Asian Journal of Engineering and Technology (ISSN: 2321 – 2462)
Volume 02 – Issue 05, October 2014

Self-Positioning of a Floating Platform

Hiroshi Isshiki[1] and Hideyuki Niizato[2]

[1] IMA (Institute of Mathematical Analysis, Osaka, Japan)
Osaka, Japan
Email: Isshiki {at} dab.hi-ho.ne.jp

[2] Business&Product Development Center, Environment, Energy & Plant Headquarters, Hitachi Zosen Corporation
Osaka, Japan

ABSTRACT— ***Self-positioning methods of a floating platform, namely methods to keep the position of a floating platform in current or wind without anchoring to the sea floor or without using externally supplied energy are discussed. Energy extracted from the current and/or wind is used for the self-positioning. More specifically, the energy is extracted from the current using the energy harvesting turbine and is used for the thrust generating turbine. The self-positioning using a method similar to the tucking of a sail boat is also possible using the thrust generating and energy harvesting turbines.***

Keywords— Self-Positioning, Floating body and ship, Current and wind, Turbine and sail, Navigation

---

1. INTRODUCTION

If we wish to keep the position of a floating platform in current and/or wind not using externally supplied energy, we usually anchor the platform to the sea floor. In the present paper, this kind of energy-free positioning is called self-positioning. Since the ocean current such as Kuroshio current meanders [1], the anchoring at a fixed place does not make sense, if the position relative to the current is important. Furthermore, the water depth where Kuroshio flows is very deep in general. This also makes the anchoring unrealistic.

A yacht can move against a wind by tucking. Hence, in an ocean current like Kuroshio, a ship can move against the current, if we place the sail in the current. This means that a floating platform can maintain the mean position in wind and current without anchoring the platform. In the present paper, we focus on the positioning of a platform used for the energy harvesting. In the case of an observation platform, the problem is much simpler.

There are two kinds of such positioning. One is to use a part of harvested energy to generate thrust to cancel the drag of the energy harvester, and the other not to use the energy harvesting turbine. The former will be realized by propeller [2,3,4] and the latter by sail. In the present paper, the principle and feasibility of such positioning are discussed.

We also discussed that the self-positioning using a method similar to the tucking of a sail boat is also possible using the thrust generating and energy harvesting turbines.

2. SELF-POSITIONING OF A FLOATING PLATFORM IN AN OCEAN CURRENT

In order to realize self-positioning of a floating platform in an ocean current, we can use a sail or propeller. The characteristics are summarized in Table 1. A tucking system is shown in Figure 1 and seems realistic as can be understood from our experience of a sail boat, but the thrusting system seems unrealistic because the water density is much bigger than the air density. This suggests that such a system is unrealistic. As shown below, if we place the propeller in sea, the power input to the propeller is bigger than the energy harvested from the current when the thrust of the propeller cancels the drag of the energy harvester. If we place the propeller in air, the diameter of the propeller becomes much bigger than that of the energy harvesting turbine in sea.

Table 1: Device for cancelling drag of energy harvesting turbine from sea current

	Tucking	Thrusting	Note
Thrust generating device	Sail	Propeller	
Place of device	Sea	Air	If thruster placed in sea, not possible theoretically
Energy supply from energy harvestor	No	Yes	
Size of thruster	Order of energy harvesting turbine	Possible, but more than 10 times bigger	The water density is much bigger than air density.

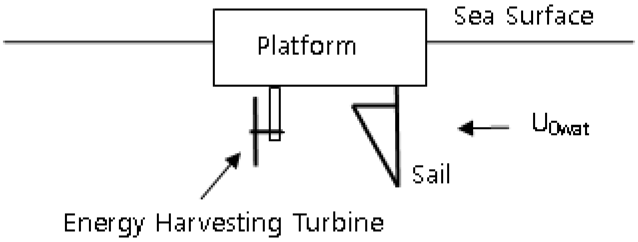

Figure 1: Self-Positioning of sea current generator

In the following, we verify that we can't realize self-positioning using the energy harvested from the sea current, if we place the propeller in the sea.

2.1. A case when the propeller is placed in sea

As shown in Figure 2, if we place the propeller in the sea, it is shown theoretically that the self-positioning can't be realized.

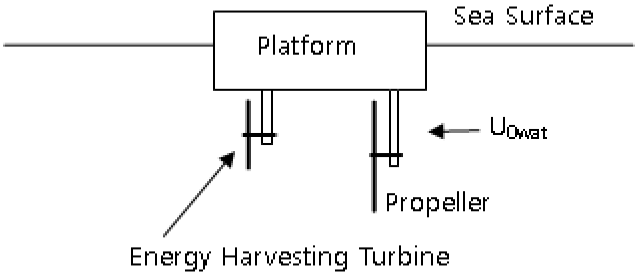

Figure 2: Positioning of sea current generator

Momentum theories of an energy harvesting turbine and thrust producing turbine are given in Appendix (A) and (B)

From Eqs. (B9) and (B10) in Appendix B, the thrust T_{thr} and power P_{thr} of the thrust producing turbine or propeller with the actuator disk area A_{thr} and downstream velocity u_{1thr} in the current with velocity U_{0wat} are given by

$$T_{thr} = \frac{1}{2}\rho_{wat}A_{thr}\left(u_{1thr}{}^2 - U_{0wat}{}^2\right), \tag{1}$$

$$P_{thr} = \frac{1}{2}\rho_{wat}\left(u_{1thr}{}^2 - U_{0wat}{}^2\right)u_{thr}A_{thr} = \frac{1}{4}\rho_{wat}\left(u_{1thr}{}^2 - U_{0wat}{}^2\right)\left(U_{0wat} + u_{1thr}\right)A_{thr}$$
$$= \frac{1}{4}\rho_{wat}A_{thr}U_{0wat}{}^3\left(\left(\frac{u_{1thr}}{U_{0wat}}\right)^2 - 1\right)\left(1 + \frac{u_{1thr}}{U_{0wat}}\right). \tag{2}$$

From Eqs. (A9) and (A10) in Appendix A, we have for the drag D_{har} and power P_{har} of the energy harvesting turbine with the actuator disk area A_{har} and downstream velocity u_{1har}

$$D_{har} = \frac{1}{2}\rho_{wat}A_{har}\left(U_{0wat}{}^2 - u_{1har}{}^2\right), \tag{3}$$

$$P_{har} = \frac{1}{2}\rho_{wat}\left(U_{0wat}{}^2 - u_{1har}{}^2\right)u_{wat}A_{har} = \frac{1}{4}\rho_{wat}\left(U_{0wat}{}^2 - u_{1har}{}^2\right)\left(U_{0wat} + u_{1har}\right)A_{har}$$
$$= \frac{1}{4}\rho_{wat}A_{har}U_{0wat}{}^3\left(1 - \left(\frac{u_{1har}}{U_{0wat}}\right)^2\right)\left(1 + \frac{u_{1har}}{U_{0wat}}\right). \tag{4}$$

Using the condition that the thrust of the propeller cancels the drag of the harvester, we obtain

$$T_{thr} = D_{har} \text{ or } \frac{1}{2}\rho_{wat}A_{thr}\left(u_{1thr}{}^2 - U_{0wat}{}^2\right) = \frac{1}{2}\rho_{wat}A_{wat}\left(U_{0wat}{}^2 - u_{1har}{}^2\right). \tag{5}$$

Hence, we derive

$$u_{1thr}{}^2 = U_{0wat}{}^2 + \frac{A_{har}}{A_{thr}}\left(U_{0wat}{}^2 - u_{1har}{}^2\right) \text{ or } \frac{u_{1thr}}{U_{0wat}} = \sqrt{1 + \frac{A_{wat}}{A_{thr}}\left(1 - \left(\frac{u_{1har}}{U_{0wat}}\right)^2\right)}. \tag{6}$$

