theta_E Q_K L^2}{8}] \quad 17$$

$$\sigma(Z_{lin}) = \sqrt{(A_1 + A_2 + \dots\dots\dots\dots + A_n)} \quad 18$$

$$\text{where: } A_! = [0.167\theta_R\frac{A_S}{\rho}(h\text{-}c\text{-}\frac{\Theta}{2})]^2[\sigma_{fcu(t)}]^2 \quad 19$$

$$A_2 = [-0.167\theta_R\frac{A_S}{\rho^2}(h\text{-}c\text{-}\frac{\Theta}{2})]^2[\sigma_\rho]^2 \quad 20$$

$$A_3 = [0.167\theta_R\frac{f_{cu}(t)}{\rho}(h\text{-}c\text{-}\frac{\Theta}{2})]^2[\sigma_{As}]^2 \quad 21$$

$$A_4 = [0.167\theta_R f_{cu}(t)\frac{A_S}{\rho}(h\text{-}c\text{-}\frac{\Theta}{2})]^2[\sigma_h]^2 \quad 22$$

$$A_5 = [0.167\theta_R f_{cu}(t)\frac{A_S}{\rho}(h\text{-}c\text{-}\frac{\Theta}{2})]^2[\sigma_c]^2 \quad 23$$

$$A_6 = [\theta_E\frac{(1.35\alpha+1.5)L^2}{8}]^2[\sigma_{Qk}]^2 \quad 24$$

$$A_7 = [0.167\theta_R f_{cu}(t)\frac{A_S}{\rho}(h\text{-}c\text{-}\frac{\Theta}{2})][\sigma_{\theta R}]^2 \quad 25$$

$$A_8 = [Q_K \frac{(1.35\alpha+1.5)L^2}{8}]^2 [\sigma_{\theta S}]^2 \quad 26$$

$$A_9 = [-0.167\theta_R \frac{A_S}{2\rho} (h\text{-}c\text{-}\frac{\Theta}{2})] [\sigma_{\Theta}]^2 \quad 27$$

**Step three:** Determine β (equation 14.0)

**Step four:** Calculate $X_i^*$ given by

$$X_i^* = \mu_{Xi} - \alpha_i\beta\sigma_{Xi}, \quad 28$$

where, $$\alpha_{Xi} = \frac{A_i}{\sqrt{(\sum A_i)}} \quad 29$$

**Step five:** Repeat step two to four until the process has converged to sufficiently accurate values.

## 4. RESULT AND DISCUSSION

Fig. 1 to 3 shows the relationship between safety index and dead to live load ratio at various concrete ages slab thicknesses of 100 mm, 150 mm an. A target safety index above 3.8 was assumed base on the recommendation of Eurocode 0 [8] and JCSS Probabilistic Model Code [13]. As depicted in Fig. 1.0, an early aged loaded reinforced concrete slab with thickness of 100 mm or less is unsafe even at the least dead to live load ratio of 1.5, with maximum achievable safety index of 1.7 at 28 days. This implied that, the removal of shoring and form work within the first 28 days may expose the slab to the risk of failure. When the slab thickness is increased to 150 mm (Fig. 2.0), the target safety index was achieved at 21 and 28 days if and only if the ratio of dead to live load is not more than 1.5 and 1.75 respectively. As the slab thickness is further increased to 175 mm (Fig.3.0), the target safety index was achieved after 7, 14, 21 and 28 days of casting if the ratio of dead to live load is not more than 1.8, 2.6, 2.9 and 3.2 respectively.

