

Glimpses of Scientific Mathematic Modules in High School

João Cabral¹

¹ Department of Mathematics of the Azores University
Ponta Delgada, Portugal

ABSTRACT—*The introduction of advanced scientific modules to support teaching has an important fundamental in mathematics education. But this kind of introduction collides with the lack of time that the teachers of High School have to develop new strategies and implement them, because they are attached, in Portugal, to a bureaucratic machine that consumes valuable time that could be invested in education.*

Didactic concepts and scientific content must work together like a gear of knowledge. The didactic knowledge allows introducing concepts that are later explored scientifically enabling educational expansion, which will be reflected in the student's creativity. We assist to a dialectical progression knowledge, which will have immediate effects on motivation and capture the attention by those who learn, so in this work we present a model to introduce some advanced Scientific Modules from some graduations of University Mathematics curriculum in Portuguese High Schools.

Keywords— Mathematics; Education; Modeling; Modeling process.

1. INTRODUCTION

Each student, in Portugal, who first came into contact with their handbook that supports the discipline teaching of mathematics in secondary education, is faced with content that is not part of the program, come at the end of each chapter that will be effectively taught in class. These parts are often ignored by them, but in general the graphics draw calls for the attention of students, who then with the lack of time, completely ignores the message. They are extra contents in the spirit of trying to capture the curiosity of the student, giving those insights to explore the issues and go beyond what is taught by the teacher in the classroom, but can also contain only historical aspects that are very important in the evolution of knowledge of each lecture unit. All these contents, and all others that are introduced by the teacher in an attempt to improve its scientific - pedagogical activity, which are extracted from a knowledge that goes beyond the needs of the graduation of each program or even a teaching cycle, are designated herein by advanced scientific modules. In our case we will about the graduation in Mathematics in High School in Portugal, refereed in this work as “discipline” or “discipline of mathematics”.

The advanced scientific modules are of fundamental importance in mathematics education. An importance that becomes an urgent necessity, because it complements, in most cases, the need that the student has to realize “where you can apply it”, specifically in reality the concepts and content taught in the discipline. But this need collides with the lack of time that teachers, in Portugal, have to develop new strategies and implement them, since they become trapped in a bureaucratic machine that consumes valuable time that could be invested in education.

The didactic concepts and scientific content have to work together, like sprocket gear knowledge on the field of didactics knowledge to build key concepts which will then be explored scientifically creating scientific concepts in turn that will enable didactic expansion, which reflect in supporting and expanding the student's creativity. This allows a dialectical progress in knowledge that has immediate effects on motivation and captures the attention of the learner.

In addition to the existing scientific component, if we introduce small advanced scientific modules in the discipline, which can be presented by the teacher himself, or by invited lectures in small, worked in the classroom, in groups, or in mathematics laboratories, the result is a self-motivating work by the student. Working this way, the student reflects their own creativity, ability to learn and innovate, generating their own path ways of knowledge. As large movements, as Scouting, already use, quite effectively since the early twentieth century, “learning by doing” always has better results in the genesis of teaching.

2. THE CALCULATOR

When the calculator was introduced in mathematics education, in Portugal, has brought innovation to the discipline, in terms of technology, because much of graphics and calculation that was prepared by the students with the tools previously used more, as pencil and paper, by resorting too often to mental calculation, was replaced by the push of a button. The whole process of events that led to the execution of a task, in which there was a need to think about how all

its components should be arranged to achieve the desired result, was replaced by a collection of mechanical events, dominated by pressing keys, based on a structure internal to a manual calculator, which itself requires only a mechanical and static learning. For the wrong command calculator lead to unintended results, and all structures of the various components of a given reasoning are encapsulated in a plastic box, which works in a manner incomprehensible to the vast majority of students.

This product technological advances that society suffers continuously, has brought many changes in the classroom, and the most important was the fact that teachers had to learn to live with it. By all means a not peaceful change took place, after the introduction of this piece of technology in High School, since the lack of guidelines in this regard, in the hierarchically chain , caused reactions that led to the simplistic use of the machine, reducing it to a mere tool of algebraic calculation, type a more advanced multiplication table. Depending on the teacher education and its constant upgrade, we can see a brief resume how it evolved in Portugal in [2], however, with the passage of time, and the habituation to the new instrument, the calculator has come to be better understood and its role within the classroom improved between the school participants.

Over the past years, students learned how to handle efficiently the new tool, and even managed to create new forms to use it by creating their own programs, assisting them beyond the intended educational agent serving almost as a memory aid. After a few bumps on its implementation, with ups and downs, today is already well accepted in schools and tasks required in the classroom, are geared towards its use, and the respective structural reformulation of concepts, caused by its introduction into education was more or less well surpassed by students and teachers. A nice view of this process of adaption is described by Rafael Borini [4].

