

# The Discharge Coefficient for a Compound Sharp Crested V-Notch Weir

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**ABSTRACT**---- *This paper presents experimental work to investigate the discharge coefficient for a compound V-notch weir. This weir is composed of two triangular parts with distinctive notch angles. Fundamental standards were utilized to connect the release to the significant geometrical and hydraulic parameters in non-dimensional form. Physical model tool was used, where more than 30 runs were executed in a rectangular flume. Five weir models were tested representing different weir characteristics. The outcomes were contrasted with previous work, the discharge coefficients were found in a decent concurrence with the comparable compound sharp-crested weir. The study demonstrated the principle point of preference of the proposed compound weir to allow a decent exactness in the flow measurement for a varied range of flows with no cutoffs as well.*

**Keywords**--- V-notch Weir, Weir Design, Hydraulic structures, Discharge Measuring Structures

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## 1. INTRODUCTION

Weir is a hydraulic structure set perpendicular to the flow direction with the objective to measure the flow discharge. The weir gives precise measurement for a different spread of flows. The lower triangular part of the weir manages the normal range of discharges at the measurement structure, the weir upper part living up to expectations for the unpredictable higher top flows. Weirs are usually used to observe rivers flow keeping in mind the end goal to shield from flooding and bolster navigation in rivers.

The V-notch weir is one of the sharp crested weirs with a triangular section, used to measure small discharge values subsequent to the water head over the weir peak that is generally touchy to changes in flow. The compound V-notch weir (composed of two triangular sections) is a new model introduced by [1], utilized for the same purposes of the established one notwithstanding recompense to pass high discharges. Consequently, the new weir model is permitted to be utilized as a part of numerous canal categories. Unfortunately, few studies were completed on this new sort of weirs, along these lines this study was done to elucidate more fine points of interest concentrating on the discharge coefficient of a compound sharp crested triangular (CSCT) weir model.

## 2. BACKGROUND

The structural parameter of the weir (crest height, notch angle, weir width) affects the weir performance. The geometrical parameters involved in the hydraulic operation of weirs are the length of the weir crest and the shape of the flow control section [2]. Various studies have been executed concerning flow of water over weirs, as [3], [4], [5], [6], [7] and [8].

The discharge coefficient speaks to the impacts not considered in the determination of the mathematical statements used to estimate discharge from flow depth. Per to [9] and [10] these effects include surface tension, capillarity, viscosity, velocity distribution in the approach section, and streamline curvature attributable to weir contraction.

Sharp-crested weirs have been extensively investigated ([11], [12], [13], [14]). The sharp-crested weir is the most commonly used device in channels for flow measurement and flow regulation due to its simplicity, [15]. [16] Contended that data is accessible with respect to both the discharge coefficient and water surface profiles for the sharp crested weirs.

([17];[18];[1]) argued that from practical engineering point of view, a compound weir composed of rectangular and/or triangular parts in the shape of cross section is also a common device for flow control in canals and mountainous gullies.

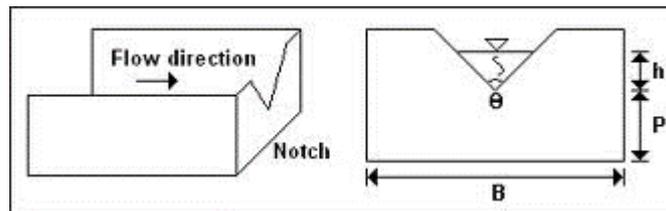
One of the offered crested weirs is the V-notch model. The triangular or V-notch sharp-crested weir is frequently utilized for flow measurement, particularly when a precise estimation of low flow rates is required. The compound sharp crested V-notch weir was mulled over by [19], focusing on the bed profile downstream the weir. [20] tentatively assessed the effect of vertical flow curvature on the discharge coefficient. He found that the discharge coefficient is conversely corresponding to the V-notch angle ( $\theta$ ) and directly proportional to the relative head ( $h=P$ ), Figure1.

