Conversion of Low Speed Mechanical Energy into Electric Energy - New Electrostatic Generator

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ABSTRACT — In power generation from natural energy, especially such as wave and tidal energies, the velocity of the moving body or medium is very slow. In the traditional energy conversion, we are obliged to convert the slow speed motion into high speed motion, and this causes a big energy loss. Accordingly, an energy conversion suited for the low speed motion is required. Namely, the energy conversion of not velocity but displacement type is important in these cases. In the present discussion, we propose a new electrostatic generator based not on electromagnetic but on electrostatic phenomena. We use the charge storage effect of a capacitor. In the present discussion, we discuss not only on the principle but also on the application to wave and tidal generators. If we use electrical double-layer capacitors, the capacitance increase dramatically. This suggest us the possibility of an electrostatic generation using capacitors.

Keywords— Energy transformation, Displacement-type, Electric double-layer capacitor, Electrostatic generator, Wave/Tidal generation

1. INTRODUCTION

The traditional converter of kinetic energy into electric energy mostly utilizes the electromagnetic phenomenon and assumes a high speed displacement or rotation of a body or medium. However, in case of energy conversion from natural energies, especially in case of wave and tidal generation, the motion is reciprocal and the velocity is very slow. In case of traditional conversion of energy, the low speed reciprocal motion must be transformed from the low speed reciprocal motion into the high speed rotational motion. This invites an energy loss. Hence, we must develop a new energy conversion method for low speed motion. Namely, a not velocity-type but displacement-type energy converter is required.

In the present paper, we propose a new energy converter using not electromagnetic but electrostatic phenomenon. Electrostatic generators such as the Van de Graff generator is well known [1]. Kelvin water dropper is also well known [2-5]. However, they may not be utilized for large scale power generation. In short, we utilize the electricity storage effect of a capacitor, and this convert displacement into electricity. Recently, a electric double layer capacitor has appeared, and the storage performance of the capacitor is drastically increased [6]. Since a capacitor with capacitance 100F has been developed, this suggests a possibility of high-performance electrostatic generator. In the present paper, we discuss not only the principle of the electrostatic generator but also the application to the wave and/or tidal generator.

Although the output of the wave and wind power generation is very irregular, that of the tidal power is constant. Furthermore, if we install the tidal power generator into a container connected to sea, we can avoid the damage due to strong wind and big wave. The reliability of the tidal power generation is high, though the output is small.

Table 1: List of symbols.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>Area of capacitor plate</td>
<td>m²</td>
<td>ε</td>
<td>permittivity</td>
<td>F/m</td>
</tr>
<tr>
<td>Δ</td>
<td>Distance between capacitor</td>
<td>m</td>
<td>t</td>
<td>Time</td>
<td>s</td>
</tr>
</tbody>
</table>

(If there is plurality of capacitors, we discriminate using subscripts. For example, \( V_i \) refers to the voltage between capacitor plates of capacitor \( i \).)
2. PRINCIPLE OF ELECTROSTATIC GENERATOR

2.1. Parallel plate capacitor

Figure 1 illustrates a parallel plate capacitor. If we apply voltage on the facing electrodes, the electric charge is stored in the capacitor.

![Figure 1: A parallel plate capacitor](image)

The relationship between the voltage $V$ and charge $Q$ is given by

$$Q = CV,$$

where the capacitance $C$ is given by the permittivity $\varepsilon$, area $S$ of the electrode and distance $\Delta$ between electrodes by

$$C = \frac{\varepsilon_0 S}{\Delta},$$

The stored electric energy $U$ is obtained as

$$U = \frac{1}{2} CV^2 = \frac{1}{2} CVQ.$$  

Rewriting, we have

$$U = \frac{1}{2} CVQ = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2}{\varepsilon S}.$$  

The strength $E$ of the electric field and force $F$ acting on the electrode are given by

$$E = \frac{V}{\Delta} = \frac{Q}{C\Delta} = \frac{Q}{\varepsilon S},$$  

$$F = \frac{1}{2} \frac{Q E}{\Delta} = \frac{1}{2} \frac{Q}{\varepsilon S} = \frac{1}{2} \frac{Q^2}{\varepsilon S} = \frac{1}{2} \frac{C^2 (\Delta)V^2}{\varepsilon S}.$$  

It is necessary to pay attention to $F \neq QE$. Since the charge on the same electrode does not contribute on $F$, and the strength of the electric field due to the other electrode is a half of the strength of the electric field due to both electrodes.