Substituting Eq. (6) into Eq. (4). We obtain

$$P_{thr}=\frac{1}{4}\rho_{wat}A_{har}U_{0wat}^{3}\left(1-\left(\frac{u_{1har}}{U_{0wat}}\right)^{2}\right)\left(1+\sqrt{1+\frac{A_{wat}}{A_{thr}}\left(1-\left(\frac{u_{1har}}{U_{0wat}}\right)^{2}\right)}\right). \quad (7)$$

Then, Eqs. (7) and (4) give

$$\frac{P_{thr}}{P_{wat}}=\frac{1+\sqrt{1+\frac{A_{har}}{A_{thr}}\left(1-\left(\frac{u_{1har}}{U_{0wat}}\right)^{2}\right)}}{1+\frac{u_{1har}}{U_{0wat}}}. \quad (8)$$

Since u_{1har} is smaller than U_{0wat}, we obtain

$$\frac{P_{thr}}{P_{har}}\geq 1. \quad (9)$$

Hence, if we place the propeller in the sea, the self-positioning using the energy harvested from the sea current is impossible. We may try to realize the energy harvesting and thrust generation in one turbine placed in the sea. The self-positioning by the turbine is also impossible as explained in Appendix C.

2.2. A case when the propeller is placed in air

As shown in Figure 3, if we place the propeller in the air, it is shown theoretically that the self-positioning is possible but unrealistic.

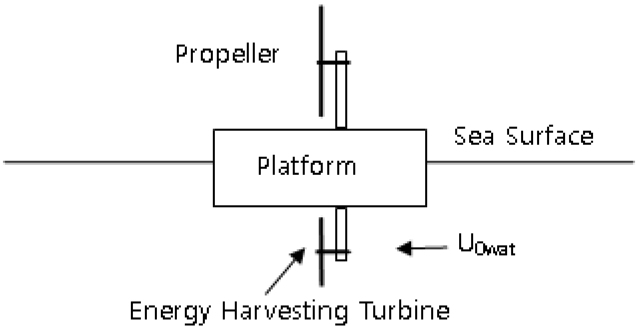

Figure 3: Positioning of sea current generator

If we assume U_{0air} is zero, then, from Eqs. (A9) and (B16), we have

$$D_{har}=\frac{1}{2}\rho_{wat}A_{har}\left(U_{0wat}^{2}-u_{1har}^{2}\right), \quad (10)$$

$$T_{thr}=\frac{1}{2}\rho_{air}A_{thr}u_{1thr}^{2}. \quad (11)$$

From the condition of the self-positioning, we require

$$D_{har}=T_{thr}\ \text{ or }\ \frac{1}{2}\rho_{wat}A_{wat}\left(U_{0wat}^{2}-u_{1har}^{2}\right)=\frac{1}{2}\rho_{air}A_{thr}u_{1thr}^{2}. \quad (12)$$

From Eq. (12), we obtain

$$u_{1thr}=\sqrt{\frac{\rho_{wat}A_{har}}{\rho_{air}A_{thr}}\left(U_{0wat}^{2}-u_{1har}^{2}\right)}. \quad (13)$$

Substituting Eq. (13) into Eq. (B18), we have

$$P_{thr}=\frac{1}{4}\rho_{air}A_{thr}u_{1thr}^{3}=\frac{1}{4}\rho_{air}A_{thr}\left[\sqrt{\frac{\rho_{wat}A_{har}}{\rho_{air}A_{thr}}\left(U_{0wat}^{2}-u_{1har}^{2}\right)}\right]^{3}. \quad (14)$$

Finally, from Eqs. (14) and (A10), we obtain

$$\frac{P_{thr}}{P_{har}}=\frac{\frac{1}{4}\rho_{air}A_{thr}u_{1thr}^{3}}{\frac{1}{4}\rho_{wat}\left(U_{0wat}^{2}-u_{1har}^{2}\right)\left(U_{0wat}+u_{1har}\right)A_{har}}=\frac{\frac{1}{4}\rho_{air}A_{thr}\left[\sqrt{\frac{\rho_{wat}A_{har}}{\rho_{air}A_{thr}}\left(U_{0wat}^{2}-u_{1har}^{2}\right)}\right]^{3}}{\frac{1}{4}\rho_{wat}\left(U_{0wat}^{2}-u_{1har}^{2}\right)\left(U_{0wat}+u_{1har}\right)A_{har}}$$

$$= \frac{1}{1+\dfrac{u_{1har}}{U_{0wat}}}\sqrt{\frac{\rho_{wat}A_{har}}{\rho_{air}A_{thr}}\left(1-\left(\frac{u_{1har}}{U_{0wat}}\right)^2\right)}. \tag{15}$$

For the self-positioning, we require

$$\frac{P_{thr}}{P_{har}} < 1. \tag{16}$$

The relationship between power ratio P_{thr}/P_{har} and velocity ratio u_{1har}/U_{0wat} is shown in Figure 4, where a parameter X is given by

$$X = \frac{\rho_{air}A_{thr}}{\rho_{wat}A_{har}}. \tag{17}$$

Since the water density is about 1000 times bigger than the air density. The actuator area A_{thr} of wind turbine must be very big. This suggests that the self-positioning is possible but almost unrealistic in this system.

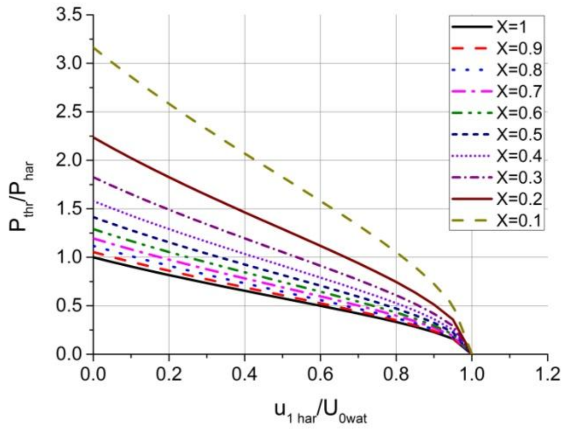

Figure 4: Relationship between power ratio P_{thr}/P_{har} and velocity ratio u_{1har}/U_{0wat}

(ρ_{wat}=1035 kg/m^3, ρ_{air}=1.205kg/m^3)

From Eq. (A12), the maximum efficiency of the energy harvesting turbine is obtained at

$$a_{har} = \frac{1}{3}. \tag{18}$$

From Eq. (A8), we have

$$\frac{u_{1har}}{U_{0wat}} = (1-2a_{har}) = \frac{1}{3}. \tag{19}$$

Substituting this into Eq. (15), we obtain

$$\frac{P_{thr}}{P_{har}} = \frac{1}{1+\dfrac{u_{1har}}{U_{0wat}}}\sqrt{\frac{\rho_{wat}A_{har}}{\rho_{air}A_{thr}}\left(1-\left(\frac{u_{1har}}{U_{0wat}}\right)^2\right)} = \frac{3}{4}\sqrt{\frac{\rho_{wat}A_{har}}{\rho_{air}A_{thr}}\frac{8}{9}} = \frac{1}{\sqrt{2}}\sqrt{\frac{\rho_{wat}A_{har}}{\rho_{air}A_{thr}}}. \tag{20}$$

The graph of P_{thr}/P_{har} versus $\sqrt{A_{thr}/A_{har}}$ at maximum efficiency of energy harvesting turbine is shown in Figure 5. The size of a wind thruster must be very big, if we want to make the power ratio P_{thr}/P_{har} small. However, if P_{har} is small as in the case of an observation platform, the self-positioning may become realistic.