Like any introduction of new tools in mathematics education, we can always point out advantages and disadvantages. In this classification there will always be voices for and against. Supporters argue the advantages of time savings, that otherwise would not be possible, enabling the exploration of more content in the discipline, and the ability to generate activities in the classroom, since, without the use of the calculator, even with the strong will teacher, there would never be time to implement them. A supporter of the disadvantages claims that it causes regressive effects, because when the calculator was brought to the class room, it dominated the mental calculus, creating a gap in the systematic articulation of algebraic calculation, by students. When using the calculator the student ignores important properties and rules in the evolution of many scientific content of the course. They become limited to just writing something on a small screen and wait for the timely response of the machine. In fact, analyzing the entire current development and restructuration of the discipline of Mathematics in the Portuguese curriculum, we can see that both sides are right, and to find the balance point has always been a chore for the discipline teaching evolution. Clearly, we can also conclude that the calculator is an excellent auxiliary on didactics, but a lousy tool in science if its use is not well managed. The calculator itself is, in itself, an authentic advanced concept that has found its space in the discipline, which was once dominated by paper, the pencil, the ruler, the square, and many other materials which their existence depended on the strength of teacher will create in them to be used in the classroom. Basically, if used effectively, represents an equivalent to those shown at the end of each chapter in the textbook, allowing the exploration, develop knowledge, advanced scientific module and go further, by the student.

3. DYNAMICS OF A ADVANCED SCIENTIF MODULE

There is no need to change many parameters in school dynamics to introduce an advanced module on teaching the discipline of mathematics, but this can cause processes contrary to the intended if its introduction is not well executed and controlled. The calculator is a good example of this effect.

Then we propose an implementation in three stages, as seen in figure 1, which minimizes the negative effect of introducing it and maximize its potential as a driver and enabler of the development of the student's reasoning. This implementation is seen only from a theoretical point of view, because it lacks a practical experimentation integrated into the school network, which has not yet been effected.

In the first phase of implementation, the advanced module is used only as a tool, an aid to allow the gain of time in specific units of mathematics, especially if it is built with a view to simplification of rules of logic or deductive calculation. Its use moves parallel to traditional modules way, only used as a supply for the need in obtain a quickly progress to a result that is significantly more important than any algorithmic, becoming only a mechanical support. The advanced module can be seen here as the "glue" that binds various segments of reasoning, effectively, be built over the result, taking the student opportunity to learn of the different components of the process, without ever getting lost in too many calculations prepared and/or strenuous forming disperse their attention from the didactic component held by the teacher.

In a second implementation phase, the advanced module gradually replaces the traditional modules, in that it actually becomes more efficient. The teacher can introduce some concepts to enable the student to creatively manipulate the module, which can be based only for a few rules, allowing the student the opportunity to restructure and adapt to their way of thinking and acting on the contents. Basically, here are dictated the basic rules of a game, in which the student is the main player, and the teacher only needs to arbitrate the use of the same. The teacher should let the student play, and take pleasure in doing so.

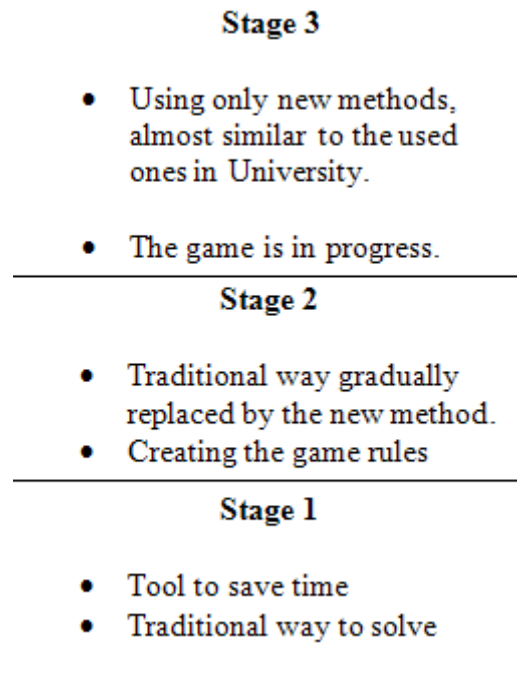


Figure 1: Stages of the proposed model

The third phase of implementation of advanced module is recommended to be performed in the laboratories of mathematics outside the classroom, in group work or individual with the ultimate goal of full implementation of the contents of the support module, in scientific terms. At this stage the teacher acts as a dynamic agent about student learning by introducing the concepts missing, and helping the student in his walk, orienting it so that there is significant progress in the understanding of the advanced scientific module. The student should always be encouraged, and encouraged to share their findings and find ways to communicate them to their colleagues. This communication is where the student must use all their creativity. For example, the public presentation of the project to the school community in cyclical exposures integrated into the school project is a good chance, or else the presentation, more guarded, to other groups, other laboratories Mathematics from other schools.