2.2. Principle of a new electrostatic generation using capacitor

Let $S$ be constant. The state of a capacitor can be described by $(\Delta, E)$. The initial state is described by $(\Delta_0, E_0)$. $Q$, $E$, $V$ and $F$ are functions of $\Delta$ and $E$. Since $C$ is a function of $\Delta$, we express $C$ as $C(\Delta)$. Namely, we have

$$Q = C(\Delta)\Delta E = \frac{\varepsilon S}{\Delta} \Delta E = \varepsilon SE,$$

$$V = E\Delta,$$
\[ F = \frac{1}{2} Q E = \frac{1}{2} \varepsilon S E^2. \quad (9) \]
\[ U = \frac{1}{2} C V^2 = \frac{1}{2} \varepsilon S (\Delta E)^2 = \frac{1}{2} \varepsilon S \Delta E^2, \quad (10) \]
\[ \partial W = F \partial \Delta = \frac{1}{2} \varepsilon S E^2 \partial \Delta. \quad (11) \]

From Eq. (10), the contour plot of the stored energy \( U \) is shown in Figure 2. As shown by an arrow in the figure, the gradient of the contour becomes steep, as \( U \) increases. We consider the transition of states \((\Delta, E)\) as shown in Figure 3.

![Figure 2: Contour of energy \( U \) of a capacitor](image)

**Step (1):** We cut off the external electric circuit. Namely, applying the external force \( F = (1/2)\varepsilon S E_0^2 \), we increase the distance between the electrodes until \( \Delta = \Delta_0 \) under \( Q = Q_0 \). The stored electric energy \( U \) becomes large by an amount corresponding to the work done by the external force: 
\[ W = F(\Delta_1 - \Delta_0) = (1/2)\varepsilon S E_0^2(\Delta_1 - \Delta_0) \] . The voltage increases from \( V_0 = E_0 \Delta_0 \) to \( V_1 = E_0 \Delta_1 \). Namely
\[ \Delta_0 \rightarrow \Delta_1 \text{ under } Q = Q_0 \text{ then } (\Delta_0, E_0) \rightarrow (\Delta_1, E_0) \quad (12) \]
\[ U_1 = U_0 + W = \frac{1}{2} \varepsilon S \Delta_0 E_0^2 + \frac{1}{2} \varepsilon S E_0^2(\Delta_1 - \Delta_0) = \frac{1}{2} \varepsilon S \Delta_1 E_0^2 \quad (13) \]

**Step (2):** We connect the external electric circuit. Under \( \Delta = \Delta_1 \), the electricity is discharged to the external electric circuit until \( E \) decreases to \( E_2 \). The discharged electric energy is 
\[ (1/2)\varepsilon S \Delta_1 E_0^2 -(1/2)\varepsilon S \Delta_1 E_2^2 \] . The stored electric energy is 
\[ U_2 = (1/2)\varepsilon S \Delta_1 E_2^2 \] . The voltage drops to \( V_2 = E_2 \Delta_1 \). Namely
\[ Q_0 \rightarrow Q_2 = C(\Delta)V_2 = \varepsilon S E_2 \text{ under } \Delta = \Delta_1 \text{ then } (\Delta_1, E_0) \rightarrow (\Delta_1, E_2) \quad (14) \]
Step (3): We cut off the external electric circuit. Under \( Q = Q_0 \), applying the external force \( F_1 = Q_0^2 \frac{\varepsilon}{r} = (1/2) e SE_2^2 \) to the inverse direction as in step (1), the distance between electrodes is reduced to \( \Delta = \Delta_0 \). The work done by the external force is \( W_2 = F_2(\Delta_1 - \Delta_0) = (1/2) e SE_2^2(\Delta_1 - \Delta_0) \). The voltage drops to \( V_3 = E_2 \Delta_0 \). Namely
\[
\Delta_1 \rightarrow \Delta_0 \text{ under } Q = Q_0 \text{ then } (\Delta_1, E_1) \rightarrow (\Delta_0, E_2)
\] (15)