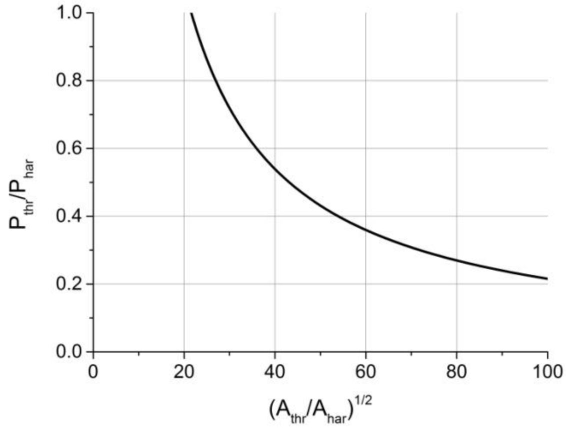

Figure 5: Relationship between the size of a wind thruster and the input power (ρ_{wat}=1035 kg/m^3, ρ_{air}=1.205kg/m^3)

2.3. Tucking using thrust generating and energy harvesting turbines

Now, we discuss how to realize tucking using rotating wings instead of a sail. Let's consider a platform or boat moving obliquely to the flow U_0 as shown in Figure 6. For example, a boat is crossing a river flowing with velocity U_0 as shown in Figure 7. The geometrical direction of the platform or boat is specified by the angle θ. The velocity vector of the platform or boat is denoted by (U_P, V_P), where U_P and V_P are the components parallel and orthogonal to the direction of the platform or boat, respectively.

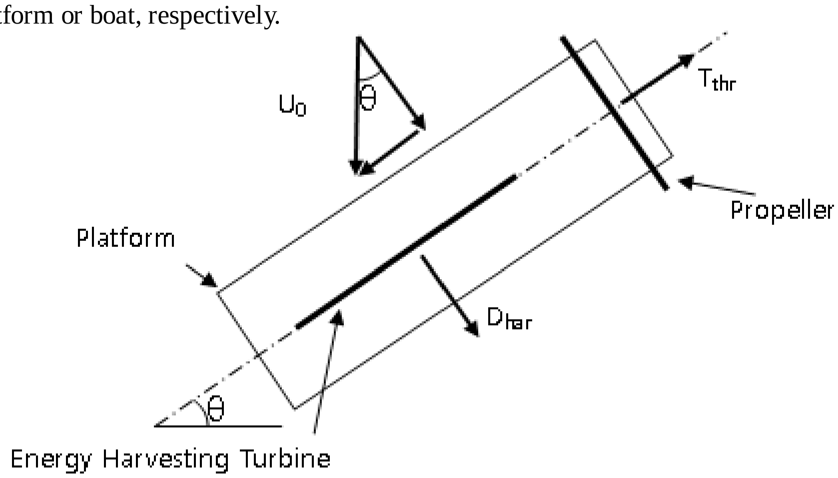

Figure 6: Platform moving obliquely to the flow

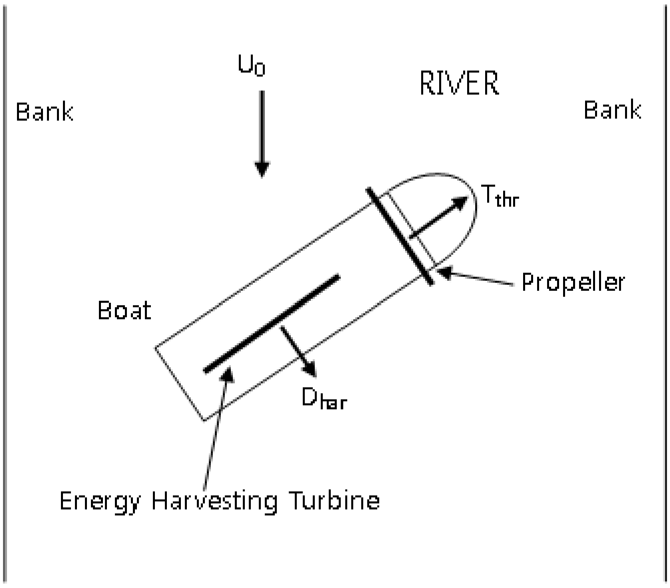

Figure 7: Ship crossing a river obliquely to the flow

Now, the inflow velocities $U_{0\,thr}$ and $U_{0\,har}$ to the propeller and energy harvesting turbine are given by

$$U_{0\,thr} = U_0 \sin\theta + U_P,\; U_{0\,har} = U_0 \cos\theta - V_P. \qquad (21a,b)$$

Applying Eqs. (B9) and (A9), we have for the thrust T_{thr} of the propeller and drag D_{har} of the energy harvesting turbine

$$T_{thr} = \frac{1}{2}\rho_{wat}A_{thr}\left(u_{1\,thr}{}^2 - U_{0\,thr}{}^2\right) = \frac{1}{2}\rho_{wat}A_{thr}\left(u_{1\,thr}{}^2 - (U_0 \sin\theta + U_P)^2\right), \qquad (22a)$$

$$D_{har} = \frac{1}{2}\rho_{wat}A_{har}\left(U_{0\,har}{}^2 - u_{1\,har}{}^2\right) = \frac{1}{2}\rho_{wat}A_{har}\left((U_0 \cos\theta - V_P)^2 - u_{1\,har}{}^2\right), \qquad (22b)$$

respectively. From Eqs. (B10) and (A10), the input power T_{thr} to the propeller and output power from the energy harvesting turbine are given by

$$P_{thr} = \frac{1}{2}\rho_{wat}\left(u_{1\,thr}{}^2 - U_{0\,thr}{}^2\right)u_{thr}A_{thr} = \frac{1}{2}\rho_{wat}\left(u_{1\,thr}{}^2 - (U_0 \sin\theta + U_P)^2\right)u_{thr}A_{thr}, \qquad (23a)$$

$$P_{har} = \frac{1}{4}\rho_{wat}\left(U_{0\,har}{}^2 - u_{1\,har}{}^2\right)\left(U_{0\,har} + u_{1\,har}\right)A_{har} = \frac{1}{4}\rho_{wat}\left((U_0 \cos\theta - V_P)^2 - u_{1\,har}{}^2\right)\left((U_0 \cos\theta - V_P) + u_{1\,har}\right)A_{har}, \qquad (23b)$$

respectively. The equations of the motion of the platform or boat would be obtained as

$$M\frac{dU_P}{dt} = T_{thr} - \frac{1}{2}C_{D\,thr}\rho_{wat}S_{thr}(U_0 \sin\theta + U_P)^2, \qquad (24a)$$

$$M\frac{dV_P}{dt} = D_{har} - \frac{1}{2}C_{D\,har}\rho_{wat}S_{har}(U_0 \cos\theta - V_P)^2, \qquad (24b)$$

where $C_{D\,thr}$ and $C_{D\,har}$ are the drag force coefficients of the platform or boat in the longitudinal and perpendicular directions. The added mass and lift forces acting on the platform are neglected.

