

## 3-D Thick Cylinder under Internal Pressure

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**ABSTRACT----** *This paper described the results of a nonlinear static mode within ELFEN of elastic and elastic-plastic behaviour of thick cylinder that is subjected to an internal pressure and therefore a linear stress analysis performed using ELFEN implicit code. Such an analysis is important because the shape of most structures under internal pressure is cylindrical.*

*In this paper is considered only. Elastic and elastic-plastic finite element analysis is used to predict the principle stresses, effective stress and results are compared with those obtained from theoretical equations in order to predict the limit and failure loads for this type of loading also the relationships between radial, hoop stresses and displacement has been used to develop a through understanding. The analysis was completed using ELFEN Version 3.0.4 (a finite element program for Microsoft Windows NT). The program allows pre-processing, analysis and post-processing stages to be completed within a single application. The program can be used to model a large number of situations including buckling, plastic deformation, forming and stress analysis problems.*

*In this study, a thick cylinder of internal radius 100.0 mm and of external radius 200.0 mm is subjected to an internal pressure which is gradually increased to near the ultimate load that may be sustained by the cylinder. The cylinder is modelled as an elasto-plastic material using the Von Mises yield criterion which is normally used for metallic materials[2]. The specification of the load in several increments enables the spread of the plasticity to occur gradually and its effect on the stress distribution to be assessed.*

**Keywords---** finite element analysis, elastic-plastic behavior, thick walled cylinder and equivalent stress

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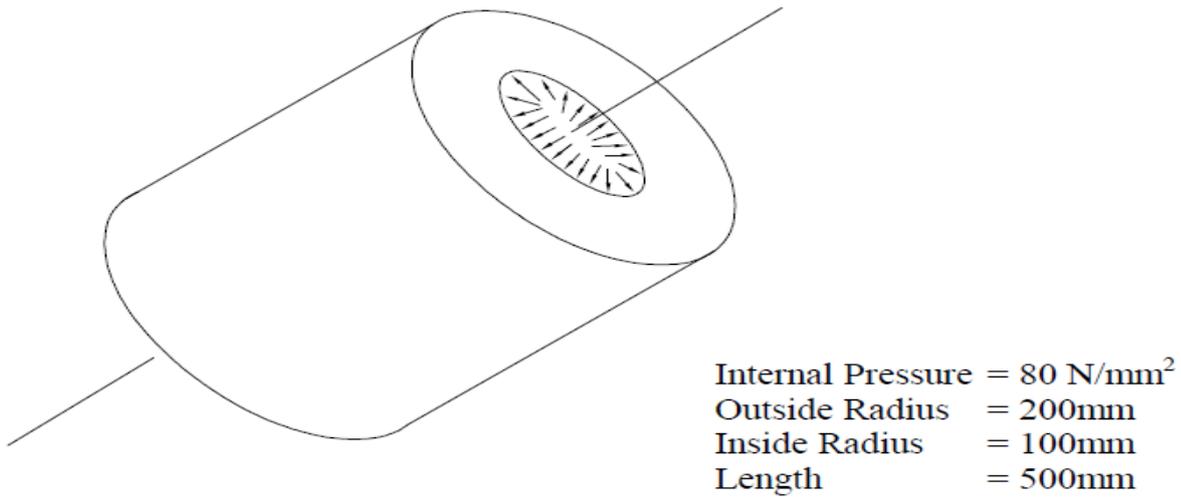
### 1. INTRODUCTION

The most structures under internal pressure take cylindrical shape like as the boilers in thermal power plant, aerosol cans and gas cylinders, in this study one of the kind of this shape[3].

Elastic and elastic-plastic finite element analysis is used to predict the equivalent and principle stresses and results are compared with those obtained from theoretical equations. The researcher most understand mechanics properties of materials and stress-strain diagram and the principle stresses and strains because all the design of mechanical machines must have high strength to support external loads before it collapse.