Step (4): We connect the external electric circuit. Under \( \Delta = \Delta_0 \), we supply the electricity from the external electric circuit until \( Q = Q_0 \). The supplied electric energy is \( (1/2) e SE_2^2(\Delta_0 - \Delta_0)E_0^2 = (1/2) e SE_0^2(\Delta_0^2 - E_0^2) \). The voltage returns to \( V_4 = E_0 \Delta_0 \) from \( V_3 = E_2 \Delta_0 \). Namely
\[
Q_4 \rightarrow Q_0 \text{ under } \Delta = \Delta_0 \text{ then } (\Delta_0, E_2) \rightarrow (\Delta_0, E_0)
\] (16)

We show below that the net electric energy supplied to the external electric circuit is equal to the net work done by the external force:

Net electric energy supplied to the external electric circuit
= Electric energy discharged to the external circuit in Step (2)
- Electric energy supplied from the external circuit in Step (4)
\[
= \frac{1}{2} e S(\Delta_1 - \Delta_0)E_0^2 - \frac{1}{2} e S(\Delta_1 - \Delta_0)E_2^2 = \frac{1}{2} e S(\Delta_1 - \Delta_0)(E_0^2 - E_2^2)
\] (17)

Net work supplied from the external force
= Work supplied from the external force in Step (1)
- Work done to the external force in Step (3)
\[
= \frac{1}{2} e S E_2^2(\Delta_1 - \Delta_0) - \frac{1}{2} e S E_0^2(\Delta_1 - \Delta_0) = \frac{1}{2} e S(\Delta_1 - \Delta_0)(E_0^2 - E_2^2)
\] (18)

However, if the work done to the external force is thrown away, the efficiency \( \eta \) becomes
\[
\eta = \frac{\text{Net electric energy}}{\text{Work done by external force}} = \left( \frac{U_3 - U_2}{U_1 - U_0} \right) = \frac{U_3 - U_2}{U_1 - U_0} = 1 - \frac{W_2}{W_1} = 1 - \frac{1 - \eta}{1 - \eta} = \frac{1 - \eta}{1 - \eta}
\] (19)

3. VERIFICATION OF PRINCIPLE OF ELECTROSTATIC GENERATOR BY NUMERICAL SIMULATION

3.1. Verification of principle 1
As shown in Figure 4, we consider a device having a pair of capacitors charged initially. If we change the distance of the electrodes of one of the capacitors, the voltage between the electrodes changes, and an alternating current is generated. We need work done from the outside to change the distance of the electrodes. Then, electricity corresponding to the mechanical input energy is generated and consumed by a resistor in the circuit. We conduct simulation calculations and verify the principle of the electrostatic generator.

![Figure 4: A system for verifying the principle of an electrostatic generator](Image)

In this case, the distance \( \Delta_0 \) of capacitor 0 is driven, and the distance \( \Delta_1 \) of capacitor 1 is held constant. The basic equations are given by
\[
V_0 + ri = V_i
\] (20)
\[
\frac{dQ_0}{dt} = i, \quad \frac{dQ_i}{dt} = -i,
\]
(21)
\[
Q_0 = C_0(\Delta_0) V_0, \quad Q_i = C_i(\Delta_i) V_i,
\]
(22a, b)
\[
\Delta_0 = \overline{\Delta} + \sin \left( \frac{2\pi}{T} t \right), \quad \Delta_1 = \overline{\Delta}.
\]
(23a, b)