If we introduce the parameters α_{thr} and α_{har} defined as

$$u_{1\,thr} = \alpha_{thr}(U_0 \sin\theta + U_P),\; u_{1\,har} = \alpha_{har}(U_0 \cos\theta - V_P), \qquad (25a,b)$$

Eqs. (22-24) are rewritten as

$$T_{thr} = \frac{1}{2}\rho_{wat}A_{thr}(U_0 \sin\theta + U_P)^2\left(\alpha_{thr}{}^2 - 1\right),\; D_{har} = \frac{1}{2}\rho_{wat}A_{har}(U_0 \cos\theta - V_P)^2\left(1 - \alpha_{har}{}^2\right), \qquad (26a,b)$$

$$P_{thr} = \frac{1}{2}\rho_{wat}(U_0 \sin\theta + U_P)^3\left(\alpha_{thr}{}^2 - 1\right)\alpha_{thr}A_{thr},\; P_{har} = \frac{1}{4}\rho_{wat}(U_0 \cos\theta - V_P)^3\left(1 - \alpha_{har}{}^2\right)(1 + \alpha_{har})A_{har}, \qquad (27a,b)$$

$$M\frac{dU_P}{dt} = \frac{1}{2}\rho_{wat}(U_0 \sin\theta + U_P)^2\left[A_{thr}\left(\alpha_{thr}{}^2 - 1\right) - C_{D\,thr}S_{thr}\right], \qquad (28a)$$

$$M\frac{dV_P}{dt} = \frac{1}{2}\rho_{wat}(U_0 \cos\theta - V_P)^2\left[A_{har}\left(1 - \alpha_{har}{}^2\right) - C_{D\,har}S_{har}\right]. \qquad (28b)$$

If we consider the steady state:

$$dU_P/dt = 0\,,\ dV_P/dt = 0\,, \tag{29a,b}$$

then, we have

$$A_{thr}\left(\alpha_{thr}{}^2 - 1\right) - C_{D\,thr}S_{thr} = 0\,,\ A_{har}\left(1 - \alpha_{har}{}^2\right) - C_D S_{har} = 0\,. \tag{30a,b}$$

Hence, if α_{thr}, α_{har}, $C_{D\,thr}$, $C_{D\,har}$, S_{har} and S_{thr} are given, then, we can determine A_{thr} and A_{har} using

$$A_{thr} = \frac{C_{D\,thr}S_{thr}}{\alpha_{thr}{}^2 - 1}\,,\ A_{har} = \frac{C_{D\,har}S_{har}}{1 - \alpha_{har}{}^2}\,. \tag{31a,b}$$

Furthermore, if we move the platform or boat using only the power harvested from the current and require:

$$P_{har} = P_{thr}\,, \tag{32}$$

we have from Eq. (27)

$$\left(U_0\cos\theta - V_P\right)^3\left(1 - \alpha_{har}{}^2\right)\left(1 + \alpha_{har}\right)A_{har} = 2\left(U_0\sin\theta + U_P\right)^3\left(\alpha_{thr}{}^2 - 1\right)\alpha_{thr}A_{thr} \tag{33}$$

or

$$V_P = -\left(U_0\sin\theta + U_P\right)\left(\frac{2\left(\alpha_{thr}{}^2 - 1\right)\alpha_{thr}A_{thr}}{\left(1 - \alpha_{har}{}^2\right)\left(1 + \alpha_{har}\right)A_{har}}\right)^{\frac{1}{3}} + U_0\cos\theta\,. \tag{34}$$

Hence, if U_0, θ and U_P are given, we can determine V_P from Eq. (34). This shows us how to realize tucking with the thrust generating and energy harvesting turbines.

The numerical examples are shown below. The parameters used for the calculation in the first example are $\alpha_{thr} = 1.2$, $\alpha_{har} = 0.667$, $C_{D\,thr} = 0.1$, $C_{D\,har} = 1.0$, $S_{thr} = 50.0\mathrm{m}^2$, $S_{har} = 500\mathrm{m}^2$, $\theta = 45\mathrm{deg}$ and $U_P = 4\mathrm{m/s}$. In this case, we obtained $A_{thr} = 11.36m^2$ and $A_{har} = 900.7m^2$. The effects U_0 on V_P and $\mathrm{atan}(V_P/U_P)$ are shown in Figure 8. When $U_0 \approx 9.5\mathrm{m/s}$, $\mathrm{atan}(V_P/U_P)$ becomes about 45degs. Hence, very interestingly, the platform or boat moves perpendicular to the current with angle θ 45degs.

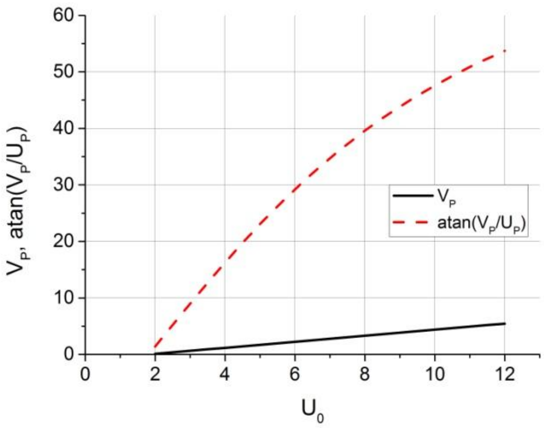

Figure 8: Effects U_0 on V_P and $\mathrm{atan}(V_P/U_P)$

In the second example, we assume $C_{D\,har} = 0.5$ and $S_{har} = 100\mathrm{m}^2$, and the other data same. In this case, we obtained $A_{thr} = 11.36m^2$ and $A_{har} = 90.07m^2$. The effects U_0 on V_P and $\mathrm{atan}(V_P/U_P)$ are shown in Figure 9. When $U_0 \approx 18.3\mathrm{m/s}$, $\mathrm{atan}(V_P/U_P)$ becomes about 45degs. Hence, the platform or boat moves perpendicular to the current with angle θ 45degs very interestingly.

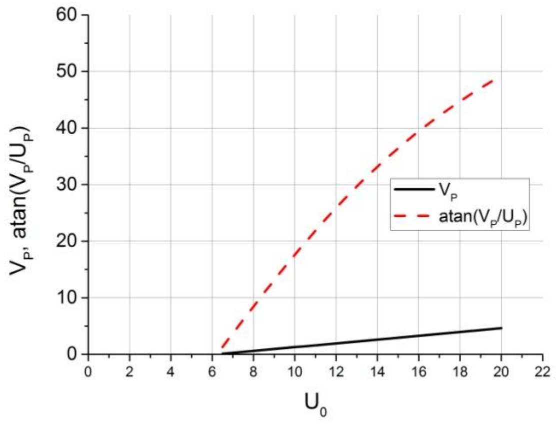

Figure 9: Effects U_0 on V_P and $\text{atan}(V_P/U_P)$

3. POSITIONING OF A FLOATING PLATFORM IN WIND

In order to realize self-positioning of a floating platform in a wind, we can use a sail or propeller. The characteristics are summarized in Table 2. A tucking system is shown in Figure 10 and seems realistic as can be understood from our experience of a sail boat, and the thrusting system also seems realistic because the air density is much smaller than the water density. This suggests that such a system is realistic. As shown below, if we place the propeller in sea, the power input to the propeller is smaller than the energy harvested from the wind when the thrust of the propeller cancels the drag of the energy harvester. If we place the propeller in sea, the diameter of the propeller becomes much smaller than that of the energy harvesting turbine in air. Hence, if the water depth is very big and anchoring is not realistic, the self-positioning would be competitive.

Table 2: Device for cancelling drag of energy harvesting turbine from air flow

	Tucking	Thruster	Note
Thrust generating device	Sail	Propeller	
Place of device	Air	Sea	If thruster placed in air, not possible theoretically
Energy supply from generator	No	Yes	
Size of thruster	Order of energy harvesting turbine	Possible, 1/10 smaller than windmill	The air density is much smaller than water density.

