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Non-uniqueness of Strassen's sub-cubic matrix multiplication formula

Jacob Adopley
Ghana Technology University College
Tesano-Accra, Ghana

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Abstract

An alternative expression for Strassen's matrix multiplicaton formula is derived to show that, the original expressions for the formula is not unique.

1 Introduction

The divide and conquer principle, is one the major principles in Algorithm Design. In 1969, Strassen gave his famous formula for reducing the matrix multication complexity from the order of B $O(n^3)$ to the order of $O(n^{\ln(7)/\ln(2)})$ or $n^{2.807355}$, where n is the size of the matrix. Even though in recent times others, [2],[3], [4], [5], [1], [7] and [6] have developed new formulas that can outperform Strassen's algorithm in principle, they are of little practical importance. It is based on the fact that, for these formulas to outperform Strassen's formula, n has to be so large that, present modern computers cannot handle them. Based on the fact that all these modern exortic new formulas are of very limited practical importance, the current alternative expression will improve the Strassen's algorithm by offering an option when the original compution will cost more based on the number of significant digits in each factor. Below is a brief review of the Strassen's formula.

2 Strassen's Matrix Multiplication Formula

Here we begin with just multiplying two matrices, namely

$$X = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} E & F \\ G & H \end{bmatrix} \tag{1}$$

Then multiplying the two matrices X and Y using matrix multiplication rules from grade school, we get

$$X * Y = \begin{bmatrix} (AE + BG) & (AF + BH) \\ (CE + DG) & (CF + DH) \end{bmatrix} \tag{2}$$

We noticed that, it rquires 8 or n^3 multiplications to compute a 2×2. In 1969, Strassen reduced the number of multiplications for the 2×2 from 8 to 7 with a clever combination of the matrix elements as follows:

$$\begin{array}{ll} P_1 = A(F - H) & P_5 = (A + D)(E + H) \\ P_2 = (A + B)H & P_6 = (B - D)(G + H) \\ P_3 = (C + D)E & P_7 = (A - C)(E + F) \\ P_4 = D(G - E) & \end{array} \tag{3}$$

Then as given by Strassen,

$$X * Y = \begin{bmatrix} P_5 + P_4 - P_2 + P_6 & P_1 + P_2 \\ P_3 + P_4 & P_1 + P_5 - P_3 - P_7 \end{bmatrix} \tag{4}$$

The intention here is not to show a better algorithm, rather to demostrate that, Strassen's formula is not unique. Indeed, from simple re-arrangement of the multiplication of the matrix elements, it quite easy to derive the following alternative to Strassen's Formula as shown below:

$$\begin{array}{ll} Q_1 = (A - B)E & Q_5 = (B + C)(E + H) \\ Q_2 = B(E + G) & Q_6 = (A + C)(E - F) \\ Q_3 = C(F + H) & Q_7 = (B + D)(G - H) \\ Q_4 = (C - D)H & \end{array} \tag{5}$$

This then gives alternative to Strassen's formula as:

$$X * Y = \begin{bmatrix} Q_1 + Q_2 & Q_5 + Q_1 - Q_3 - Q_6 \\ Q_5 - Q_4 - Q_2 + Q_7 & Q_3 - Q_4 \end{bmatrix} \tag{6}$$

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As of now, equations 6 has not appeared anywhere in the literature. It was communicated to Tim Roughgarden of Standford University, the first time this was proved to be true by the author. The significance is that, there is an alternative to te original Strassen's formula with the same running time complexity.

3 Conclusion

We have been able to provide a new alternative to Strassen's matrix multiplication formula which is of practical interest and value and can be used instead of the original formula. It is not an improvement or just an academic exercise. It is an alternative formula that can be used inplace of Strassen's formula.

References

[1] Cohn, H; Kleinberg, R: Szegedy, B and Umans, C. Group-theoretic algorithms for matrix multiplication. *46th Annual IEEE Symposium on Foundations of Computer Science, (FOCS 2005). p 379*, 1970.

[2] Coppersmith, Don and Winograd, Shmuel. Matrix multiplication via arithmetic progressions. *Journal of Symbolic Computation 9 (3): 251, Vol. 20, 1-24*, 1990.

[3] Davie, A. M. and Stothers, A. J. Improved bound for matrix multiplication. *Proceedings of the Royal Society of Edinburg 143A, 351-370*, 2013.

[4] Le Gall, Francois. *Power of Tensors and fast matrix multiplication.* Proceedinds of the 39th International Symposium on Symbolic and Algebraic Computation(ISSAC, 2014.

[5] Robinson, Sara. Towards an optimal algorithm for matrix mutiplication. *SIAM News 38 (9)*, 2005.

[6] Stothers, Andrew. *On the Complexity of Matrix Multiplication.* 2010. Thesis.

[7] Williams, Virginia. Breaking the coppersmith-winograd barrier. 2011.