From Eq. (20), we have
\[
i = \frac{1}{r} (V_i - V_0).
\]
(24)

Substituting Eqs. (22) and (24) into (21), we obtain
\[
\frac{dQ_0}{dt} = \frac{1}{r} (V_i - V_0), \quad \frac{dQ_i}{dt} = -\frac{1}{r} (V_i - V_0).
\]
(25)

Then, differential equations for the voltages \( V_0 \) and \( V_i \) are derived as
\[
\frac{dV_0}{dt} + \left( \frac{dC_0(\Delta_0)}{dr} \right) V_0 = \frac{1}{rC_0(\Delta_0)} (V_i - V_0),
\]
(26a)
\[
\frac{dV_i}{dt} + \left( \frac{dC_i(\Delta_i)}{dr} \right) V_i = -\frac{1}{rC_i(\Delta_i)} (V_i - V_0).
\]
(26b)

Since \( \Delta_0 \) and \( \Delta_1 \) are given by Eq. (23), we are required to solve Eq. (26) under the initial condition:
\[
\Delta_0 = \Delta_1 = \overline{\Delta}, \quad V_0 = V_i = V_B, \quad Q_0 = Q_i = C(\overline{\Delta}) V_B, \quad i = 0 \text{ at } t = 0.
\]
(27)

The electric force \( F_0 \) and \( F_i \) acting on electrodes 1 and 2 are given by
\[
F_0 = \frac{1}{2} Q_0 E_0 = \frac{1}{2} C_0^2(\Delta_0) V_0^2, \quad F_i = \frac{1}{2} Q_i E_i = \frac{1}{2} C_i^2(\Delta_i) V_i^2.
\]
(28)

The average electric output power \( \text{avg}(EP) \) and average mechanical input power \( \text{avg}(MP) \) are defined as
\[
\text{avg}(EP) = \lim_{t \to \infty} \frac{1}{2} \int_0^t r i^2 dt, \quad \text{avg}(EP) = \lim_{t \to \infty} \frac{1}{2} \int_0^t F_0 \frac{d\Delta_0}{dt} dt.
\]
(29)

As proved in Appendix A, the conservation of energy gives
\[
\text{avg}(EP) = \text{avg}(MP).
\]
(30)

Numerical results are shown in Figure 5. Parameters used in the simulation are shown in Table 2. In this case, \( \text{avg}(EP) = 199 \) and \( \text{avg}(MP) = 246 \), the difference between them is not small.

**Table 2: Parameters used in the simulation**

<table>
<thead>
<tr>
<th>A</th>
<th>1</th>
<th>( S_o )</th>
<th>10</th>
<th>( V_B )</th>
<th>100</th>
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<tr>
<td>T</td>
<td>20</td>
<td>( S_i )</td>
<td>10</td>
<td>( R )</td>
<td>4</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>2</td>
<td>( \varepsilon )</td>
<td>10</td>
<td>( dt )</td>
<td>0.02</td>
</tr>
</tbody>
</table>
Figure 5: One pair of electrodes is fixed and the other pair of electrodes is driven sinusoidally. (\(r = 0.4\); (a) Distance \(\Delta\) between electrodes and the time rate of change \(d\Delta/dt\); (b) Voltages \(V_0\) and \(V_1\) between electrodes; (c) Stored charges \(Q_0\) and \(Q_1\); (d) Current \(i\); (e) Electric input power \(EP\); (f) Mechanical output power \(MP\))

3.2. Verification of principle 2

We consider a different system from 3.1. As shown in Figure 6, we consider a device having a pair of capacitors charged initially. If we change the distance between electrodes of one capacitor sinusoidally and that of the other capacitor in opposite phase, an alternating current is generated in the circuit, since the voltage between electrodes of one capacitor goes up and down with that of the other capacitor in opposite phase. Then, electricity corresponding to the mechanical input energy is generated and consumed by a resistor in the circuit. We conduct simulation calculations and verify the principle of the electrostatic generator.

