The Solution of Two Dimensional and Time Dependent Burger Equations Using Finite Difference Method

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ABSTRACT—*There is a differential equation governing all engineering phenomena. For some of these phenomenas, there are mathematical models that define the behavior pattern of that phenomena in different conditions. The equation we used in this study was the two dimensional and time dependent burger equation. This equation is discontinued through using finite differentiation and solved using newton Robinson technique in Matlab program. Then sensitivity analysis has been performed on time passage and their effect on sedimentation velocity have been studied and analyzed. The results that showed that in the most of times, the skewness diagrams of length and depth velocity in depth direction to is to the left which shows the maximum velocity of falling particle at the time of study and near the bed.*

Keywords—Burger Equation, Finite Difference, Newton-Raphson Method, Velocity Distribution

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1. INTRODUCTION

Most of the mathematical models are partial differential equations. In order to study the different behaviors of a phenomena in response to different conditions, the mathematical models will be solved and effective parameters will be studies for these conditions.

The differential equations can be solved through different methods, some through analytical solutions while others have not been resolved through analytical methods due to problem complexities.

Although the numerical methods have some errors, but in many equations in which analytical methods do not have a proper result, many numerical methods like finite element, finite difference, finite volume are other remedies, the equation we used in this study was the two dimensional burger equation.

The two dimensional Burger equations are nonlinear and time dependent as follows.

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\nabla^2 u$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\nabla^2 v \tag{1}$$

υ is kinematic viscosity and u and v are flow velocity in two vertical and depth directions.

This equation is discontinued through using finite differentiation and solved using newton Robinson technique in Matlab program. Then sensitivity analysis has been performed on time passage and their effect on sedimentation velocity have been studied and analyzed. In this study, the sedimentation phenomena of residues have been studied and analyzed.

In most fluids, there are material and particles that do not solve in the solvent, and they move where the fluids goes, thus they have movement, these particles are called suspended particles, so in supposing a particle moving with the fluid, we see gravity effect on the particle, if we suppose the particle stops in a place and time for any reason, in this stance, this particle does not move anymore by the power of the fluid but by the power of gravity and specifically it travels from its premier location where the fluid left it to the direction that gravity attracts it and then residues in the bottom. This phenomena is called sedimentation.

The actual example of this phenomena happens when the water stream is blocked behind the dam, thus the accumulation of water behind dam makes it stop and in this case, the suspended particles move vertically and sediment on the dam wall.

This phenomena is modeled through Berger equation in a dimensionless manner and the Berger equation which is a differential nonlinear and time depended equation is used as the mathematical model of this phenomena.

2. LITERATURE

In 2005, Abdou and Soliman solved and analyzed this equation using Variational Iteration Method [1]. Wazwaz used Multiple Kink Solutions in 2010 to analyze and solve this equation [2]. Wei and Gu in 2002, studied this Burger equation and used Conjugate Filter Approach to solve it [3]. In 2011, Dag et al. used Taylor-Galerkin and Taylor-Collocation methods to solve this equation and also used B-Splines in this process [4]. In 2011, Shao et al. solved Burger equation using zero border conditions using finite element Local Galrkin method [5]. In 2004, this equation was solved using Decomposition method, analytically by Dogan and released its response through special boundary, and set off conditions. It should be mentioned that he studied and solved this equation during different boundary and initial conditions. Soliman who has produced several articles on solving Burger equation, analyzed KDVB and solved this equation in 2006, using The Modified Extended Tanh-Function method by specific initial and boundary conditions [6]. In 2009, Haq et al. numerically solved this equation (KDVB) and studied and solved this using Mesh-Free method [7]. In addition, Mittal and Arora solved this problem in 2011, using Crankowicolsori and Cubic B-Spline methods [8]. The Burger method was solved using Quandritcher method by Vaghefi in 2012, which was the advent of solving different Burger equations using Quandritcher method [9]. The two dimensional Burger equation was solved by Hongqing and Hongyan in 2007 using New Direct Ansatz [10]. The three dimensional equation of Burger was solved in 2004 by Feng et al. using the following equation, and the detailed solves this equation was provided [11]. In 2009, Dai et al. studied and solved the Burger equation in a three dimensional manner and they used Enp-Function to solve this equation and produced a highly accurate reply for this equation [12]. In 2009, Christou et al. studied the three dimensional equation of Burger and described it through physics practical. They completed the equation introduced by Lie and they used the following two and three dimensional algorithmic equations of Lie [13]. These scholars solved the same equation in 2011 using numerical method and they used Newton Kentorovich method for that [14]. In 2010, Rady et al. introduced Boussinesq-Burger equation and they produced an accurate reply for this equation using Tanh method [15]. In 2012, Dehghan et al. produced the numerical response to this problem using interpolation scaling functions and analyzed its results using accurate methods and calculated its errors [16]. In 2012, this form of Burger equation (Fisher Equation) was solved by Zhao et al. using Chebysher-Legender Pseudo-Spectral, and they provided numerical results in three different boundry conditions [17]. In 2012, Korpusov studied Benjamin-Bona-Mahony–Burgers with proper boundary conditions and this was the first blow-up space response to this problem [18]. In 2012, Hag used MMOL to numerically solve the nonlinear equation of Burger [19].