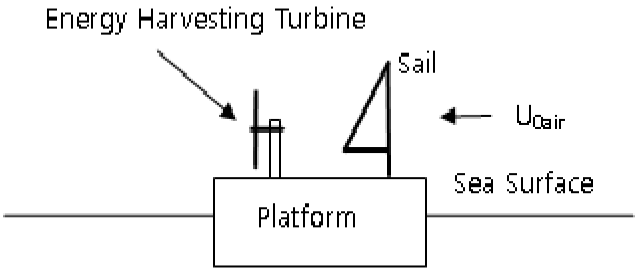

Figure 10: Positioning of wind generator

As shown in Figure 11, if we place the sail by the propeller in the air, it is shown theoretically that the self-positioning can't be realized as already shown in section 2.1.

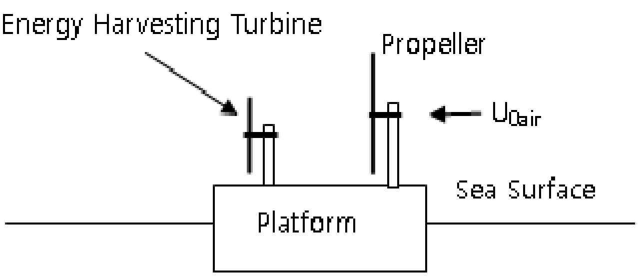

Figure 11: Positioning of sea current generator

As shown in Figure 12, if we place the propeller in the sea, it is shown theoretically that the self-positioning is possible and realistic.

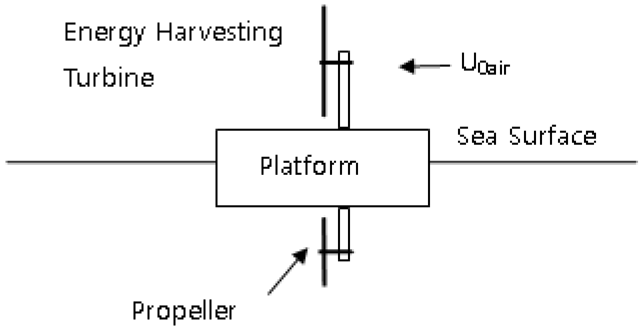

Figure 12: Positioning of sea current generator

If we assume U_{0wat} is zero, then, from Eqs. (A9) and (B16), we have

$$D_{har} = \frac{1}{2}\rho_{air}A_{har}\left(U_{0air}{}^{2} - u_{1\,har}{}^{2}\right),\ T_{thr} = \frac{1}{2}\rho_{wat}A_{thr}u_{1\,thr}{}^{2}\,. \qquad (35,36)$$

From the condition of the self-positioning, we require

$$D_{har} = T_{thr} \text{ or } \frac{1}{2}\rho_{air}A_{har}\left(U_{0air}{}^{2} - u_{1\,har}{}^{2}\right) = \frac{1}{2}\rho_{wat}A_{thr}u_{1\,thr}{}^{2}\,. \qquad (37)$$

From Eq. (37), we obtain

$$u_{1\,thr} = \sqrt{\frac{\rho_{air}A_{har}}{\rho_{wat}A_{thr}}\left(U_{0air}{}^{2} - u_{1\,har}{}^{2}\right)}\,. \qquad (38)$$

Substituting Eq. (38) into Eq. (B18), we have

$$P_{thr} = \frac{1}{4}\rho_{wat}A_{thr}u_{1\,thr}{}^{3} = \frac{1}{4}\rho_{wat}A_{thr}\left[\sqrt{\frac{\rho_{air}A_{har}}{\rho_{wat}A_{thr}}\left(U_{0air}{}^{2} - u_{1\,har}{}^{2}\right)}\right]^{3}\,. \qquad (39)$$

Finally, from Eqs. (39) and (A10), we obtain

$$\frac{P_{thr}}{P_{har}} = \frac{\frac{1}{4}\rho_{wat}A_{thr}\left[\sqrt{\frac{\rho_{air}A_{har}}{\rho_{wat}A_{thr}}\left(U_{0air}{}^{2} - u_{1\,har}{}^{2}\right)}\right]^{3}}{\frac{1}{4}\rho_{air}\left(U_{0air}{}^{2} - u_{1\,har}{}^{2}\right)\left(U_{0air} + u_{1\,har}\right)A_{har}} = \frac{1}{1 + \frac{u_{1\,har}}{U_{0air}}}\sqrt{\frac{\rho_{air}A_{har}}{\rho_{wat}A_{thr}}\left(1 - \left(\frac{u_{1\,har}}{U_{0air}}\right)^{2}\right)}\,. \qquad (40)$$

For the self-positioning, we require

$$\frac{P_{thr}}{P_{har}} \le 1\,. \qquad (41)$$

The relationship between power ratio P_{thr}/P_{har} and velocity ratio $u_{1\,har}/U_{0air}$ is shown in Figure 13, where a parameter X is given by

$$X = \frac{\rho_{wat}A_{thr}}{\rho_{air}A_{har}}\,. \qquad (42)$$

Since the air density is about 1/1000 times smaller than the water density. The actuator area A_{thr} of energy thrusting turbine becomes very small. This suggests that the self-positioning is possible and very realistic in this case.

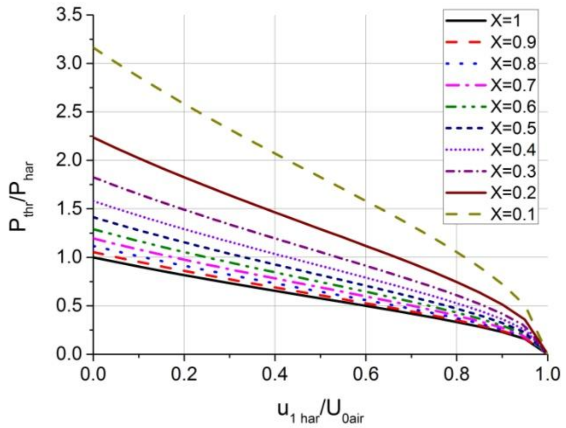

Figure 13: Relationship between power ratio P_{thr}/P_{har} and velocity ratio $u_{1\,har}/U_{0air}$

(ρ_{wat}=1035 kg/m^3, ρ_{air}=1.205kg/m^3)

From Eq. (A12), the maximum efficiency of the energy harvesting turbine is obtained at

$$a_{har} = \frac{1}{3}. \tag{43}$$

From Eq. (A8), we have

$$\frac{u_{1\,har}}{U_{0air}} = (1 - 2a_{har}) = \frac{1}{3}. \tag{44}$$

Substituting Eq. (44) into Eq. (40), we obtain

$$\frac{P_{thr}}{P_{har}} = \frac{1}{\sqrt{2}}\sqrt{\frac{\rho_{air}A_{har}}{\rho_{wat}A_{thr}}}. \tag{45}$$

As shown in Figure 14, the size of a propeller can be small, even if we want to make the power ratio P_{thr}/P_{har} small.

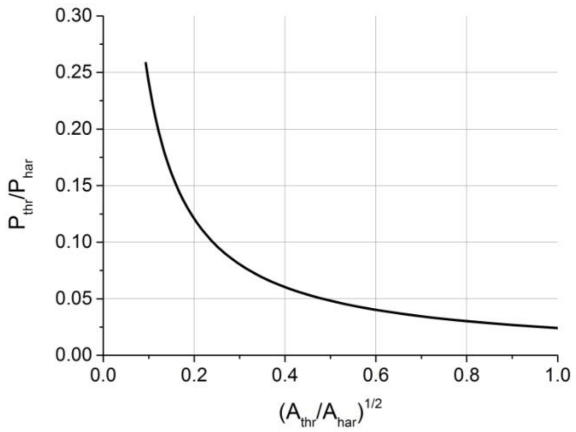

Figure 14: Relationship between the size of a wind thruster and the input power

(ρ_{wat}=1035 kg/m^3, ρ_{air}=1.205kg/m^3)

4. CONCLUSIONS

We usually anchor the platform to the sea floor to keep the position of the platform. Since the ocean current such as Kuroshio current meanders, the anchoring at a fixed place does not make sense, if the position relative to the current is important. Furthermore, the water depth where Kuroshio flows is very deep in general. This also makes the anchoring unrealistic. In the present paper, the position keeping of a floating platform in current and/or wind not using externally supplied energy or self-positioning was discussed.