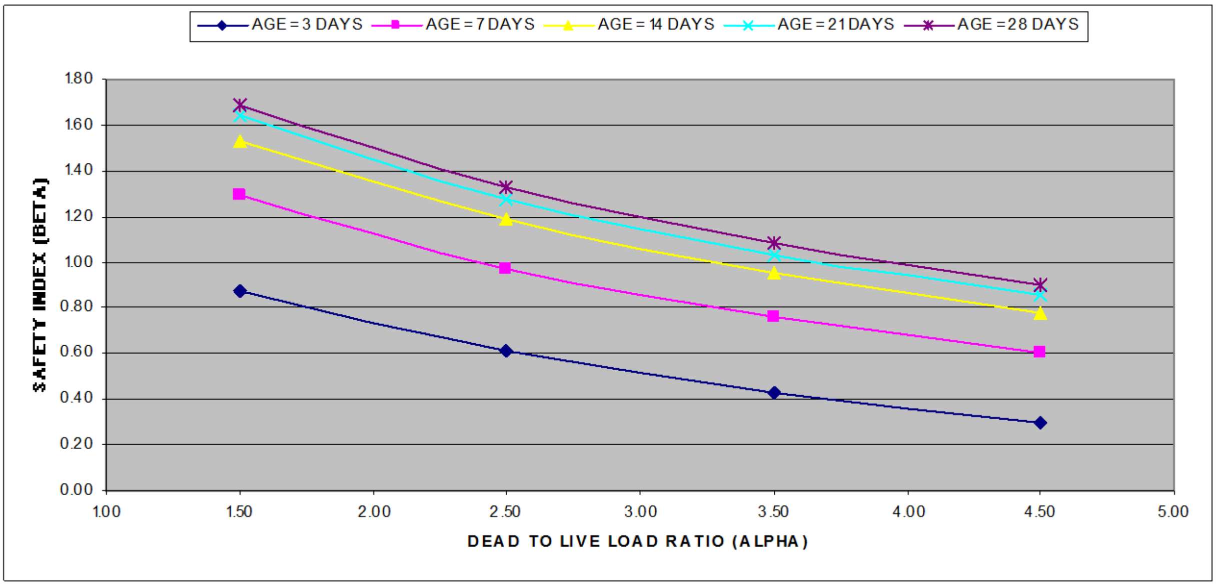
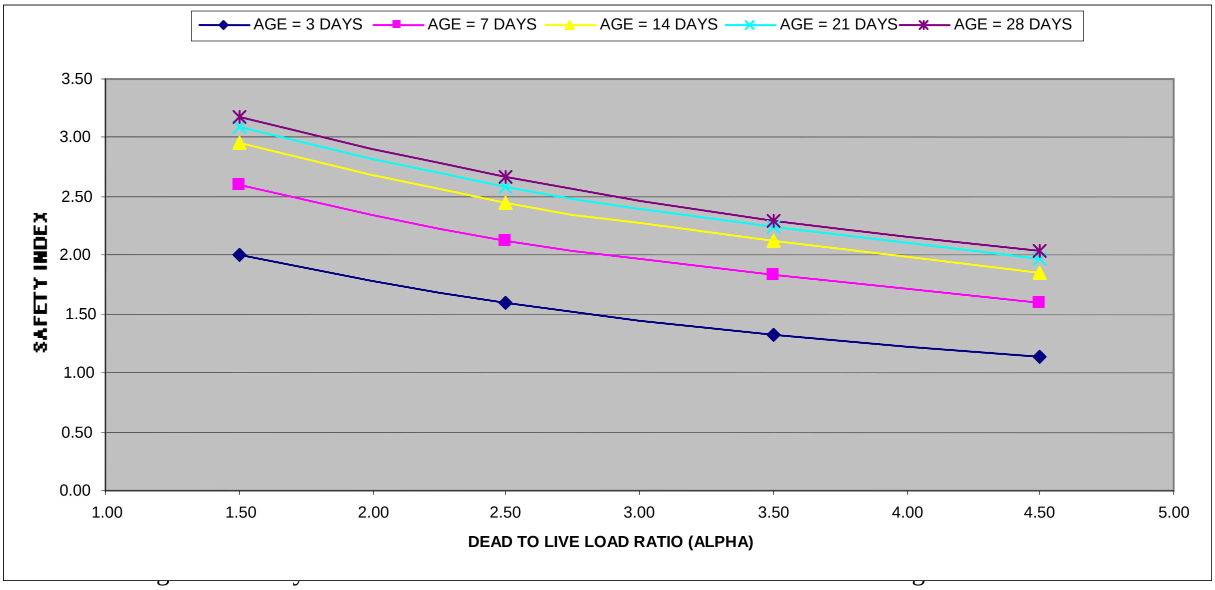