Based on [21], the flow equation for triangular weirs is expressed in the form below:

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\left(\frac{\theta}{2}\right) h^{\frac{5}{2}} \dots\dots\dots(1)$$

Where:

- Q: flow discharge
- $c_d$ : the discharge coefficient
- g: gravitational acceleration
- h:flow head above weir vertex



**Figure 1:** Schematic diagram for V-notch triangular weir

Where:

- B: weir width equal to flume width
- P: vertical distance from weir base to its vertex
- $\theta$ : weir V-notch angle

Most studies fixating on triangular weir as a SCW were inhibited to vertex angle of  $90^\circ$ , and constrained number of research studies considered an extensive variety of vertex angle. Flow equations for sharp-crested weirs are usually obtained by the mathematical integration of elemental flow strips over the nappe developed by [7], where he introduced the following equations:

$$C_d = 0.6085 - 0.05254 * \theta + 0.02135 * \theta^2 \dots\dots\dots(2)$$

$$k_h = 3.9058 - 3.8558 * \theta + 1.1940 * \theta^2 \dots\dots\dots(3)$$

Where:

- $\theta$ : the notch angle in rad.
- $c_d$ : the discharge coefficient
- $k_h$ : the correction factor for the head

[1] carried out experimental tests on fully contracted weir with distinctive vertex angles, furthermore they analyzed the compound weir with diverse vertex angle( $\theta_2$ ). Theoretical discharge equations have been acquired for this kind of weir and a system for estimation of the discharge coefficients has been proposed and tentatively accepted. Toward the end they contended that the upside of this kind of weir is that it can provide truthful measurement for a various range of flows without any gaps. They proposed the following equation for the global discharge coefficient:

$$C_d = \frac{c_{d1} \tan\left(\frac{\theta_1}{2}\right) \left[ \frac{1}{\left(1 - \frac{h_0}{h}\right)^{\frac{2}{5}} - 1} \right] + c_{d2} \tan\left(\frac{\theta_2}{2}\right)}{\tan\left(\frac{\theta_1}{2}\right) \left[ \frac{1}{\left(1 - \frac{h_0}{h}\right)^{\frac{2}{5}} - 1} \right] + \tan\left(\frac{\theta_2}{2}\right)} \dots\dots\dots(4)$$

$c_d$ : the discharge coefficient  
 $\theta_1, \theta_2$ : weir V-notch angles  
 $h$ : operating head;  
 $h_0$ : height of lower part

Where  $c_{d1}$  and  $c_{d2}$  are calculated based on the LMNO equation [22],

$$C_d = 0.6072 - 0.000874 * \theta + 6.1 * 10^{-6} * \theta^2 \dots\dots\dots(5)$$

$$k = 4.42 - 0.1035 * \theta + 1.005 * 10^{-3} * \theta^2 - 3.24 * 10^{-6} * \theta^3 \dots\dots\dots(6)$$

Where,

$c_d$ : the discharge coefficient  
 $k$ : is the correction factor for the head  
 $\theta_1, \theta_2$ : weir V-notch angles  
 $h$ : operating head;  
 $h_0$ : height of lower part

After careful study for the previous work; the primary goal of this research study is to investigate the discharge coefficient related to diverse V-notch weir angles in practically identical to the discharge coefficients associated to the classical V-notch weir under the same flow conditions. For this reason; five weir models (Figure 2) were tested to develop a design equation for the V-notch weir in terms of the discharge coefficient.

### 3. EXPERIMENTAND RESULTS

The physical model tool was used in this study. Experiments were performed in a flat rectangular laboratory flume located at the Hydraulics Research Institute (HRI) experimental hall of the National Water Research Center, Egypt. The experimental flume is 21 m long, 0.6 m wide, 0.5 m deep, where water was supplied to the channel through an overhead tank provided with an overflow arrangement to maintain constant head. Steel wooden gate with an orifice with a rectangular shape is armed within the flume. Versatile downstream gate is situated toward the end of the flume. Centrifugal pump compelled by induction motor to re-circulated the flow from a reservoir to the flume. The inflow discharge was measured using an ultra-sonic flow meter. Steel plates of 0.02m thick 0.3 m height and 0.6m width each were used to create the designed weir models.