A yacht can move against a wind by tucking. Hence, in an ocean current like Kuroshio, a ship can move against the current, if we place the sail in the current. This means that a floating platform can maintain the position in wind and current without anchoring the platform. In the present paper, we focused on the positioning of a platform used for the energy harvesting. In case of an observation platform, the problem is much simpler.

There are two kinds of such positioning. One is to use a part of harvested energy to generate thrust to cancel the drag of the energy harvester, and the other not to use the energy harvesting turbine. The former will be realized by propeller and the latter by sail. In the present paper, the principle and feasibility of such positioning were discussed.

If we want to realize the self-positioning in a sea current, the former method is unrealistic because of the too big difference of the densities between the water and the air. However, if the energy extracted from water flow is small as in the case of an observation platform, the self-positioning may become realistic. Very interestingly, we showed that the self-positioning using a method similar to the tucking of a sail boat is also possible using the thrust generating and energy harvesting turbines are possible.

If we want to realize the self-positioning in a wind, the both methods are realistic, since the big difference of the densities between the water and the air favors the realization of the self-positioning.

5. ACKNOWLEDGEMENTS

The cooperation of Prof. T. Kinoshita of Nihon University is highly acknowledged. His judgement based on his long career as a yacht man was very valuable.

6. REFERENCES

[1] Wikipedia, Kuroshio current
http://en.wikipedia.org/wiki/Kuroshio_Current
[2] Bose, N., Windmills - Propulsion for a hydrofoil trimaran, Symp. on Wind Propulsion of Commercial Ships, RINA, 1980
[3] Bose, N., The design of a wind turbine propulsion system, Int. Symp. on Ship Hyrodynamics and Energy Saving (ISSHES-83), (1998), pp. IX.1-1 1-11.
http://www.cehipar.es/__files/users/publicaciones/117.pdf
[4] Eirik Bøckmann1, Sverre Steen1, Wind Turbine Propulsion of Ships, Second International Symposium on Marine Propulsors smp'11, Hamburg, Germany, June (2011).
http://www.marinepropulsors.com/smp/files/downloads/smp11/Paper/FA1-2_Bockmann.pdf
[5] Wikipedia, Betz's law
http://en.wikipedia.org/wiki/Betz's_law
[6] Wikipedia, Blade element momentum theory
http://en.wikipedia.org/wiki/Blade_element_momentum_theory
[7] Wikipedia, Momentum theory
http://en.wikipedia.org/wiki/Momentum_theory
[8] MIT, 11.7 Performance of Propellers - MIT
http://web.mit.edu/16.unified/www/FALL/thermodynamics/notes/node86.html

APPENDIX A. MOMENTUM THEORY OF AN ENERGY HARVESTING TURBINE [5,6]

In the following, we summarize Betz's momentum theory on a windmill [,]. Betz obtained a limit of the theoretical efficiency called Betz limit. He treated the rotor of the windmill as an actuator disk as shown in Figure 1. He applied the conservation laws of mass, momentum and energy to obtain his theory.

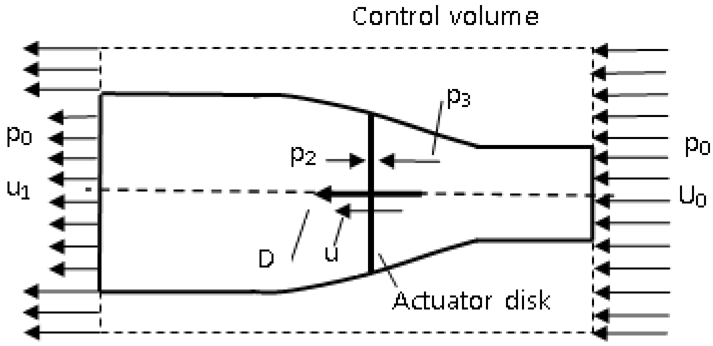

Figure A1: Energy harvesting turbine

From the continuity law, we have

$U_0 A_0 = u_1 A_1 = uA$, (A1)

where A_0, A and A_1 are the areas of the upstream inlet, actuator disk and downstream outlet, respectively. U_0, u and u_1 are the axial velocities in A_0, A and A_1, respectively. The pressures in A_0, A^-, A^+ and A_1 are denoted by p_0, p_2, p_3 and p_1, respectively. A^- and A^+ refer to the suction and pressure sides of the actuator disk, respectively. D is a drag forve acting on the actuator disk.

Applying the conservation of momentum, we have

$$D = \rho\left(U_0^2 A_0 - u_1^2 A_1\right) = \rho u_1 A_1 (U_0 - u_1), \quad \text{(A2)}$$

where ρ is the density of fluid.

Conservation of Energy or Bernoulli's theorem gives

$$\frac{1}{2}\rho U_0^2 + p_0 = \frac{1}{2}\rho u^2 + p_3, \ \frac{1}{2}\rho u^2 + p_2 = \frac{1}{2}\rho u_1^2 + p_0. \quad \text{(A3a,b)}$$

Adding Eqs. (A3a) and (A3b), we obtain

$$p_3 - p_2 = \frac{1}{2}\rho\left(U_0^2 - u_1^2\right). \quad \text{(A4)}$$

Then, we have

$$D = A(p_3 - p_2) = \frac{1}{2}\rho A\left(U_0^2 - u_1^2\right). \quad \text{(A5)}$$

From Eqs. (A1), (A2) and (A5), we obtain

$$u = \frac{1}{2}(U_0 + u_1). \quad \text{(A6)}$$

Introducing the axial induction coefficient a:

$$a = \frac{U_0 - u}{U_0} = 1 - \frac{u}{U_0} \ \text{or} \ u = (1-a)U_0, \quad \text{(A7)}$$

we obtain from Eqs. (A6) and (A7)

$$u_1 = (1-2a)U_0. \quad \text{(A8)}$$

Substituting Eq. (A8) into Eq. (A5), the drag D is given by

$$D = \frac{1}{2}\rho A\left(U_0^2 - u_1^2\right) = \frac{1}{2}\rho U_0^2 A\left(1 - \left(\frac{u_1}{U_0}\right)^2\right) = \frac{1}{2}\rho U_0^2 A\left(1-(1-2a)^2\right) = 2\rho U_0^2 Aa(1-a). \quad \text{(A9)}$$

The power P extracted from the flow is obtained as

$$P = \frac{1}{2}\rho\left(A_0 U_0^3 - A_1 u_1^3\right) = \frac{1}{2}\rho\left(U_0^2 - u_1^2\right)uA = \frac{1}{2}\rho U_0^3\left(1 - \frac{u_1}{U_0}\right)\left(1 + \frac{u_1}{U_0}\right)\frac{u}{U_0}A$$
$$= \frac{1}{2}\rho U_0^3(1-(1-2a))(1+(1-2a))(1-a)A = 2\rho U_0^3 a(1-a)^2 A. \quad \text{(A10)}$$

Then, the efficiency C_P becomes

$$C_P \equiv \frac{P}{\frac{1}{2}\rho U_0{}^3 A} = 4a(1-a)^2 \,. \tag{A11}$$

From Eq. (A11), the maximum efficiency is obtained at

$$a_{\max} = 1/3 \,, \tag{A12}$$

with

$$C_{P\max} = \frac{16}{27} = 0.593 \,. \tag{A13}$$

APPENDIX B. M)MENTUM THEORY OF A THRUST PRODUCING TURBIME [7,8]

In the following, the momentum theory of a thrust producing turbine is summarized. Almost same notations as in Appendix A are used except the axial force acting on the actuator. We use T for the thrust produced by the actuator disk.