3. METHODOLOGY

During the infinite differentiation, only one interval is considered for the problem. In this equation, the number of divisions are obtained through endeavor in a linear mode, the reason for which is the size of the matrix and its effect on the convergence of the response.

For viscosity of 0.05, 58 points has been considered for the longitudinal and in the traverse axises and 150 intervals have been considered for the time axis, thus the resulting matrix is a 6728*6728.

The set of equation type (2) is a nonlinear equation that is solved forming the matrix form of the equation and using the Newton-Raphson method by MATLAB software.

$$\frac{U(m,n,k+1)-U(m,n,k)}{\Delta t}+U(m,n,k)\times\frac{U(m+1,n,k)-U(m,n,k)}{\Delta x}+V(m,n,k)\times\frac{U(m,n+1,k)-U(m,n,k)}{\Delta y}$$

$$=\left\{\left(\frac{U_{m+1,n,k}+U_{m-1,n,k}-2U_{m,n,k}}{\Delta x^2}\right)+\left(\frac{U_{m,n+1,k}+U_{m,n-1,k}-2U_{m,n,k}}{\Delta y^2}\right)\right\}$$

$$\frac{V(m,n,k+1)-V(m,n,k)}{\Delta t}+U(m,n,k)\times\frac{V(m+1,n,k)-V(m,n,k)}{\Delta x}+U(m,n,k)\times\frac{V(m,n+1,k)-V(m,n,k)}{\Delta y}$$
$$=\left\{\left(\frac{V_{m+1,n,k}+V_{m-1,n,k}-2V_{m,n,k}}{\Delta x^2}\right)+\left(\frac{V_{m,n+1,k}+V_{m,n-1,k}-2V_{m,n,k}}{\Delta y^2}\right)\right\} \quad (2)$$

In Figure (1) it is seen the schematic outrider finite difference method.

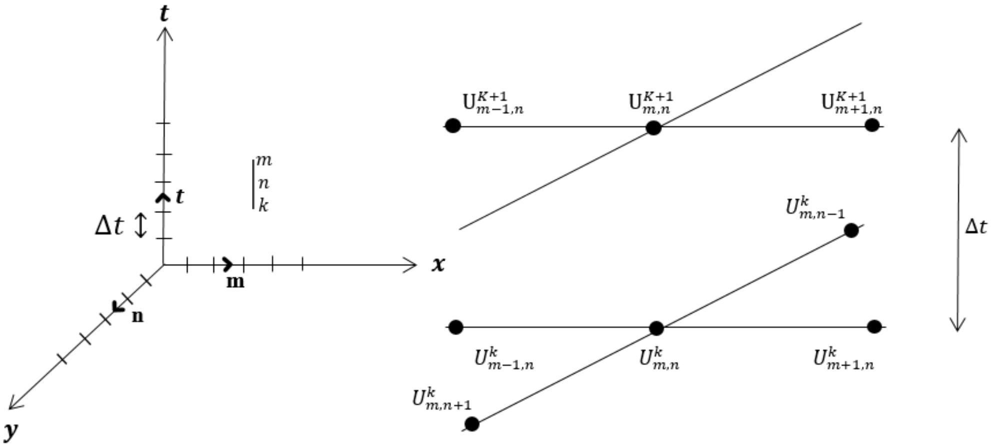

Figure 1: The Schematic Progressive Finite Difference Method

Most of phenomenons that happen in the nature, are simplified by nonlinear equations. Although most of these nonlinear equations are solvable with proper accuracy and locally by introducing similar linear models, but are as well phenomenon which can only be described using nonlinear models. Finding the resolve of a series of nonlinear equations is much harder that linear equations, and there may be nonlinear equations with no real answer. The Newton-Raphson method is a type of consequence proximity methods for solving nonlinear equations.