### 2. THICK- WALLED CYLINDER

A thick-walled cylinder is one where the thickness of the wall is greater than one-tenth of the radius. In this study a thick cylinder made from steel of internal radius 100, external radius 200mm, length 500mm and thickness 100mm subjected to an internal pressure of 80N/mm<sup>2</sup> as shown in figure 1.



Figure|:1 thick walled cylinder

### 2.1 Stresses in thick walled cylinder

A thick walled cylinder subject to an internal pressure has three principal stresses will be  $\sigma_r$  is radial stress,  $\sigma_a$  axial stress and  $\sigma_t$  tensile tangential stress [3] as shown in fig1. The stress conditions occur throughout the section and vary primarily relative to the radius r.

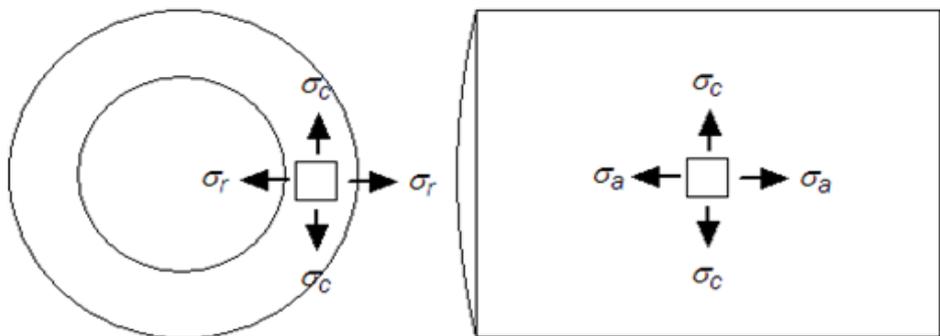


Figure 1 stresses in thick walled cylinder

#### 2.1.1 Stress in radial Direction

The stress in radial direction at a point in the cylinder wall can be expressed as:

$$\sigma_r = \frac{p_i r_i^2}{r_o^2 - r_i^2} \left( 1 + \frac{r_o^2}{r_i^2} \right) \dots (1)$$

Where

- $p_i$  internal pressure
- $r_o$  outer radius of then cylinder
- $r_i$  inner radius of the cylinder
- $\sigma_r$  radial stress

### 2.1.2 Stress in axial direction

The stress in axial direction at a point in the cylinder wall can be expressed as:

$$\sigma_a = \frac{p_i r_i^2}{r_0^2 - r_i^2} \dots (2)$$

Where

$\sigma_a$  stress in axial direction

### 2.1.3 Stress in tangential direction

The stress in tangential direction at a point in the cylinder wall can be expressed as:

$$\sigma_t = \frac{p_i (r_0^2 + r_i^2)}{r_0^2 - r_i^2} \dots (3)$$

Where

$\sigma_t$  stress in tangential direction

### 3 Theoretical solution (Lam's solution [3])

$$\begin{aligned} \sigma_r &= \frac{p_i r_i^2}{r_0^2 - r_i^2} \left( 1 + \frac{r_0^2}{r_i^2} \right) \\ &= \frac{80 \times (100)^2}{(200)^2 - (100)^2} (1 + (200)^2 / (100)^2) = 133.3 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \sigma_a &= \frac{p_i r_i^2}{r_0^2 - r_i^2} \\ &= \frac{80(100)^2}{(200)^2 - (100)^2} = 26.6 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \sigma_t &= \frac{p_i (r_0^2 + r_i^2)}{r_0^2 - r_i^2} \\ &= \frac{80(200)^2 + (100)^2}{(200)^2 - (100)^2} = 133.3 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \sigma_{eq} &= \frac{1}{\sqrt{2}} \sqrt{(\sigma_a - \sigma_t)^2 + (\sigma_t - \sigma_r)^2 + (\sigma_r - \sigma_a)^2} \\ &= 163.5 \text{ N/mm}^2 \end{aligned}$$

### 3. GEOMETRIES

A typical geometry is shown in Fig 3. The basic shape of the component is a three-dimensional cylinder.