#### 3.1. Run duration

The experimental work started by testing the duration time of runs using distinctive weir models and diverse discharges. The overall time was taken as four hours and the upstream water head was recorded every 10 minutes. It was found that following 20 minutes from beginning, no varieties of water levels were recorded. Thusly, the run time of any test equivalent to 4 hours underscores that there is no obvious changes in water levels and the flow came to the quasi-equilibrium state.

#### 3.2. Run procedure

The test methodology was as per the following: (1) The selected weir model was fixed carefully in the flume in its place; (2) The tail gate was completely closed; (3) The pump was activated and the discharge was adjusted using a control valve. (4) The exact water discharge was measured using an ultra-sonic flow meter with an accuracy of  $\pm 1\%$ ; (5) The tail gate was screwed gradually until the required downstream water depth was reached using the point gauge with an accuracy of  $\pm 0.1$  mm; (6) The running time of the test is started; (7) After 4 hours (where there is no appreciable changes in water levels), the upstream water level measurements were recorded; (8)The pump was switched off; (9) The previous steps were repeated for each run.

### 4. DIMENSIONAL ANALYSIS

In the analysis of the discharge coefficient of V-notch weir, the considered variables were:  $h$ = the water head above the vertex of the V-notch weir,  $Q$  = water discharge through the flume,  $P$  = vertical distance from weir base to its vertex,  $S_o$ = flume bed slope,  $B$ = flume width,  $\theta_1$ = weir upper angle,  $\theta_2$ = weir lower angle,  $L_{ap}$ = length of apron

The relationships for the discharge coefficient  $c_d$ , could be expressed as follow:

$$\phi(B, P, h, Q, S_o, \theta_1, \theta_2) = 0 \dots\dots\dots(7)$$

In this study  $P, B, S_o, \theta_2, L_{ap}$  were kept constant. Then, the Eq. (7) might be written in the following form: Using  $\pi$ -theorem and applying the properties of dimensional analysis, it yields;

$$c_d = \phi\left(\frac{h}{P}, \frac{h}{B}, Q, \theta_1\right) \dots\dots\dots(8)$$

Three Vertex angles of  $90^\circ, 120^\circ$ , and  $150^\circ$  were tested. The water head over the weir was measured using the point gauge. More than 30 test keep running of the examinations were executed with diverse weir setup. The discharge for each test was measured notwithstanding water depth upstream of the weir, and from these measurement the discharge coefficients for each mode was assessed. The consequences of the tests completed on the V-notch weir are displayed in table 1, and figures 3-5, where the head versus flow was plotted.