As example of application of this simple method, and to illustrate these three phases, we consider part of the content of the discipline of mathematics whose primary goal is learning methods of solving linear equations systems, with three unknown variables, which is integrated in the study of the relative position of planes, more specifically in Geometry. Existing here, are the traditional methods of resolution: orderly replacement and addition of equations, which are taught in the 11th and 12th years of schooling method in Portugal, the equivalent to K11 and K12 in USA. The advanced scientific module is introduced as part of the curriculum of the discipline of Linear Algebra in the 1st year of a university course of Mathematics, which is to solve systems of equations using the elimination method of Gauss.

Consider then the problem of solving the following system of linear equations, whose equations are already in

$$\text{canonical form: } \begin{cases} 2x - y + 3z = 1 \\ x + y - z = 1 \\ 3x - y + z = 0 \end{cases} .$$

Stage 1 - Based on the syllabus of the course, the resolution of such systems is performed using the method of ordered addition, where the equations of the system are multiplied limb from limb by nonzero constants, and added two by two so that the coefficient of terms of a variable, this sum is set aside. This process generates always a bit of confusion in writing it by the student, often getting halfway to the process due to faults in algorithmic organization. This mental clutter and writing often goes hand in student's reasoning to the need to use the least common multiple, $lcm(a,b)$, between two numbers, as an aid to simplify the calculation as well as the polynomial addition.

Thus, introducing new organization of the coefficients in matrix form so that they remain arranged in the column respectively associated with each of the variables the variable x in the 1st column, the variable y in the 2nd column and the variable z in the 3rd column, positioning the constant terms in the fourth column, you gain advantage in terms of writing

and also organization. The resulting matrix would be the type $\begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 1 & -1 & 1 \\ 3 & -1 & 1 & 0 \end{bmatrix}$. Now begins the process of adding

ordered using only the coefficients by multiplying each of the lines by a non-zero constant and adding them two by two, by replacing one of the lines created with the new addition. The process should be guided by the teacher so that one line is secured, preferably a line that has three non-zero coefficients simultaneously, and then the addition must be addressed in order to undo a column of the coefficients of a particular variable. For example, choosing the 2nd row as fixed, we can eliminate the coefficients in y , adding the 1st line with the 2nd and 3rd to 2nd.

The result would be a matrix $\begin{bmatrix} 3 & 0 & 2 & 2 \\ 1 & 1 & -1 & 1 \\ 4 & 0 & 0 & 1 \end{bmatrix}$. Now run up the ordered only with the addition two new lines

created by fixing one and eliminated one of the other affections coefficients to the variables x and z . Here in this example, we can see that the coefficient on z has been canceled on the 3rd line. The goal will be to build at least one line in which two affections zero coefficients arise to variables x , y and z . Shortly after the system should be rebuilt in its usual format and solved the system by simple substitution.

Stage 2 - The professor details the basics of handling the method of Gaussian elimination, see Giralde [3], identifying the diagonal of the matrix, giving the exchange properties of rows and columns, so that the diagonal arises whenever possible nonzero elements. The ordered item must now follow the usual format of the method of Gaussian elimination, which lies in fixing the 1st line, eliminating the coefficients in x , except the 1st line after setting the 2nd line, eliminating the coefficient y of 3rd line. Passing, in the end, the original format of the system, then the system is solved by substitution.

At this stage it is important that the method is feasible and trained with certain systems, and only then begin to be exploited to solve impossible and possible systems, uncertain systems. This classification should be part of the rules imposed on the game, so that students see that it is a byproduct of the rules themselves.

Stage 3 - In the laboratory can now be crafted the concept of matrix, and each element is represented by usual $a[i,j]$ notation as well as a more technical writing that is as close as possible to what is usual in the method of Gauss elimination from the initial matrix until the final matrix. The teacher encourages the full use of $lcm(a,b)$ between numbers, as well as writing the type of operations carried $L_i = L_i + K * L_j$, to signify that the line i was replaced by row i added a multiple k of row j .

4. CONCLUSIONS

The exemplification of these three stages is obviously not prepared exhaustively here in this article, for their integral development, in all its variants in the proposed exercise would lead to an excessive number of pages in relation to the limit for this paper, but the author hopes that it became clear the distinction between the three phases of implementation to the reader, at a level that, then for future developments, the whole process becomes more noticeable. So, in the near future, more examples will follow, perfecting the method creating the suitable background to discuss with educators from around the world, through the share of experiences in other countries, in order to fulfill the adequate formulization of this method.

The author is fully aware that the introduction of advanced scientific modules with even a hint of strategic introduction of same, is always a sensitive issue, because we can always find some resistance from educators, but if we take the example of the introduction of calculator in teaching, this resistance will dissipate over time, as the modules will bring the discipline new tools and new ways of thinking and acting on the content taught in secondary education.

5. REFERENCES

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