Figure 6: A system for verifying the principle of an electrostatic generator. ((a) Electric circuit; (b) Mechanism) The distance \(\Delta_0\) between electrodes of capacitor 0 and the distance \(\Delta_1\) between electrodes of capacitor 1 are driven with reverse phase
3.3.1. A case when capacitors 1 and 2 are connected always

In Eqs. (20–26), we replace Eq. (23) by

\[
\Delta_0 = \Delta + A \sin\left(\frac{2\pi t}{T}\right); \quad \Delta_1 = \Delta - A \sin\left(\frac{2\pi t}{T}\right).
\]

(31a, b)

We are required to solve Eq. (26) under the initial condition (27)

Parameters used in the simulation are given in Table 3. The numerical results are shown in Figure 7. In this case, \( \text{avg}(EP) = 1003 \) and \( \text{avg}(MP) = 1116 \), the difference between them is small.

**Table 3:** Parameters used in simulation

<table>
<thead>
<tr>
<th>( A )</th>
<th>1</th>
<th>( S_0 )</th>
<th>10</th>
<th>( V_B )</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>20</td>
<td>( S_B )</td>
<td>10</td>
<td>( \sigma )</td>
<td>4</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>2</td>
<td>( \varepsilon )</td>
<td>10</td>
<td>( \Delta t )</td>
<td>0.02</td>
</tr>
</tbody>
</table>

![Graphs](image)

**Figure 7:** Two pairs of electrodes are driven sinusoidally with opposite phase each other. (4 = \( r \); (a) Distance \( \Delta \) between electrodes and the time rate of change \( d\Delta/dt \); (b) Voltages \( V_0 \) and \( V_1 \) between electrodes; (c) Stored charges \( Q_0 \) and \( Q_1 \); (d) Current \( i \); (e) Electric input power \( EP \); (f) Mechanical output power \( MP \)).

The numerical results in which \( r = 4 \) is replaced by \( r = 0.04 \) in Table 3 are shown in Figure 8. In this case, \( EP = 11151 \) and \( MP = 11149 \), they are almost equal.
Two pairs of electrodes are driven sinusoidally with opposite phase each other. \( r = 0.04 \); (a) Distance \( \Delta \) between electrodes and the time rate of change \( d\Delta/dt \); (b) Voltages \( V_0 \) and \( V_1 \) between electrodes; (c) Stored charges \( Q_0 \) and \( Q_1 \); (d) Current \( i \); (e) Electric output power \( EP \); (f) Mechanical input power \( MP \).

Effects of resistance \( r \) and base voltage \( V_B \) on the average electric output power \( \text{avg}(EP) \) and mechanical input power \( \text{avg}(MP) \) are shown in Figure 9. Effects of wave amplitude \( A \) and period \( T \) on the average electric output power \( \text{avg}(EP) \) and mechanical input power \( \text{avg}(MP) \) are shown in Figure 10.

**Figure 9:** Effects of resistance \( r \) and base voltage \( V_B \) on average electric output power \( \text{avg}(EP) \) and mechanical input power \( \text{avg}(MP) \). (a) Effects of resistance \( r \); (b) Effects of base voltage \( V_B \)
3.3.2. A case when capacitors 1 and 2 repeatedly connected and disconnected

In case of the tidal generation, since the wave period $T$ becomes very long, we may need the following idea. For example, we connect the electric circuit when $nT+0.25T < t < nT+0.35T$ and $nT+0.75T < t < nT+0.85T$, $n = 0, 1, 2, \cdots$ and disconnect otherwise. Namely, we replace Eqs. (26a) and (26b) with

\begin{align}
\frac{dV_o}{dt} + \left( \frac{dC_o(\Delta_0)}{dt} / C_o(\Delta_0) \right) V_o &= \begin{cases} \frac{1}{rC_o(\Delta_0)} (V_i - V_o) & \text{for } nT + \{0.25, 0.75\} T \leq t < nT + \{0.35, 0.85\} T, \\ 0 & \text{otherwise} \end{cases} \tag{32a} \\
\frac{dV_i}{dt} + \left( \frac{dC_i(\Delta_i)}{dt} / C_i(\Delta_i) \right) V_i &= \begin{cases} -\frac{1}{rC_i(\Delta_i)} (V_i - V_o) & \text{for } nT + \{0.25, 0.75\} T \leq t < nT + \{0.35, 0.85\} T, \\ 0 & \text{otherwise} \end{cases} \tag{32b}
\end{align}