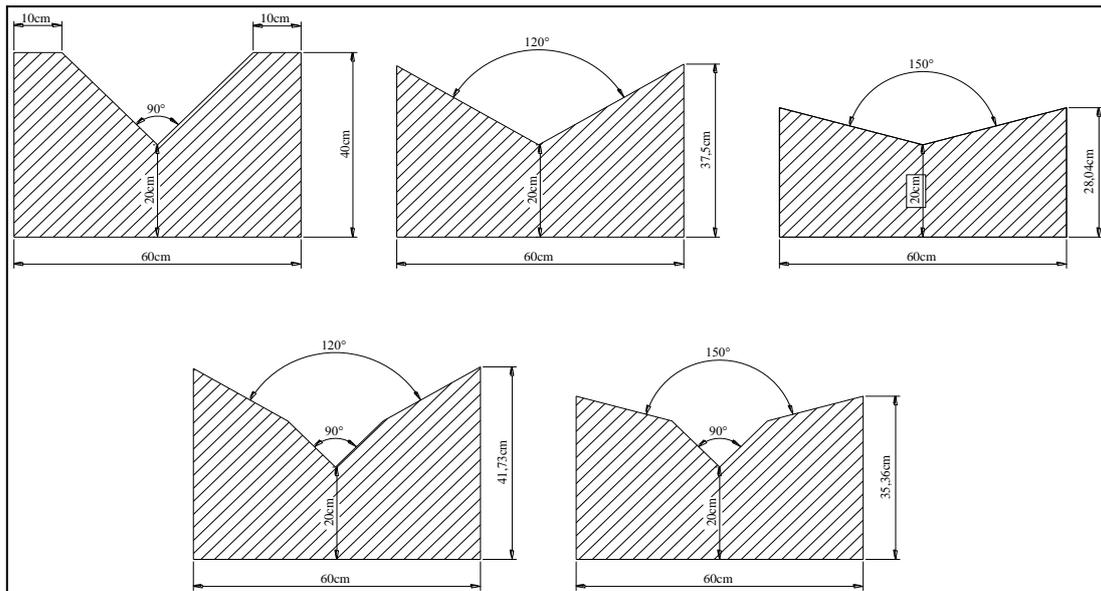


Figure 2: The tested weir models

Table 1: Test conditions and results

| Vertex angle                           | h/P       | Q (L/s)<br>Measured | h (m)<br>Measured | h <sub>e</sub> (mm) | C <sub>d1</sub> | C <sub>d2</sub> | C <sub>d</sub> Calculated |
|--|-----------|---------------------|-------------------|---------------------|-----------------|-----------------|---------------------------|
| $\theta_1=90^\circ$                    | 0.2-0.75  | 0.46-12.1           | 0.04-0.15         | 40.9-150.9          | -               | -               | 0.576-0.581               |
| $\theta_1=120^\circ$                   | 0.55-0.75 | 9.9-21.37           | 0.11-0.15         | 110.9-150.9         | -               | -               | 0.59-0.591                |
| $\theta_1=150^\circ$                   | 0.55-0.75 | 21.99-47.62         | 0.11-0.15         | 110.6-150.6         | -               | -               | 0.613-0.614               |
| Vertex angle                           | h/B       | Q (L/s)<br>Measured | h (m)             | h <sub>e</sub> (mm) | C <sub>d1</sub> | C <sub>d2</sub> | C <sub>d</sub> Calculated |
| $\theta_2/\theta_1=90^\circ/120^\circ$ | 0.18-0.3  | 5.5-20.42           | 0.11-0.18         | 111.75-181.75       | 0.578           | 0.59            | 0.5824-0.5887             |
| $\theta_2/\theta_1=90^\circ/150^\circ$ | 0.18-0.3  | 5.7-21.53           | 0.11-0.18         | 111.45-181.45       | 0.578           | 0.613           | 0.5973-0.6112             |

Where,  $h_e$ : is effective head of the weir  $=h+K_h$

The global discharge coefficient ( $C_d$ ) has been calculated from the experimental flow discharge and head. Figures 3 and 4 demonstrated the relationship between the water head and the discharge for different tested weir models. In figure (3) the measurements of classical V-notch weir were plotted. It was proclaimed that under settled head, the release was shifted straightforwardly with the weir angle. Likewise, the discharge was discovered straightforwardly relative to the water head for settled weir point, which was totally concurred with [21]. Focusing on the new compound V-notch weir, figure (4) was plotted. In this figure, the weir vertex angle (the lower angle  $\theta_2$ ), was altered however the upper angle ( $\theta_1$ ) was 120 and 150 to delineate the influence of upper part of the compound weir with diverse angles. It was deduced

that, the weir of  $150^\circ$  upper angle passes a discharge higher than the weir of  $120^\circ$  upper angle under the same head. The findings were coordinated with [1]. Henceforth, that guarantees the adequacy of the composite weir in passing on higher discharges contrasted with the traditional V-notch under the same head.

Figure (5) illustrates the relation between the discharge coefficient and the water head for the classical V-notch weir. The discharge coefficient was presented in three formulas; the calculated, the reasoned by [22] exhibited in (eq.5), and [7]. The computed discharge was found in a decent consent to alternate equations with much similarity to eq.3. That guarantees the dependability of the study estimations.