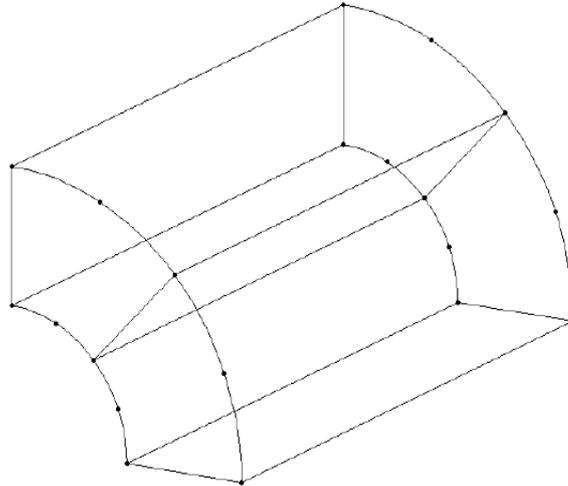


Figure:3 Finite element geometry

### 4. LOADING AND BOUNDARY CONDITIONS

A face loading applied to the inside surface of the cylinder. End of the cylinder is restrained in the Z directions no movement normal to the end surface, surface Y-Z plane restrained in X direction and surface X-Z plane restrained in Y direction as shown in figure 4.

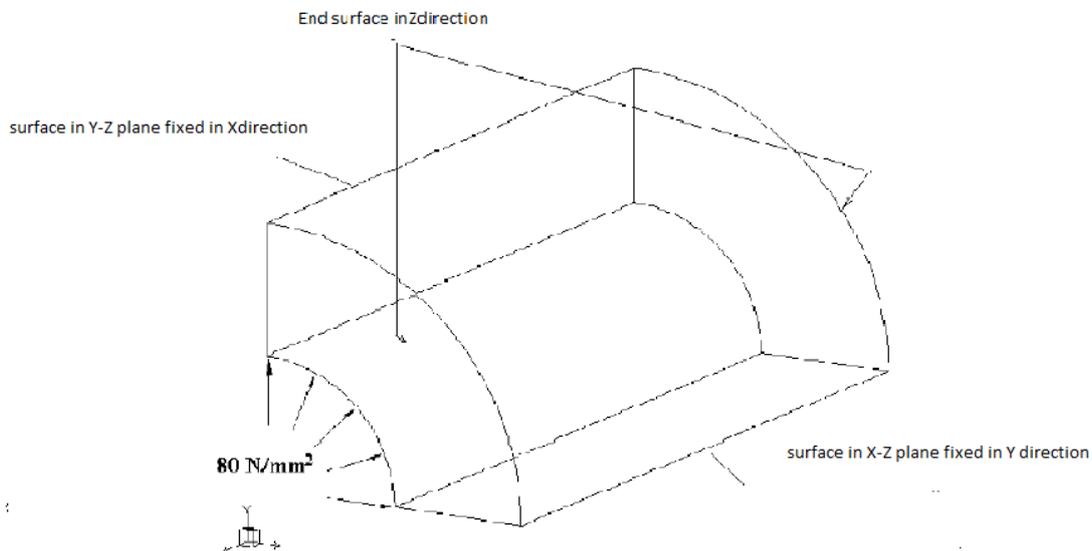


Figure: 5 Structural constraints and applied loading

## 5. MATERIAL MODEL

The multi-linear material model for steel, the materials properties as shown in table 1 was used for this analysis. A finite element mesh of 60 rectangular 4-noded elements and one element through the thickness was generated automatically using the ELFEN [1] mesh generator and the mesh study result is shown in Figure 6.