Regarding the non dimentional transformation of the study and measurement parameters are important throughout the study, for example the non dimentional transmission of velocities with average velocity for the upstream path or the velocity on the stream controlling structures and non dimentional transformation of space can be for example through the width of the stream canal or the lendth or geometrical features of a structure located in the stream canal, in addition, the non dimentional transformation of time by the proportional balanced time in the sedimention process.

In case one have numerous numbers of studied parameters in the phenomena, one can use non dimentional transformation theories including Bakinghum theory.

Anyhow, in this study, the hypothesis is the non dimentional transformation of parameters located in Burger equation in the two dimentional manner. Through this hypothesis we study the effect of non dimentionalized parameters of time, viscosity on the velocity field in different locations.

4. RESULTS

In this section, this problem was solved in four different time points using progressive finite differentiation.

Figure (2) shows the longitudinal velocity distribution fall of a particle for the axis that goes through middle bottom in different times.

As you see in Figure (2), for the longitudinal velocity of the falling particle toward the longitudinal direction and for the axis going through the depth, and as time develops, the peak point of longitudinal velocity is deflected to right, and also the this velocity is diminished, in a manner that in the time equal to balance time, and taking into consideration the initial time diminution of 70 percent, the peak speed reclines to 28%. In addition, there are two inflection points in the upward branches of figures. In addition, the average incline of the upward branch related to each vector declines until reaching the efficient velocity, but the incline of the downward branch will increase by the time.

Figure (3) shows the depth velocity distribution in the longitudinal direction and for the vector passing through the middle bottom in different times. Figure (3) shows the depth velocity of a falling particle in the longitudinal direction and for the vector passing through the middle bottom in different times. As you see again in here, the negative depth velocity is obvious until reaching balance, and which shows the upward flow.

In addition, in a time when the depth velocity of the particle stays the same through most of the sections length. The reduction of particle's depth velocity as time passes, shows the suspension of the particle.

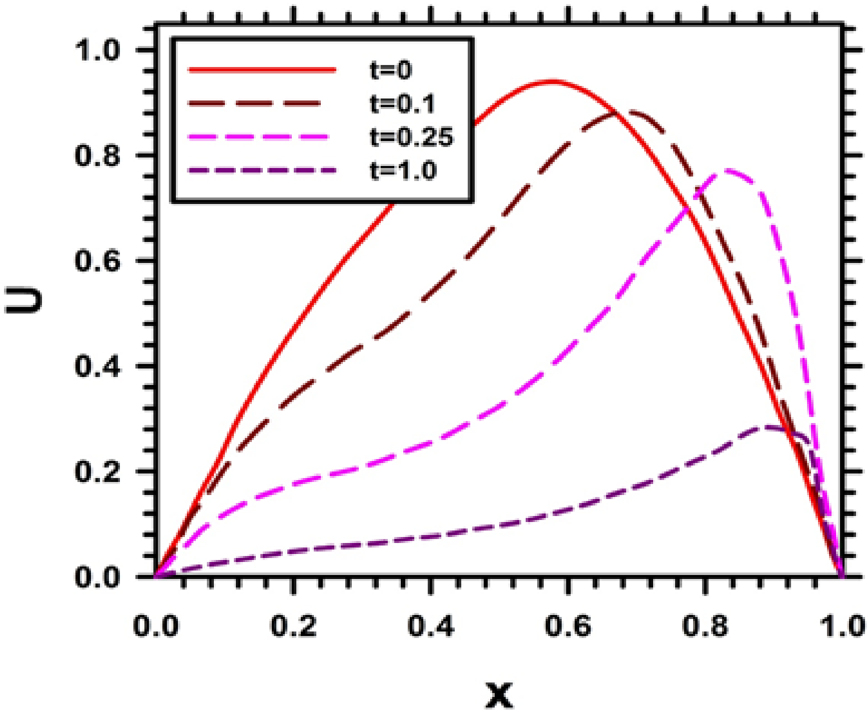

Figure 2: The Longitudinal Velocity Distribution Fall of a Particle for the Axis that goes Through Middle Bottom in Different Times (υ=0.05)