The parameters used in the numerical calculation are given in Table 4. The numerical results are shown in Figure 11. In this case, $\text{avg}(EP) = 411934.6$ and $\text{avg}(MP) = 421329.8$, the difference between them is small.

<table>
<thead>
<tr>
<th>$A$</th>
<th>1</th>
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<th>$V_B$</th>
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<tr>
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<td>20</td>
<td>$S_f$</td>
<td>10</td>
<td>$r$</td>
<td>0.001</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>2</td>
<td>$\varepsilon$</td>
<td>100</td>
<td>$dt$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

**Table 4:** Parameters used in simulation

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**Figure 10:** Effects of wave amplitude $A$ and period $T$ on average output electric power $\text{avg}(EP)$ and mechanical input power $\text{avg}(MP)$. (a) Effects of amplitude $A$; (b) Effects of period $T$. 

**Figure 11:** Numerical results for $\text{avg}(EP)$ and $\text{avg}(MP)$.
Figure 11: Two pairs of electrodes are driven sinusoidally with opposite phase each other. \( r = 0.001 \); (a) Distance \( \Delta \) between electrodes and the time rate of change \( d\Delta /dt \); (b) Voltages \( V_0 \) and \( V_1 \) between electrodes; (c) Stored charges \( Q_0 \) and \( Q_1 \); (d) Current \( i \); (e) Electric output power \( EP \); (f) Mechanical input power \( MP \); (g) Force acting on the device \( F = -F_0 + F_1 \).

Figure 12: Effects of resistance \( r \) on average electric output and mechanical input powers \( \text{avg}(EP) \) and power \( \text{avg}(MP) \). (Period \( T = 20 \) )

On the assumption that it is used for the tidal power generation, we show a result when the wave period \( T \) is 12hr or 43200sec in Figure 13. The computational conditions are shown in Table 5. In this case the average electric output power \( \text{avg}(EP) \) and \( \text{avg}(MP) \) are 172W and 180W, respectively. The difference is small.

Table 5: Parameters used in simulation

<table>
<thead>
<tr>
<th>( A )</th>
<th>( 1 )</th>
<th>( S_0 )</th>
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<th>( V_B )</th>
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<tr>
<td>( T )</td>
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<td>( S_1 )</td>
<td>10</td>
<td>( r )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>2</td>
<td>( \varepsilon )</td>
<td>100</td>
<td>( dt )</td>
<td>2.16</td>
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</table>
Figure 13: Case when wave period $T$ is 12hr or 43,200sec. ((a) Distance and time derivatives of electrodes; (b) Voltage between electrodes; (c) Charge of capacitor 0 and 1; (d) Electric current in circuit)

4. CONCLUSIONS

Principle of a new electrostatic generator using capacitors and possibility of the application to wave and/or tidal generation are discussed. In case of wave and/or tidal generation, an energy convertor suitable to the slow speed motion of a moving body or media would be required. Not velocity type but displacement type energy conversion would be required. It would reduce the energy loss in the conversion of slow reciprocating motion into high speed rotational motion. It would reduce the energy loss when slowly oscillating fluid motion is transformed into high speed rotational motion of a turbine.

We proposed a new electrostatic generator that does not use the electromagnetic phenomenon. It uses the electrostatic phenomenon. In a word, it uses the charge storage effect of a capacitor. It converts the displacement into the electricity.

We also studied the conversion cycle and efficiency when a capacitor converts the oscillatory displacement into the electric energy. Furthermore, we discussed the application to the wave and/or tidal generation. We believe that it will improve the conversion efficiency from the natural energy such as wave and tide to the electricity. It can also be applied to the power generation by animal power and human power in developing countries. It will also contribute to ensure the energy in emergency.