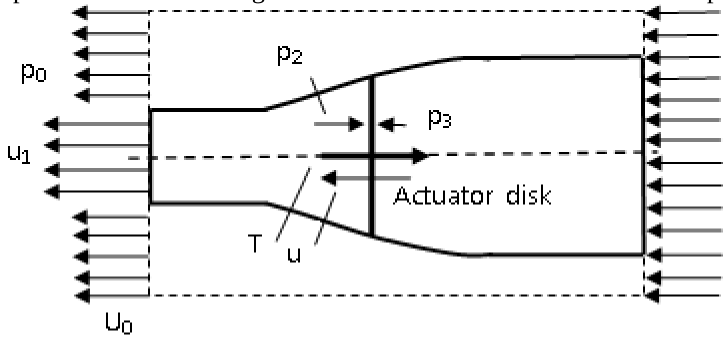

Figure B1: Propeller

From the continuity law, we have

$$U_0A_0 = u_1A_1 = uA. \tag{B1}$$

Applying the conservation of momentum, we have

$$T = \rho\left(u_1^2A_1 - U_0^2A_0\right) = \rho u_1A_1\left(u_1 - U_0\right). \tag{B2}$$

Conservation of Energy or Bernoulli's theorem gives

$$\frac{1}{2}\rho U_0^2 + p_0 = \frac{1}{2}\rho u^2 + p_3,\ \frac{1}{2}\rho u^2 + p_2 = \frac{1}{2}\rho u_1^2 + p_0. \tag{B3a,b}$$

Adding Eqs. (B3a) and (B3b), we have

$$p_2 - p_3 = \frac{1}{2}\rho\left(u_1^2 - U_0^2\right). \tag{B4}$$

Then, the thrust T is given by

$$T = A\left(p_2 - p_3\right) = \frac{1}{2}\rho A\left(u_1^2 - U_0^2\right). \tag{B5}$$

From Eqs. (B1), (B2) and (B5), we have

$$u = \frac{1}{2}\left(U_0 + u_1\right). \tag{B6}$$

Introducing the axial induction coefficient a:

$$a = \frac{u - U_0}{U_0} = \frac{u}{U_0} - 1 \text{ or } u = (1+a)U_0, \tag{B7}$$

we obtain from Eqs. (B6) and (B7)

$$u_1 = (1+2a)U_0. \tag{B8}$$

Substituting Eq. (B8) into Eq. (B5), the thrust T is given by

$$T = \frac{1}{2}\rho A\left(u_1^2 - U_0^2\right) = \frac{1}{2}\rho U_0^2A\left(\left(\frac{u_1}{U_0}\right)^2 - 1\right) = \frac{1}{2}\rho U_0^2A\left((1+2a)^2 - 1\right) = 2\rho U_0^2Aa\left(1+a\right). \tag{B9}$$

The power P given to from the flow is obtained as

$$P = \frac{1}{2}\rho\left(A_1u_1^3 - A_0U_0^3\right) = \frac{1}{2}\rho\left(u_1^2 - U_0^2\right)uA = \frac{1}{2}\rho U_0^3\left(\frac{u_1}{U_0} - 1\right)\left(\frac{u_1}{U_0} + 1\right)\frac{u}{U_0}A$$

$$= \frac{1}{2}\rho U_0^3\left((1+2a) - 1\right)\left((1+2a) + 1\right)(1+a)A = 2\rho U_0^3a(1+a)^2A. \tag{B10}$$

Then, the efficiency C_P becomes

$$C_P \equiv \frac{U_0T}{P} = \frac{2\rho U_0^3Aa\left(1+a\right)}{2\rho U_0^3a(1+a)^2A} = \frac{1}{1+a} = \frac{U_0}{u} = \frac{2U_0}{U_0 + u_1}. \tag{B11}$$

The maximum efficiency $C_{P\max} = 1$ is obtained at $a = 0$ with $T = 0$, but $C_{P\max} \approx 0.8$ actually.

At the bollard condition or $U_0 = 0$, the theory is modified slightly. Namely, the continuity equation becomes

$$u_1A_1 = uA. \tag{B12}$$

The thrust T from the momentum conservation is given by

$$T = \rho A_1 u_1^{\ 2}. \tag{B13}$$

The energy conservation or Bernoulli's theorem becomes

$$p_0 = \frac{1}{2}\rho u^2 + p_3, \ \frac{1}{2}\rho u^2 + p_2 = \frac{1}{2}\rho u_1^{\ 2} + p_0. \tag{B14a,b}$$

Hence, we obtain

$$p_2 - p_3 = \frac{1}{2}\rho u_1^{\ 2}. \tag{B15}$$

Then, the thrust T is obtained as

$$T = A(p_2 - p_3) = \frac{1}{2}\rho A u_1^{\ 2}. \tag{B16}$$

From Eqs. (B12), (B13) and (B16), we have

$$A_1 = \frac{1}{2}A \ \text{ or } \ u = \frac{1}{2}u_1. \tag{B17}$$

Now, the power P given to the flow is given by

$$P = \frac{1}{2}\rho A_1 u_1^{\ 3} = \frac{1}{4}\rho A u_1^{\ 3}. \tag{B18}$$

APPENDIX C. MOMENTUM THEORY OF A TURBINE WITH RADIALLY DISTRIBUTED LOAD

In Appendixes A and B, the axial velocities on the actuator disk and in the wake are assumed constant, namely u and u_1, respectively. Now, we assume the velocities u and u_1 are functions of the radial coordinates r:

$$u_0 = U_0 \quad \text{for} \quad 0 < r < R_0 \text{ at upstream,} \tag{C1a}$$

$$u = u(r) \quad \text{for} \quad 0 < r < R \text{ on actuator disk,} \tag{C1b}$$

$$u_1 = u_1(r) \quad \text{for} \quad 0 < r < R_1 \text{ at far downstream,} \tag{C1c}$$

where R_0, R and R_1 are the radius of the upstream inlet, actuator disk and downstream outlet, respectively. Furthermore, for simplicity, we assume

$$u(r) = \begin{cases} u_h < U_0 & \text{for } 0 < r < R_h \\ u_t > U_0 & \text{for } R_h < r < R \end{cases}, \quad u_1(r) = \begin{cases} u_{1h} < U_0 & \text{for } 0 < r < R_{1h} \\ u_{1t} > U_0 & \text{for } R_{1h} < r < R_1 \end{cases}, \tag{C2a,b}$$

where the suffixes h and t refer to the energy harvesting and thrust generating parts.

