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Examining the Differences on Comparing Fraction Size for 5th-Graders between Contextual and Numerical Problems

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ABSTRACT—*The purpose of this study was to investigate the performance and difference of fifth graders in Taiwan between contextual and numerical fractional problems. 355 students from 7 elementary schools in Southwest area of Taiwan were selected to join this study. The t-test result indicates that the fifth graders performed significantly better on numerical problems than contextual problems when comparing the fractional size. Data also indicates that there are statistically significant differences on the problems of same numerator, same denominator, and transitive problems between situated and numerical problems, except the residual problems.*

Keywords— fractional size, transitive strategy, residual strategy, contextual problems, numerical problems

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1. INTRODUCTION

Several research studies and reports (Cramer, Post, & delMas, 2002; National Council of Teachers of Mathematics [NCTM], 2000; Yang, 2005; Verschaffel, Greer, & De Corte, 2007) highlighted that recognizing the relative magnitude of numbers is an important component of number sense. The study of Yang (2005) suggested that understanding the relative magnitude of numbers will affect the development of other ability (e.g. estimation, number sense). The earlier studies (Cramer et al., 2002) showed that four different strategies (same numerators, same denominators, transitive and residual strategies) were found when children were asked to compare fractional size. They illustrated that these four different strategies are meaningful and efficient ways as compared with the standard written algorithms. Moreover, several reports and studies (NCTM, 2000; Organization for Economic Co-operation and Development [OECD], 2004; Yang, 2006; Van den Heuvel-Panhuizen, 2001; Yang & Wu, 2010) highly suggested that realistic situations should be connected with the mathematics learning. We always heard students in the class complain, "Why we need to do these boring computation?" Or "When do we need to use these formulas?"as students are asked to do mathematics exercises. The mathematics teaching and learning of students in the classroom usually has no connection with their real world (Sparrow, 2008; Yang & Wu, 2010). This brings the result that children always do not like to learn mathematics. Therefore, mathematics teaching and learning should connect with real world to prepare children to have the ability to solve mathematical problems of everyday living. Due to the importance of understanding the relationship between fractional size and the contextual, the purposes of this study were to investigate the difference of fifth graders in Taiwan on contextual and numerical problems when comparing the fractional size. The research question is as follow:

Are there any statistically significant differences between contextual and pure numerical problems when comparing fractional size?

2. BACKGROUND

2.1 The related studies on comparison of fractions

Several earlier studies (Cramer, et al., 2002; Yang, Reys, & Reys, 2009) suggested that recognizing the fractional size play a key role on the development of fraction concepts. The study of Markovits & Sowder (1994) found that about 42% of seventh graders tried to use written methods (finding the common denominator or changing the fractions to decimals) when compared $\frac{5}{7}$ and $\frac{5}{9}$. About 25% of them didn't know how to solve it or gave an incorrect answer. The study of Reys & Yang (1998) and Yang & Reys (2002) also found that there were high percentage of sixth and eighth graders could not meaningfully compare fractions. This is due to they are lacking of fractional sense.

The earlier teaching experimental studies of Cramer et al. (2002) indicated that four different strategies could be developed by students under meaningful learning and teaching when comparing fractions. These strategies are:

Same numerator strategy: This implies that two fractions with same numerator and different denominators, then the fraction is larger when the fraction's denominator is divided into smaller pieces. For example, when compare $\frac{3}{4}$ and $\frac{3}{8}$, students know that $\frac{3}{4}$ is the larger fraction due to fourths are larger than eighths and three larger pieces must be bigger that three smaller pieces (Cramer, et al., 2002).

Same denominator strategy: This implies that two fractions with same denominator and different numerators, then the fraction is larger when the fraction's numerators is bigger. For example, when compare $\frac{3}{5}$ and $\frac{2}{5}$, students know that   is the larger fraction due to the denominators were divided into parts and each part is same size, then three parts are greater than two parts (Cramer, et al., 2002).

Transitive strategy: This means that students can use external value as a benchmark to compare two fractions. For example, when students are asked to compare fractions $\frac{3}{5}$ and $\frac{4}{9}$, they know $\frac{3}{5}$ is greater than $\frac{4}{9}$ because $\frac{3}{5}$ is greater than $\frac{1}{2}$ (applying $\frac{1}{2}$ as a benchmark) and $\frac{1}{2}$ is greater than $\frac{4}{9}$ (Cramer, et al., 2002).

Residual strategy: This implies that students are able to switch back and forth between the fraction and its complement. For example, when students are asked to compare $\frac{4}{5}$ and $\frac{8}{9}$, they know $\frac{8}{9}$ is larger than $\frac{4}{5}$, because $\frac{8}{9}$ + $\frac{1}{9}$ = $\frac{4}{5}$ + $\frac{1}{5}$ and the missing part $\frac{1}{9}$ (of $\frac{8}{9}$) is less than the missing part $\frac{1}{5}$ (of $\frac{4}{5}$).