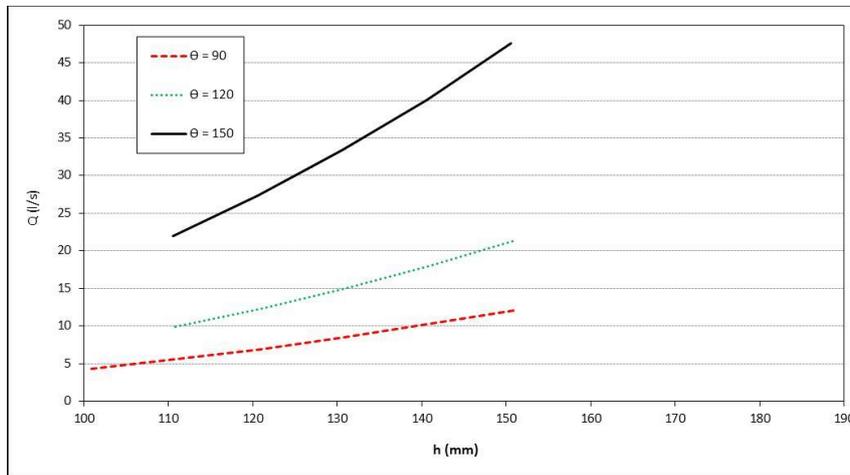


Figure 3: Relationship head-flow for classical V-notch model weirs

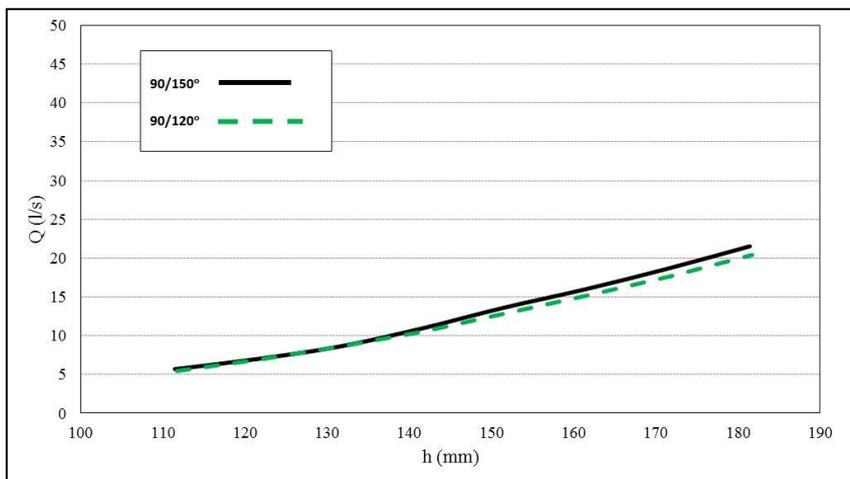


Figure 4: Relationship head-flow for compound V-notch model weirs

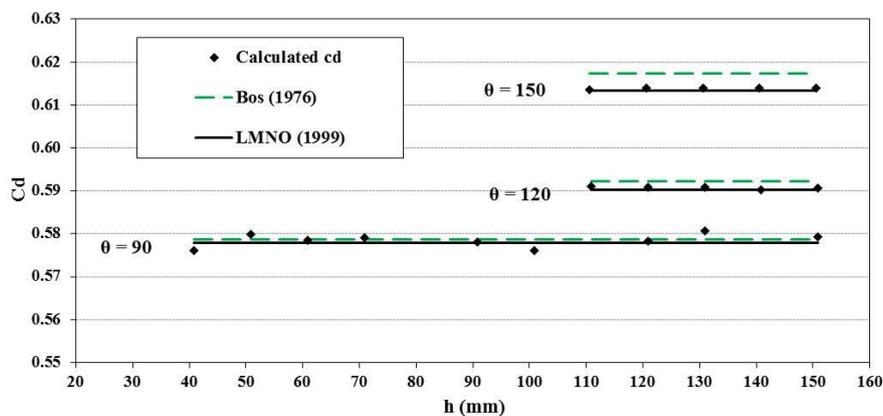


Figure 5: Discharge coefficient for the classical weir in compared to previous studies