The conservations of mass, momentum and energy give

$$M_h = 2\pi\rho\int_0^{R_{0h}} U_0 r dr = 2\pi\rho\int_0^{R_h} u_h r dr = 2\pi\rho\int_0^{R_{1h}} u_{1h} r dr, \; M_t = 2\pi\rho\int_{R_{0h}}^{R_0} U_0 r dr = 2\pi\rho\int_{R_h}^{R} u_t r dr = 2\pi\rho\int_{R_{1h}}^{R_1} u_{1t} r dr, \tag{C3a,b}$$

$$M = M_h + M_t, \tag{C3c}$$

$$D_h = 2\pi\rho\int_0^{R_{0h}} U_0^2 r dr - 2\pi\rho\int_0^{R_{1h}} u_{1h}^2 r dr, \; T_t = -2\pi\rho\int_{R_{0h}}^{R_0} U_0^2 r dr + 2\pi\rho\int_{R_{1h}}^{R_1} u_{1h}^2 r dr, \; T = -D_h + T_t, \tag{C4a,b,c}$$

$$P_h = 2\pi\rho\int_0^{R_{0h}} U_0^3 r dr - 2\pi\rho\int_0^{R_{1h}} u_{1h}^3 r dr, \; P_t = -2\pi\rho\int_{R_{0h}}^{R_0} U_0^3 r dr + 2\pi\rho\int_{R_{1h}}^{R_1} u_{1t}^3 r dr, \; P = P_h - P_t, \tag{C5a,b,c}$$

respectively, and P is the power extracted from the fluid.

Applying Bernoulli's theorem to the energy harvesting part

$$\frac{1}{2}\rho U_0^2 + p_0 = \frac{1}{2}\rho u_h^2 + p_{3h}, \; \frac{1}{2}\rho u_h^2 + p_{2h} = \frac{1}{2}\rho u_{1h}^2 + p_0. \tag{C6a,b}$$

Adding and integrating with respect to r from $r = 0$ to $r = R_h$

$$p_{2h} - p_{3h} = \frac{1}{2}\rho\left(u_{1h}^2 - U_0^2\right). \tag{C7}$$

From Eq. (C7)

$$-D_h = \pi\rho R_h^2 (p_{2h} - p_{3h}) = \frac{1}{2}\pi\rho R_h^2\left(u_{1h}^2 - U_0^2\right). \tag{C8}$$

From Eqs. (C4a) and (C8)

$$D_h = \pi\rho R_{0h}^2 U_0^2 - \pi\rho R_{1h}^2 u_{1h}^2 = \frac{1}{2}\pi\rho R_h^2\left(U_0^2 - u_{1h}^2\right). \tag{C9}$$

From Eqs. (C3a) and (C9)

$$u_h = \frac{1}{2}\left(U_0 + u_{1h}\right). \tag{C10}$$

Substituting Eq. (C10) into Eqs. (C4a)and (C5a)

$$D_h = -\frac{1}{2}\pi\rho R_h^2\left(u_{1h}^2 - U_0^2\right) = -2\pi\rho R_h^2\left(u_h - U_0\right)u_h, \tag{C11a}$$

$$P_h = \pi\rho R_{0h}^2 U_0^3 - \pi\rho R_{1h}^2 u_{1h}^3 = \pi\rho R_h^2\left(U_0^2 - u_{1h}^2\right)u_h = 4\pi\rho R_h^2\left(U_0 - u_h\right)u_h^2. \tag{C.11b}$$

Similarly, applying Bernoulli's theorem to the thrust generating part

$$\frac{1}{2}\rho U_0^2 + p_0 = \frac{1}{2}\rho u_t^2 + p_{3t}, \; \frac{1}{2}\rho u_t^2 + p_{2t} = \frac{1}{2}\rho u_{1t}^2 + p_0. \tag{C12a,b}$$

Adding and integrating with respect to r from $r = R_h$ to $r = R$

$$p_{2t} - p_{3t} = \frac{1}{2}\rho\left(u_{1t}^2 - U_0^2\right). \tag{C13}$$

From Eq. (C13)

$$T_t = \pi\rho\left(R^2 - R_h^2\right)(p_{2t} - p_{3t}) = \frac{1}{2}\pi\rho\left(R^2 - R_h^2\right)\left(u_{1t}^2 - U_0^2\right). \tag{C14}$$

From Eqs. (C4b) and (C14)

$$T_t = -\pi\rho\left(R_0^2 - R_{0h}^2\right)U_0^2 + \pi\rho\left(R_1^2 - R_{1h}^2\right)u_{1t}^2 = \frac{1}{2}\pi\rho\left(R^2 - R_h^2\right)\left(u_{1t}^2 - U_0^2\right). \tag{C15}$$

From Eqs. (C3b) and (C15)

$$u_t = \frac{1}{2}(u_{1t} + U_0). \tag{C16}$$

Substituting Eq. (C16) into Eqs. (C4b)and (C5b)

$$T_t = \frac{1}{2}\pi\rho\left(R^2 - R_h^{\ 2}\right)\left(u_{1t}^{\ 2} - U_0^{\ 2}\right) = 2\pi\rho\left(R^2 - R_h^{\ 2}\right)(u_t - U_0)u_t\,, \tag{C17a}$$

$$P_t = -\pi\rho\left(R_0^{\ 2} - R_{0h}^{\ 2}\right)U_0^{\ 3} + \pi\rho\left(R_1^{\ 3} - R_{1h}^{\ 3}\right)u_{1t}^{\ 3} = -4\pi\rho\left(R^2 - R_h^{\ 2}\right)(U_0 - u_t)u_t^{\ 2}\,. \tag{C17b}$$

From Eqs. (C4c, (C5c), (C11) and (C17)

$$T = -D_h + T_t = 2\pi\rho R_h^{\ 2}(u_h - U_0)u_h + 2\pi\rho\left(R^2 - R_h^{\ 2}\right)(u_t - U_0)u_t\,, \tag{C18a}$$

$$P = P_h - P_t = 4\pi\rho R_h^{\ 2}(U_0 - u_h)u_h^{\ 2} + 4\pi\rho\left(R^2 - R_h^{\ 2}\right)(U_0 - u_t)u_t\,. \tag{C18b}$$

Rewriting Eq. (C18)

$$\frac{T}{2\pi\rho R^2 U_0^{\ 2}} = \left(\frac{R_h}{R}\right)^2\left(\frac{u_h}{U_0} - 1\right)\frac{u_h}{U_0} + \left(1 - \left(\frac{R_h}{R}\right)^2\right)\left(\frac{u_t}{U_0} - 1\right)\frac{u_t}{U_0}\,, \tag{C19a}$$

$$\frac{P}{4\pi\rho R^2 U_0^{\ 3}} = \left(\frac{R_h}{R}\right)^2\left(1 - \frac{u_h}{U_0}\right)\left(\frac{u_h}{U_0}\right)^2 + \left(1 - \left(\frac{R_h}{R}\right)^2\right)\left(1 - \frac{u_t}{U_0}\right)\left(\frac{u_t}{U_0}\right)^2. \tag{C19b}$$

We conducted some numerical calculations. The computational condition was $(R_h/R)^2 = 0.81$, $u_t/U_0 = 1.2$. The results are shown in Figure C1. When the thrust is greater or equal to zero, the total energy extracted is negative. This means we must supply energy from the outside.

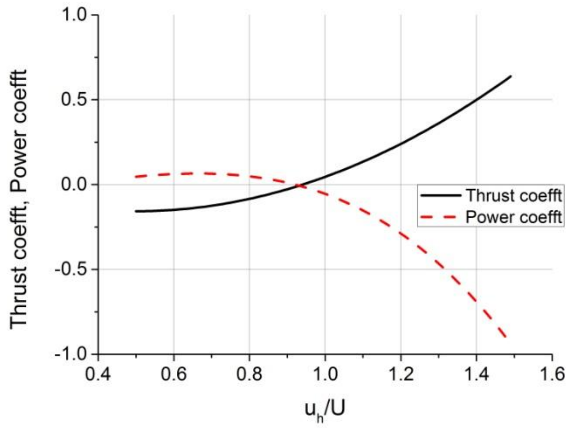

Figure C1: $T/(2\pi\rho R^2 U_0^{\ 2})$ and $P/(4\pi\rho R^2 U_0^{\ 3})$ vs u_h/U_0