Cramer, et al. (2002) believed that students can apply these strategies when comparing fractions is a good indicator of number sense. In order to examine fifth graders in Taiwan whether they can apply these strategies in comparison of fractions, this study designed the tests items based on the above framework.

2.2 Situated learning related studies

Several studies and reports (Griffin, 1995; NCTM, 2000; Yang, 2006; Yang & Wu, 2010) suggested that number sense highly relate to situations. Moreover, many studies (Yang & Huang, 2004) indicated that many students are lack of understanding on number and operations and application when solving word problems. For example, the result of 2003 TIMSS showed that Taiwanese fourth graders had poor performance on situated problems (Mullis, Martin, Gonzalez, & Chrostowski, 2004). It is difficult for children to apply the fractional knowledge they learned in school to real-life situations (Mack, 1990). Moreover, Sowder & Kelin (1990) suggested that the best way for helping children recognize the magnitude of fractions and understand the meanings of fractions is through the situated problems.

Several studies suggested that many students have better performance on solving situated problems due to realistic problems will help them to understand the meanings of problems (Griffin, C. & Jitendra, 2009; Griffin, S., 2004). However, some studies (Reuser & Stebler, 1997; Wyndhamn & Säljö, 1997; Yang & Wu, 2013) showed that children have poor performance when solving word problems. Therefore, this study was going to investigate the difference between numerical problems and contextual problems and examine the use of the strategies when comparing the fractions.

2.3 Method
2.3.1 Participants

Six schools with 355 Taiwanese fifth-graders participated into this study. Two schools were from city, suburban areas, and rural areas respectively. The students in the study come from families with a wide range of occupations, incomes, and educational levels. Therefore, the sample students included different background.

2.3.2 Instruments

Paper-and-pencil test. For testing we designed two different types of 16-item forms for the contextual problems and pure numerical problems (See appendix 1). The design of these items was based on the earlier studies (Cramer, et al., 2002) that four different strategies (same numerator, same denominator, transitive, and residual strategies) used in comparing fractional size. Each subscale included 4 items, therefore, the contextual and pure numerical problems included 16 items separately. Each test was given for a period of about 40 minutes. The two tests were parallel in contents: the numbers used in the two tests were the same but presented in different ways. For example, item 4 in the two tests as the following:

Numerical problem: 4. Which one $\frac{4}{9}$ or $\frac{4}{12}$ is larger? Please explain your reasons.

Contextual problem: 4. A bag includes 36 dumplings. Tim ate $\frac{4}{9}$ of a bag and Jane ate $\frac{4}{12}$ of a bag dumplings. Who ate more dumplings? Please explain your reasons.

Researchers conducted a pilot study with 45 fifth graders participated the paper-and- pencil test and 2 fifth graders were interviewed, to ensure that items were of appropriate difficulty and the test time for all tests and interview were appropriate. The students' backgrounds and age in the pilot were comparable to the participants in the full study.

2.4 Analysis

Paper-and-pencil tests. Students were asked to write down the answers and reasons for the tests. Therefore, a special scoring rule was determined as below:
1.) If both the answer and the reasons were correct, 3 points were given;
2.) If the answer was incorrect, but the reason was correct, 2 points were given;
3.) If the answer was correct, but the reason was incorrect, 1 point was given;
4.) If both the answer and the reason were incorrect, no score was given.
Cronbach's alpha reliability coefficients for the contextual problems and pure numerical problems were .909 and .922 respectively, calculated with the SPSS 12.0 software. The mathematics educators and three teachers were invited to review these tests. They all agreed that the items in the tests and interview were of appropriate difficulty and the contents of the tests could reflect the purposes of this study; therefore, these tests had good content validity and specialist validity.

2.5 Results

2.5.1 The performance and difference on contextual and numerical problems

Table 1 reports the results of mean scores, standard deviation, and t-test. Results show that there is a statistically significant difference between numerical problems and contextual problems when comparing fractional size for the 5^{th}-graders in Taiwan, t (354) = -3.807, p < 0.000, with the numerical problems receiving higher scores than contextual problems. Moreover, sample students' mean score was 19.96 and the correct percentage was 42% on the numerical problems which was higher than that on contextual problems (mean score was 18.18, correct percentage was 38%). We can observe that sample students' problem-solving performance on numerical problems was better than that on the contextual problems.