Figure 1: Safety index versus dead to live load ratio at various concrete ages and slab thickness, h = 100 mm)




and slab thickness, h = 150 mm)

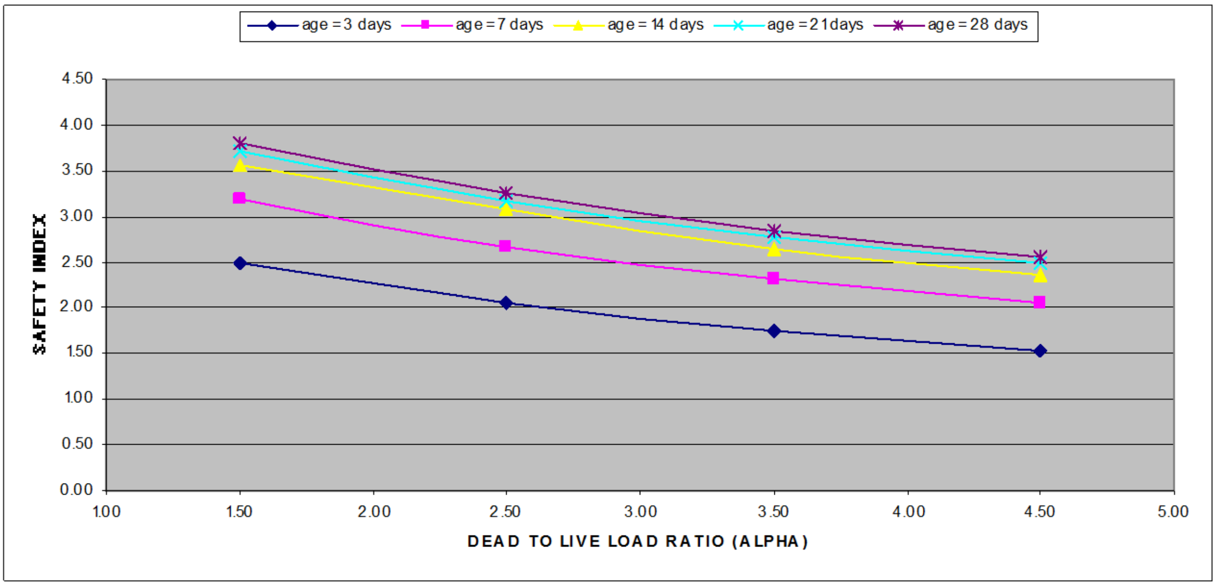



Figure 3: Safety index versus dead to live load ratio at various concrete ages
and slab thickness, h = 175 mm)

Fig. 4 to 6 displayed the relationship between safety index and design reinforcement ratio (reinforcement ratio obtained from the code-based design calculations under the applied loading) at various live to dead load ratio for three slabs of thickness 100 mm, 100 mm and 175 mm. The code prescribed minimum reinforcement ratio for this slab is 0.176%, 0.157% and 0.153% for 100mm, 150mm and 175mm thick slab respectively (Eurocode 2, 2008). For 100mm thick slab (Fig. 4), the minimum reinforcement ratio correspond to a safety indices of 1.25, 0.75, 0.25, and 0.15 for dead to live load ratio of 1.5, 2.5, 3.5 and 4.5 respectively. This implied that, a 100mm thick slab is unsafe at 28 days. When the slab thickness is increased to 150mm (Fig, 5), the code prescribed minimum reinforcement ratio correspond to a safety index of 4.0, 3.0, 2.2, and 1.5 for dead to live load ratio of 1.5, 2.5, 3.5 and 4.5 respectively. It is clear from this result that the code prescribed minimum reinforcement ratio is acceptable only when the dead to live load ratio is less than or equal to 2.5. At a slab thickness of 175mm, (Fig, 6) the code prescribed minimum reinforcement ratio correspond to 5.5, 4.0, 3.0 and 2.5 for dead to live load ratio of 1.5, 2.5, 3.5 and 4.5 respectively. This implied that the minimum reinforcement ratio is only acceptable at 28 days if the dead to live load ratio is less than or equal to 3.5.