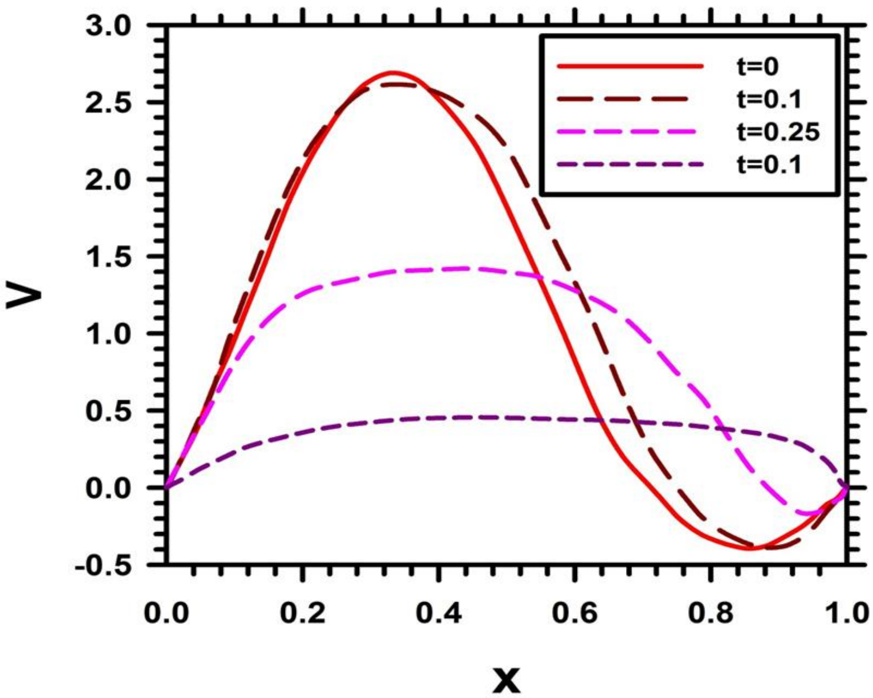

Figure 3: The Depth Velocity Distribution in the Longitudinal Direction and for the Vector Passing through the Middle Bottom in Different Times (υ=0.05)

Figure (4) shows the longitudinal velocity distribution of falling particle and for the vector passing through the middle length in different times. As it is clear, all curves are in triangle hydrograph forms, in addition, as time develops and reaching the settlement time, the peak point of each curve nears the bed bottom, in a way that in the settlement time, the peak velocity happens in the distance of 5 percent section depth from the bottom. The maximum velocity in the balance time in respect to initial time also incurred a reduction of 550 percent.

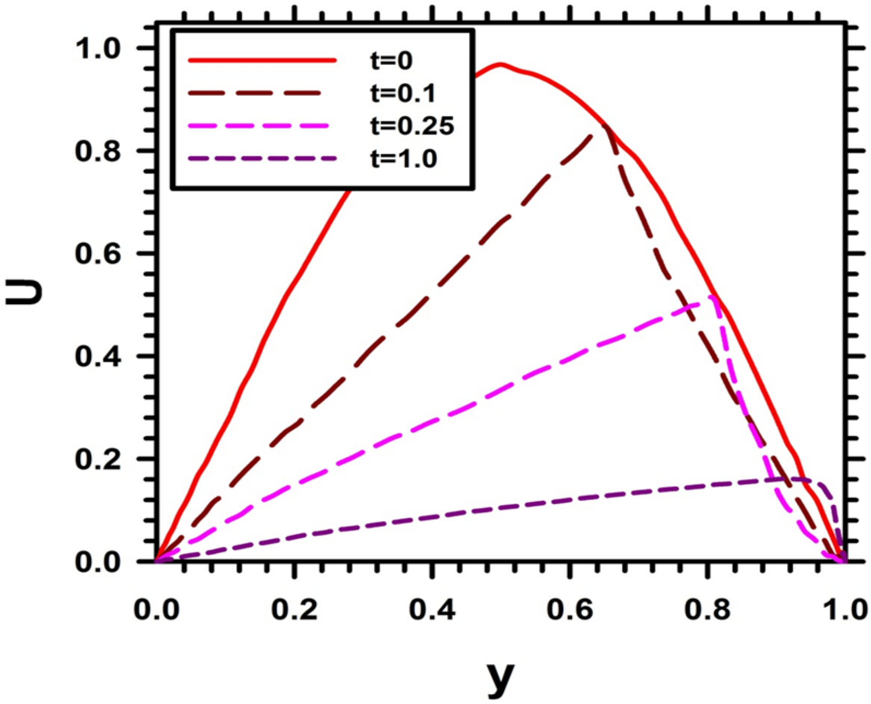

Figure 4: The distribution of Longitudinal Velocity of the Particle in the Depth Direction and for the Vector Passing through the Middle Length in Different Times (υ=0.05)