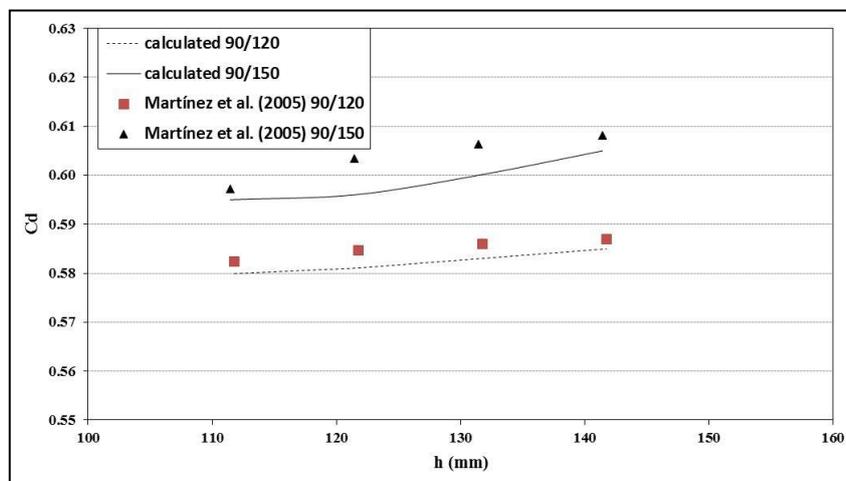
## 5. DEVELOPED EQUATION

A regression analysis was performed applying dimensional analysis strategy utilizing the gathered exploratory information to develop a practical relation using the data fit programming to characterize the relationship between the global discharge coefficients ( $C_d$ ) for the compound V-notch weir and the notch angle. The calculation of  $c_{d1}$  and  $c_{d2}$  was done in view of the recommendations by [1], to use the equation of [22] for computing the discharge coefficient for fully contracted triangular weir for ascertaining  $c_{d1}$  and  $c_{d2}$  for the compound weir consisting of two triangular parts. The developed equation for global discharge coefficient was approved utilizing the experimental data, and it is composed as beneath:

$$C_d = 0.8461 - \frac{0.1053}{\theta_1} - \frac{0.7798}{\theta_2} + \frac{0.7296}{\theta_2^2} \dots\dots\dots(9)$$

With  $R^2=0.99$

To examine the legitimacy of the deduced equation; figures 6 were plotted. Figure 6 illustrated an examination between the discharge coefficient ascertained from eq.9 and concluded from [1]. A decent understanding between the two comparisons was taken note.



**Figure 6:** Discharge coefficient for the compound weir in compared to previous studies

## 6. CONCLUSIONS

Discharge equations for flows over compound V-notch weirs that are composed of simple triangular and compound weirs were the focus of the present exploration. Experimental study was executed to assess the discharge coefficient for five weir models. Three of them for the classical V-notch weir, and rest were centered around the compound sharp-crested weir composed of two triangular parts with diverse notch angles. A statistical analysis was performed to deduce mathematically the discharge coefficient using the experiment results. The study exhibited the fundamental favorable position of the proposed compound weir which is the satisfactory to great exactness in estimation for an extensive variety of flows without any breaks. Likewise, theoretical discharge equations have been acquired for this type of weir and a strategy for estimation of the discharge coefficients has been proposed and tentatively accepted. The discharge coefficient for the examined cases was ranged between (0.573 and 0.613). The study results are constrained to the exploratory data range.

More studies were recommended for the V-notch weirs to get more knowledge and information on this new compound sharp crested triangular (CSCT) weir model. These prescribed studies could test more extensive scope of vertex angles in notwithstanding distinctive weirs board width.

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## 8. NOTATIONS

|            |   |             |
|------------|---|-------------|
| B          | width of approach channel;                        | (m)         |
| $C_d$      | discharge coefficient;                            |             |
| H          | water head;                                       | (m)         |
| h          | operating head;                                   | (m)         |
| $h_0$      | height of lower part;                             | (m)         |
| $h_e$      | effective head of the weir $=h+K_h$ ,             | (m)         |
| $k_h$      | head correction factor;                           |             |
| P          | height from bottom of notch to bottom of channel; | (m)         |
| Q          | discharge;  | ( $m^3/s$ ) |
| $L_{ap}$   | length of apron;                                  | (m)         |
| g          | gravitational acceleration,                       | ( $m/s^2$ ) |
| $S_o$      | flume bed slope                                   |             |
| $\theta_1$ | weir upper angle;                                 |             |
| $\theta_2$ | weir lower angle;                                 |             |

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