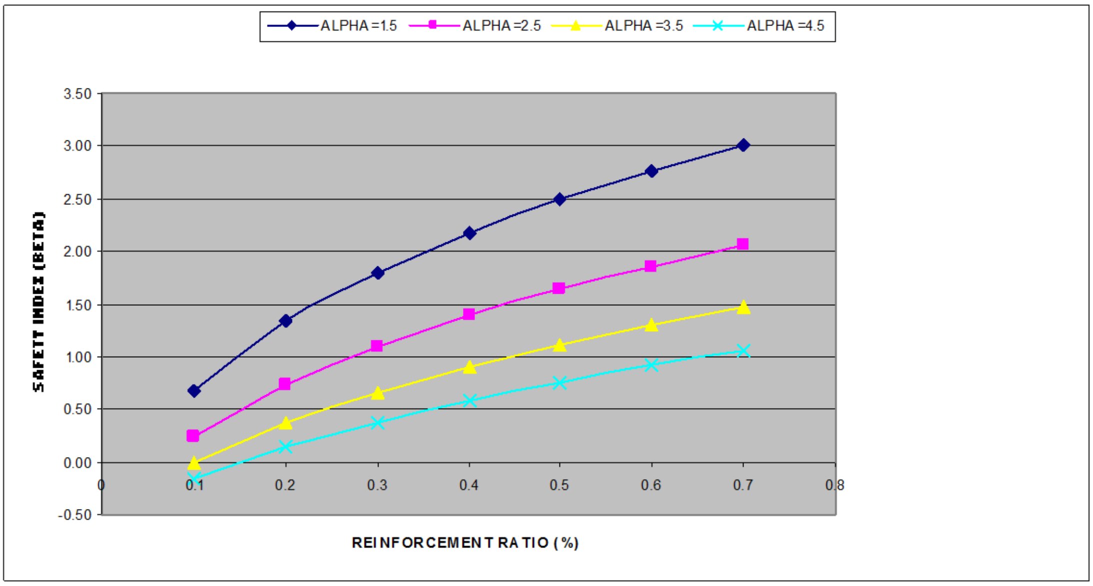



Figure 4: Safety index versus reinforcement ratio at various dead to live load ratio and slab thickness, h = 100 mm

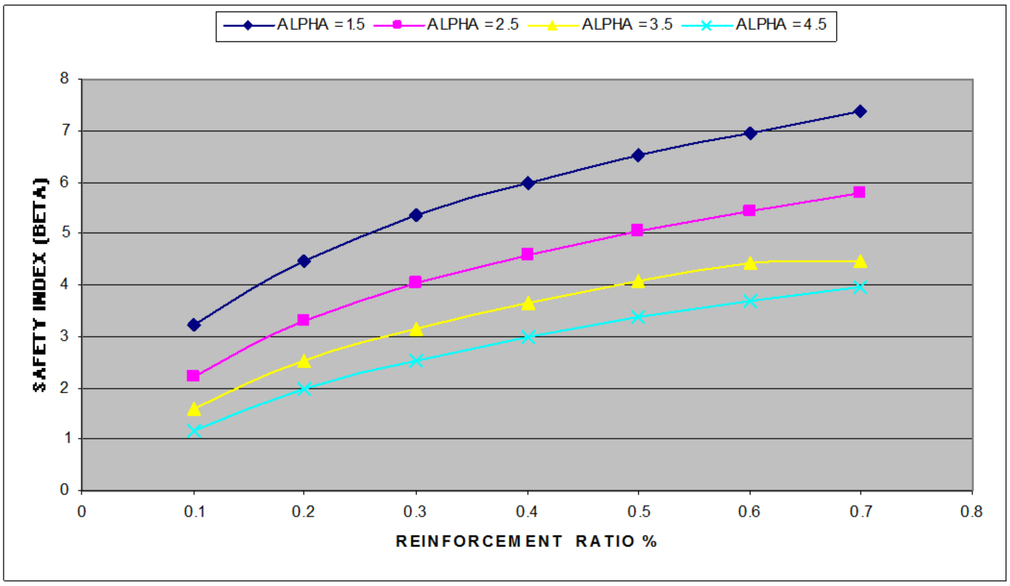



Figure 5: Safety index versus reinforcement ratio at various dead to live load ratio and slab thickness, h = 150 mm)