Figure (5) shows the depth velocity distribution of falling particle in the depth direction and for the axis passing through the middle of length. As you see, the amount of depth velocity in the depth direction increases as time develops from zero, as can be seen in the time of 0.1, the maximum depth velocity of the falling particle has the highest amount at the time of study. In addition, the negative velocities show that the upward stream in the distance of 35 percent of section depth from the bottom, that is a proof of the particles suspension during some intervals and during the time of study.

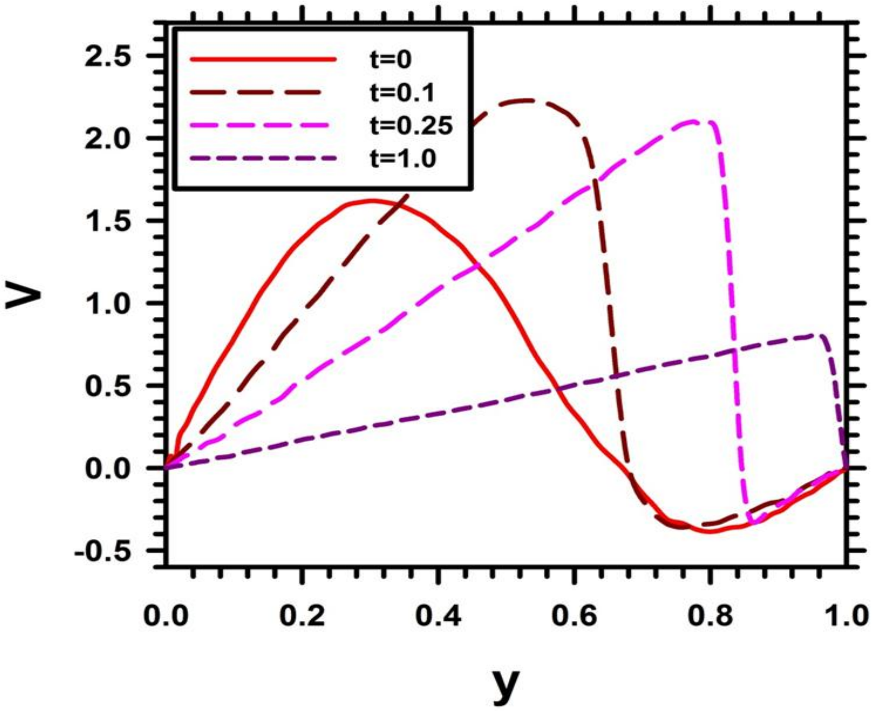

Figure 5: shows the depth velocity distribution of falling particle in the depth direction and for the axis passing through the middle length in different times

5. CONCLUTION

As time develops, the falling speed of particle in the length and depth direction decreases, which is higher for the depth direction. In some points of section especially near the bed and in time intervals, the depth velocity is negative and the stream is upwards which illustrates the suspension effect of particle.

In most of times, the skewness diagrams of length and depth velocity in depth direction to is to the left which shows the maximum velocity of falling particle at the time of study and near the bed. In addition, the finite difference method can determine the falling way of particle through Burger two dimensional equations.

The viscosity of less than 0.05 trying to solve the problem using the finite difference and achieve the desired results was achieved. Because of its low memory computer, and there is a very large matrices and the fact that there is a problem of two-dimensional non-linear.

6. REFERENCES

[1] Abdou, M. A., and Soliman, A. A., "Variational iteration method for solving Burger's and coupled Burger's equations", Journal of Computational and Applied Mathematics, Vol. 181, No. 2, pp. 245-251, 2005.