Table 1 material properties

Properties	value
Young's modulus	$2 \times 10^5 \text{ N/mm}^2$
Poisson's ratio	0.3
Density	$7.8 \times 10^{-6} \text{ kg/mm}^3$
Yield stress	$207.9 \text{ N/mm}^2$

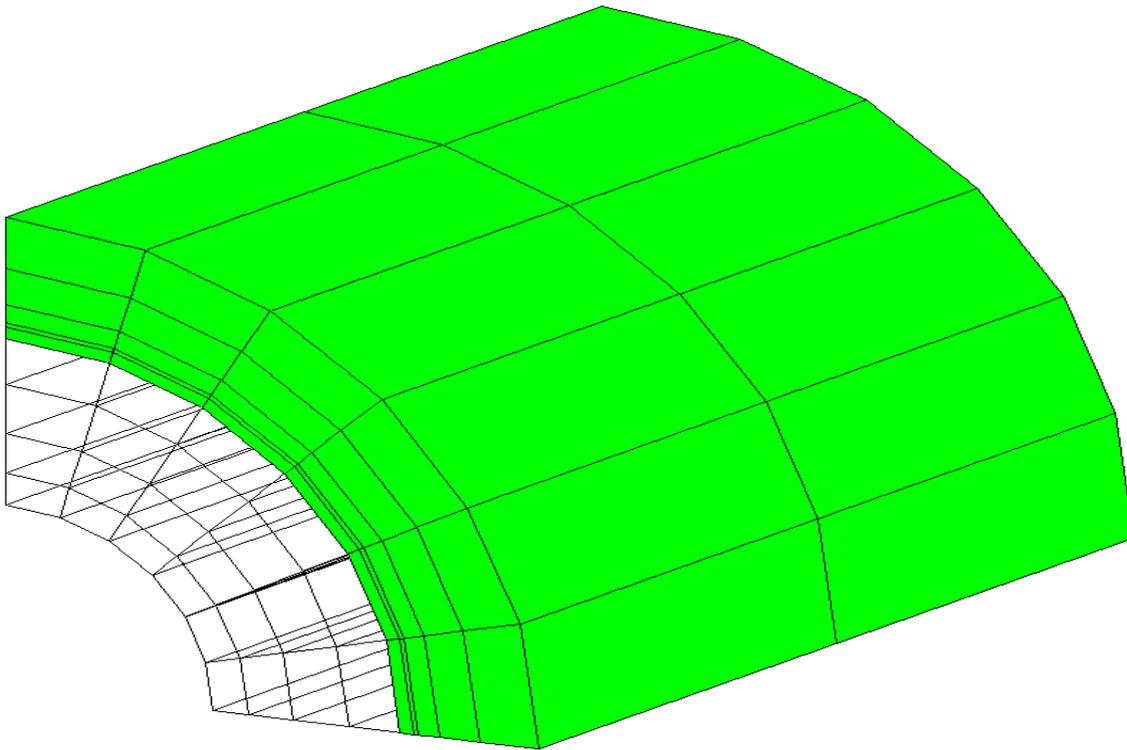
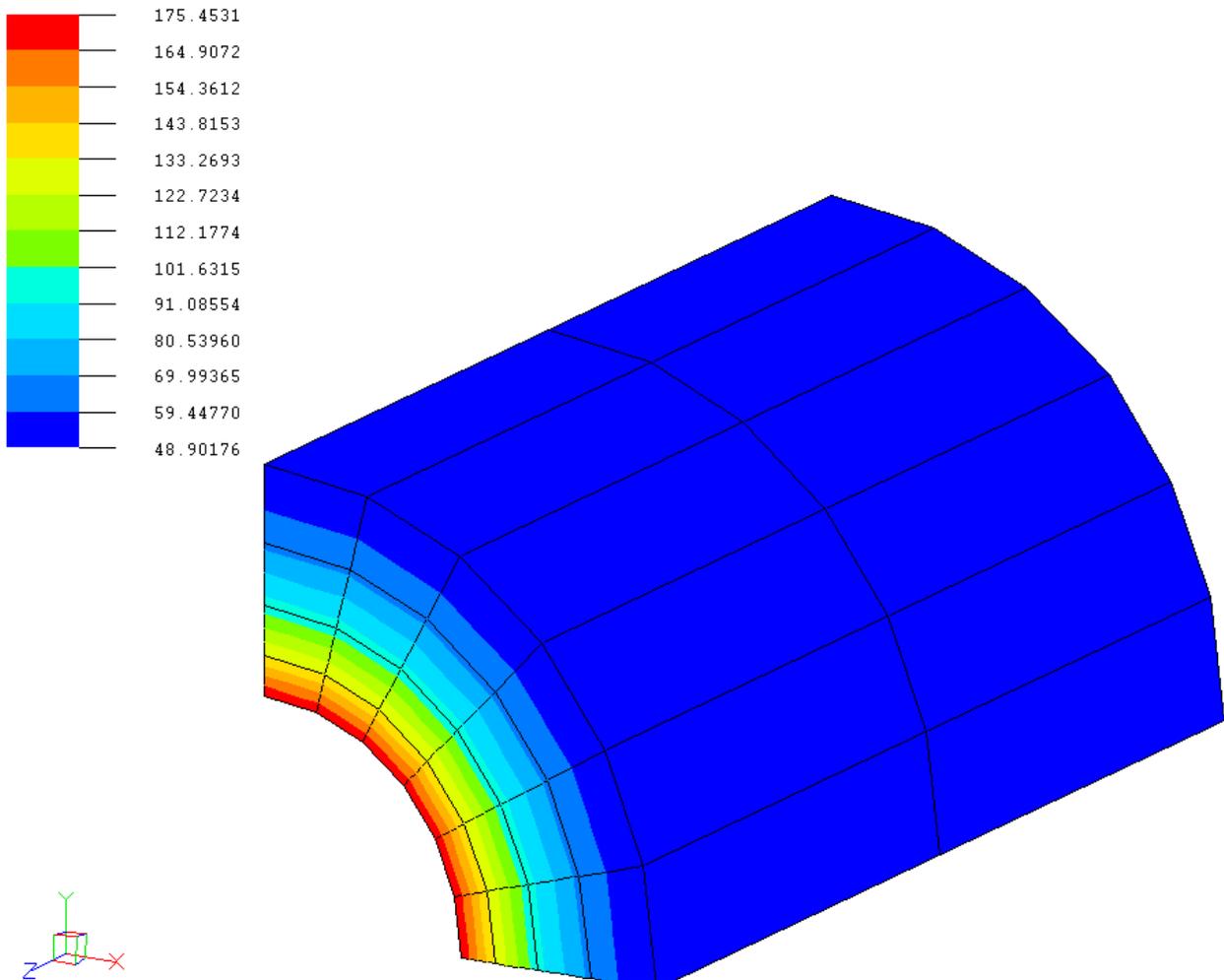


Figure 6 Finite element mesh

## 6. FINITE ELEMENT PREDICTIONS

Predictions have been obtained using the elastic and elastic-plastic analysis facilities within ELFEN [1]. The von Mises equivalent stress contour plot is shown in figure 7. It is clear that the highest stresses occur close to the neck of the cylinder.



**Figure:7**equivalent stress contour plot

## 7. DISCUSSIONS OF RESULTS

From Figure 7, it can be seen that the highest stress close to the neck of the cylinder (red colour) and the equivalent stress of 175.4N/mm<sup>2</sup> occurs, the value of the equivalent stress from the mathematical equations is 165.3N/mm<sup>2</sup> it can be seen from the results that there is excellent correlation between the two values. It is clear that the finite element analysis with elfen program has a good express for stresses analysis.

## 8. REFERENCES

- 1 **ELFEN** User Manual Version 3.0.4, 2001, Rockfield Software, Swansea, UK
- 2 **Zienkiewicz, O. C.** 1984 The finite element method, McGraw -Hill
- 3 **Black, P.H and Adams, E** 1970 Machine design
- 4 **Karwa, R** 2000 Machine Design