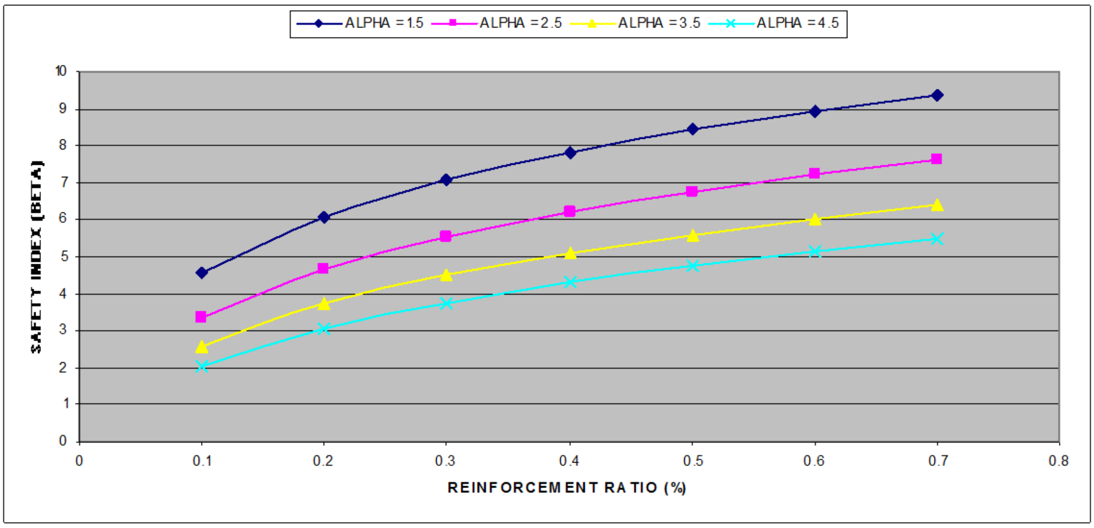



Figure 6: Safety index versus reinforcement ratio at various concrete ages
and slab thickness, h = 175 mm)

In Fig. 7, the relationship between the safety indices and the variability of concrete characteristic strength. Although, the age of concrete has a significant effect on the probability of failure of reinforced concrete structures, it is clear in Fig. 7, that the variability within concrete specimen has little effect on the probability of failure.