[2] Wazwaz, A. M., "Multiple kink solutions for M-component Burgers equations in (1+1)-dimensions and (2+1)-dimensions", Applied Mathematics and Computation, Vol. 217, No. 7, pp. 3564-3570, 2010.

[3] Wei, G. W., and Gu, Y., "Conjugate filter approach for solving Burgers' equation", Journal of Computational and Applied mathematics, Vol. 149, No. 2, pp. 439-456, 2002.

[4] Dağ, İ., Canıvar, A., and Şahin, A., "Taylor–Galerkin and Taylor-collocation methods for the numerical solutions of Burgers' equation using B-splines", Communications in Nonlinear Science and Numerical Simulation, Vol. 16, No. 7, pp. 2696-2708, 2011.

[5] Shao, L., Feng, X., and He, Y., "The local discontinuous Galerkin finite element method for Burger's equation. Mathematical and Computer Modelling", Vol. 54, No. 11, pp. 2943-2954, 2011.

[6] Soliman, A. A., "The Modified Extended Tanh-function method for solving Burgers-type equations", Physica A: Statistical Mechanics and its Applications, Vol. 361, No. 2, pp. 394-404, 2006.

[7] Haq, S., and Uddin, M., "A mesh-free method for the numerical solution of the KdV–Burgers equation", Applied Mathematical Modelling, Vol. 33, No. 8, pp. 3442-3449, 2009.

[8] Mittal, R. C., and Arora, G., "Numerical solution of the coupled viscous Burgers' equation", Communications in Nonlinear Science and Numerical Simulation, Vol. 16, No. 3, pp. 1304-1313, 2011.

[9] Vaghefi, M., Rahideh, H., khaksar, A., and Golbahar Haghighi, M.R., "Distributed approximating Functional approach to Burgers' equation using differential quadratuve method", Journal of Applied Sciences and Environmental Management, Vol. 16, No. 1, pp. 5-10, 2012.

[10] Hongyan, Z., and Hongqing, Z., "New ration solitary wave Solutions of (2+1)-dimensional Burger's equation", Nonlinear Analysis, Vol. 66, No. 34, pp. 2264-2273, 2007.

[11] Feng, S., Liang, P., and Jun, Z., "New Exact solution to (3+1) Dimensional Burger's Equation", Communications in Theoretical Physics (Beijing, China), Vol. 42, No. 26, pp. 49-50, 2004.

[12] Dai, C. Q., and Wang, Y. Y., "New exact solutions of the (3+1)-dimensional Burgers system", Physics Letters A, Vol. 373, No. 2, pp. 181-187, 2009.

[13] Christou, M. A., Ivanova, N. M., and Sophocleous, C., "Similarity reductions of the (1+3)-dimensional Burgers equation", Applied Mathematics and Computation, Vol. 210, No. 1, pp. 87-99, 2009.

[14] Christou, M. A., and Sophocleous, C., "Numerical similarity reductions of the (1+3)-dimensional Burgers equation", Applied Mathematics and Computation, Vol. 217, No. 18, pp. 7455-7461, 2011.

[15] Abdel Rady, A. S., Osman, E. S., and Khalfallah, M., "Multi-soliton solution, rational solution of the Boussinesq–Burgers equations", Communications in Nonlinear Science and Numerical Simulation, Vol. 15, No. 5, pp. 1172-1176, 2010.

[16] Dehghan, M., Nematisaray, B., and Lakestani, M., "Three Methods based on the interpolation scaling Functions and the mined colloction Finite Difference schemes for the numerical solution of the nonlinear generalized Burger's-Huxley equation", Mathematical and computer Modelling, Vol. 55, No. 21, pp. 1129-1142, 2012.

[17] Zhao, T., Li, C., Zang, Z., and Wu, Y., "Chebyshev–Legendre pseudo-spectral method for the generalised Burgers–Fisher equation", Applied Mathematical Modelling, Vol. 36, No. 3, pp. 1046-1056, 2012.

[18] Korpusov, M. O., "On the blow-up of solutions of the Benjamin–Bona–Mahony–Burgers and Rosenau–Burgers equations", Nonlinear Analysis: Theory, Methods & Applications, Vol. 75, No. 4, pp. 1737-1743, 2012.

[19] Haq, S., Hussain, A., and Uddin, M., "On the numerical solution of nonlinear Burgers'-type equations using meshless method of lines", Applied Mathematics and Computation, Vol. 218, No. 11, pp. 6280-6290.