Recently, a electric double layer capacitor has appeared, and the storage performance of the capacitor is drastically increased. Since a capacitor with capacitance 100F has been developed, this suggests a possibility of high-performance electrostatic generator.

5. REFERENCES

APPENDIX A. CONSERVATION OF ELECTRIC CHARGE AND ENERGY

First, we obtain the conservation law of charge. Rewriting Eqs. (21a) and (21b), we have

$$\frac{dQ_0}{dt} = \frac{1}{r}(V_1 - V_0), \quad \frac{dQ_1}{dt} = \frac{1}{r}(V_1 - V_0) \tag{A1.a, b}$$

Adding Eqs. (A1.a) and (A1.b), we obtain the conservation law of charge:

$$\frac{d(Q_0 + Q_1)}{dt} = 0. \tag{A2}$$

Next, we obtain the conservation law of energy. Subtracting Eqs. (A1.a) and (A1.b), we have

$$\frac{d(Q_0 - Q_1)}{dt} = \frac{2}{r}(V_1 - V_0). \tag{A3}$$

Multiplying $V_1 - V_0$ on both sides, we obtain

$$(V_1 - V_0) \frac{d(Q_0 - Q_1)}{dt} = \frac{2}{r}(V_1 - V_0)^2 \tag{A4}$$

Rewriting the left hand side, we derive

$$(V_1 - V_0) \frac{d(Q_0 - Q_1)}{dt} = V_1 \frac{dQ_0}{dt} + V_0 \frac{dQ_1}{dt} - V_1 \frac{dQ_0}{dt} - V_1 \frac{dQ_1}{dt} = V_1 \frac{dQ_0}{dt} + V_0 \frac{dQ_1}{dt} - V_1 \frac{dQ_0}{dt} - V_1 \frac{dQ_1}{dt} = \frac{d}{dt} \left[ \frac{\varepsilon_0 S_0 V_0}{\Delta_0} - \frac{\varepsilon_1 S_1 V_1}{\Delta_1} \right]$$

Multiplying $V_1 - V_0$ on both sides, we obtain

$$(V_1 - V_0)^2 \frac{dQ_0}{dt} + V_1 \frac{dQ_0}{dt} - V_0 \frac{dQ_0}{dt} + V_1 \frac{dQ_1}{dt} - V_0 \frac{dQ_1}{dt} - V_1 \frac{dQ_1}{dt} = \frac{d}{dt} \left[ \frac{\varepsilon_0 S_0 V_0}{\Delta_0} - \frac{\varepsilon_1 S_1 V_1}{\Delta_1} \right]$$

Substituting Eq. (A5) into Eq. (A4), we have

$$-2(-F_0 + F_1) \frac{dU_0}{dt} + 2 \frac{dU_1}{dt} = \frac{2}{r}(V_1 - V_0)^2. \tag{A6}$$

Rewriting, we obtain

$$(F_0 - F_1) \frac{d\Delta_0}{dt} + \frac{dU_0}{dt} + \frac{dU_1}{dt} = \frac{2}{r}(V_1 - V_0)^2. \tag{A7}$$

Integrating Eq. (A7) with respect to time, the conservation law of energy:

$$\int_0^T (F_0 - F_1) d\Delta_0 = \left[ U_0(t_2) + U_1(t_2) - U_0(t_1) - U_1(t_1) \right] + \int_0^T \frac{1}{r} (V_1 - V_0)^2 dt \tag{A8}$$

is obtained. Taking the average of one period $T$, we finally derive

$$\frac{1}{T} \int_0^T (F_0 - F_1) d\Delta_0 = \frac{1}{T} \int_0^T \frac{1}{r} (V_1 - V_0)^2 dt. \tag{A9}$$

The left hand side is the average mechanical input power $avg(MP)$ and the right hand side is the average electric output power $avg(EP)$.