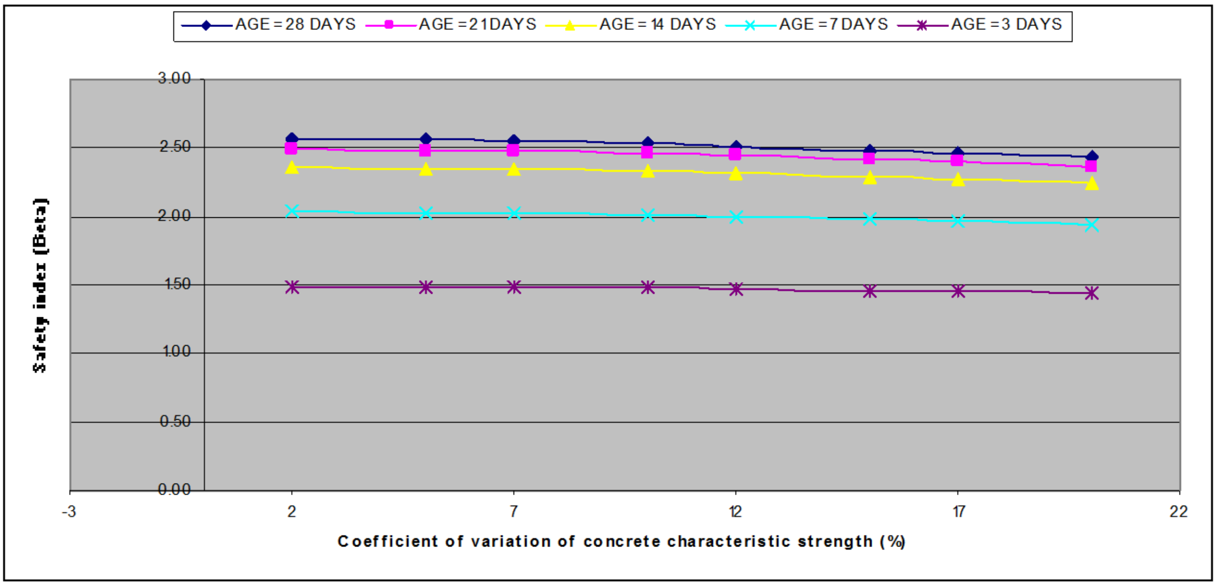



Figure 7: Safety index versus coefficient of variation of concrete characteristic strength.

Fig. 8 show the relationship between safety index and imposed floor loading at various concrete ages. The increase in imposed load during construction is as a result of construction activities. Especially during the construction of subsequent levels of a multi-storey building, which may occur at the early age of the lower levels. It is clear that variability in imposed floor load has a very serious effect on the safety of concrete structures in general. At 28 days change in coefficient of variation from 5% 0t 40% result to drop in the level of safety from β = 7.0 to β = 4.5. This occurs at the design load ratio. At higher load ratio, the result is expected to be catastrophic.

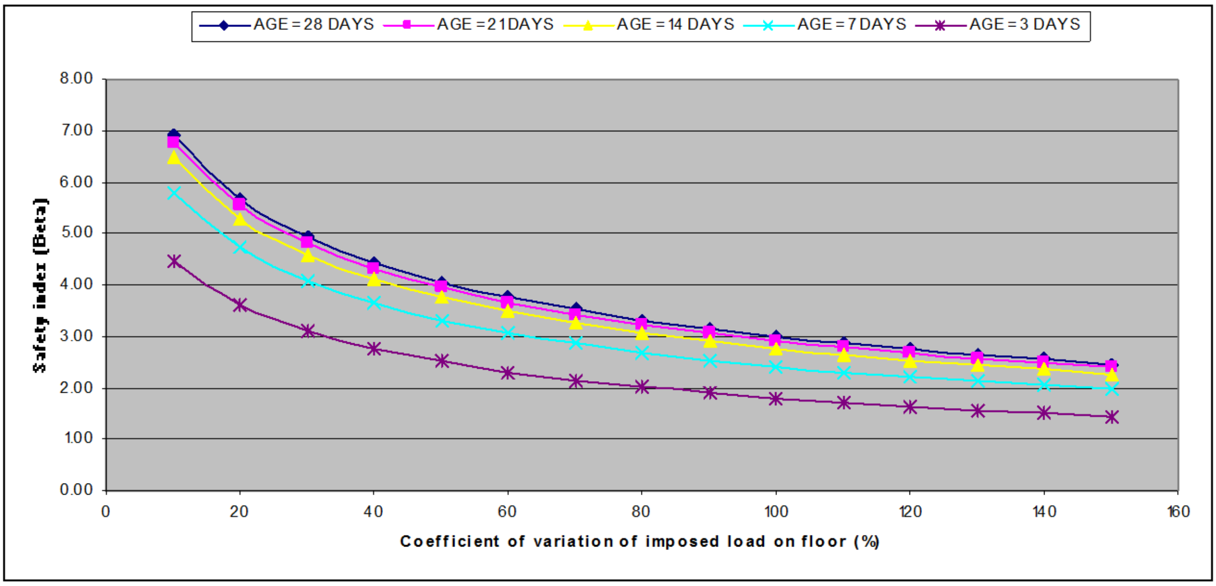



Figure 8: Safety index versus coefficient of variation of imposed load on floor.

## 5. CONCLUSION

Reliability assessment of reinforced concrete members subjected to construction load (early age load) designed based on the requirement of Eurocode 2 (2008) was undertaken in this research work. The uncertainties associated with both the variables that defined the applied loading as well as those that defined the structural resistance were fully accommodated. In the research it was clearly established that overloading concrete structures prior to twenty eighth days after casting is unsafe, and may lead to dramatic lost of stability. The outcome of this study is a clear testimony that some recent failure cases in Nigeria may be attributed to excessive construction live load imposed on the structures prematurely. The following recommendations are made.

1. The minimum age of a concrete member before proceeding to next stage of construction should be twenty eight days, which corresponds to the days of attainment of design specified twenty day cube strength. On no account should construction workers and material be imposed on a premature concrete members.
2. Very strong formwork and shoring system should always be provided for reinforced concrete members during construction.
3. Further research need to be tailored to other important failure modes, such as buckling of early age loaded reinforced concrete columns.

## 5. REFERENCES

[1] Abubakar, I. “Reliability analysis of structural design parameters of strip footings”. Journal of Applied Sciences and Research. 2(7) pp 497-401, 2006

[2] Afolayan J. O. "Reliability-Based Analysis and Design", Departmental Seminar, Department of Civil Engineering, Ahmadu Bello University Zaria, 1992

[3] Ang, H. S. and Tang, W.H. “Probability Concepts in Engineering Planning and Design” Volume 1 – Basic principles. John Wiley and Sons, New York, 1975.

[4] Ditlevsen O. “Uncertainty Modelling with Application to Multidimensional Civil Engineering Systems”. McGraw-New York, 1981

[5] Ditlevsen O. and Madsen H. O. “Structural Reliability Methods.” John Wiley & Sons, Chichester, 2005

[6] Ditlevsen, O. Der Kiureghian, A. “ Aleatory or epistemic? Does it matter?” Journal of Structural safety, 31: 105-112, 2009

[7] Dimitri, V. V. and Mark, G. S. “Safety factors assessment of existing structures”. Journal of Structural Engineering. Vol 128 (2) pp 258-265, 2002

[8] Eurocode 1990 “Eurocode-Basis of structural design”. British standards. CEN Brussels , 2002

[9] Eurocode 2 “Eurocode 1992 1-1: Design of Reinforced Concrete structures, part 1-1, General Rules and Rules foe Buildings” CEN, Brussels, 2008

[10] Gollwitzer S, Abdo T., and Rackwitz R. “First Order Reliability Method: FORM Software”. User Manual.Munich, 1988

[11] Hakan, F. L. “ Uncertainty and Risk Analysis in Fire Safety Engineering”. Lund University Institute of Technology. Department of Fire Safety Engineering. Report LUTVDG (TVBB-1016), 1998

[12] Ike A. C. “Case Histories of Building Collapse in Nigeria” Proceedings on National Technical Workshop on Building Collapse in Nigeria, pp 55-77, 2012

[13] JCSS “Probabilistic Model Code: Part 1- Basis on design”. Joint Committee on Structural Safety. 12th draft, internet version: http://www.jcss.ethz.ch, 2000

[14] Melchers, R. E. “Structural reliability analysis and prediction”. Chichester, John Wiley, 1999

[15] MacGregor, J.G. “Safety and limit state design for reinforced concrete”, Can. J., Civ,. Engg. Vol 3 (40) pp 484-513, 1976

[16] Rao, K. D. Kushwaha, H. S. Verma, A. K. and Srividya, A. Quantification of Epistemic and aleatoru uncertainties in Level-1 probabilisric safety assessment tudies. Reliability Engineering and System Safety vovol 92 pp 947-956, 2007.

[17] Rosowsky D. V., Steward M. G. and Epaarachchi D. C. “Structural Reliability of Multistorey Buildings Duing Construction”, Jnl of Struct. Engrg, Vol. 128, Issue 2, pp. 205-213, 2002

[18] Rosowsky D. V., Steward M. G. and Khor, E. H. (2002), “Early-age loading and long term deflection of reinforced concrete beams. Structural Journal. Vol 93 (7), 2002