Table 1:
The results of mean scores, standard deviation, and t-test for contextual and numerical problem (N=355)

Tests	Mean Scores	Standard Deviation	Correct percentage	T-value
Contextual	18.18	11.43	38%	-3.807*
Numerical	19.96	12.54	42%	

Note. * *p* < 0.05, Each test included 16 items and Total scores=48

Since there is a statistically significant difference between contextual and numerical problems when comparing fractional size, this study further examined the difference between contextual and numerical problems for the four different aspects. Table 2 reports the results of the t-test for the four different aspects. Data shows that there are statistically significant difference between contextual and numerical problems for the same numerator, same denominator, and transitive strategies, t (354) = -3.881, p < 0.000, t (354)= -3.663, p < .000, and t (354) = -3.669, p< .000, with the numerical problems receiving higher scores than contextual problems for the same numerator, same denominator, and transitive related problems, respectively. However, there is no significant difference between the two tests on the use of residue related problems at α=.05 level. Except the performance on the use of residue strategy, Taiwanese 5^{th}-graders significantly performed better on the numerical problems than the contextual problems for the same numerator, same denominator, and transitive strategies. Data also reports that students had worst performance on the use of residual strategy among the four different aspects. Especially, these students had better performance on the contextual problems (30%) than the numerical problems (28%) for the residual strategy.

Table 2:
The T-test results of contextual and numerical problems for 4 different aspects (N=355)

		MS	SD	Correct %	t-value
Same numerator	Contextual	4.41	4.33	37%	-3.881*
	Numerical	5.12	4.42	43%	
Same denominator	Contextual	5.94	3.19	50%	-3.663*
	Numerical	6.64	3.76	55%	
Transitive	Contextual	4.24	3.68	35%	-3.669*
	Numerical	4.87	3.99	41%	
Residual	Contextual	3.59	3.34	30%	1.737
	Numerical	3.32	3.54	28%	

Note. $* \, p < .05$

Furthermore, Table 2 also shows that these students had the best performance on the use of same denominator strategy among the four strategies no matter on numerical or contextual problems. Data also shows that sample students had higher standard deviations on the use of same numerator strategy. This implies that these students had higher difference than other aspects when solving same numerator problems for both numerical and contextual problems.

3. DISCUSSION AND CONCLUSION

The result of t-test shows that there is a statistically significant difference between numerical problems and contextual problems for 5th-graders in Taiwan when comparing fractional size. Moreover, the data also shows that the correct percentage on contextual (38%) and numerical problems (42%) are less than a half. It seems reasonable to believe that it is difficult for these students to apply the strategies, such as same numerator, same denominator, transitive, or residual strategies, when comparing fractional size. Data further shows that sample 5th-graders performed better on numerical problems than contextual problems. This finding is consistent with the earlier studies (Griffin, C., & Jitendra, 2009; Wyndhamn & Säljö, 1997; Yang & Huang, 2004) that children have poor performance on solving word problems. In fact, this finding is different from the earlier studies that many students have better performance on solving situated problems due to realistic problems will help them to understand the meanings of problems (Wagner, 2000, 2002).

Data also show that there are statistically significant differences between numerical problems and contextual problems when comparing fractional size in the dimension of "the same numerator", "the same denominator", and "transitive" strategy for the 5th-graders in Taiwan at .05 level. Only "residual strategy" did not reach difference between numerical problems and contextual problems when comparing fractional size at .05 level. According to the statistic data analysis, the sample students' mean score and correct percentage on numerical problem test was better than that on contextual problem test in each dimension, except for "residual strategy" (3.32＜3.59；28％＜30％).This specific situation might be not only due to the residual related problems are more difficult to these students, but also due to the numbers used in the residual strategy related problems are more complex. Furthermore, there is no similar question taught in the class when we reviewed the textbooks used by these students. All of the teachers whose students participated the tests responded: "Our textbooks do not tell us to teach children to solve this kind of problems by using residual strategy. We taught similar problems, such as "Which one of $\frac{8}{9}$ and $\frac{9}{10}$ is bigger?" we usually taught our kids to find the same denominator $\frac{8}{9} = \frac{80}{90}$ and $\frac{9}{10} = \frac{81}{90}$, and knew the $\frac{9}{10}$ is larger. Therefore, it is difficult for kids to use the residual strategy." It is reasonable to believe that they had poor performance on using the residual strategy. Moreover, the reasons why these students had higher percentage on the use of residual strategy for the contextual problems (30%) than numerical problems (28%)? It probably needs more studies to investigate it.

4. IMPLICATIONS

The poor performance on the use of transitive strategy and residual strategy when comparing fraction size suggest that the teaching of transitive strategy and residual strategy should be discussed in mathematics textbooks. This study shows that 5^{th}-graders in Taiwan can apply the same denominator strategy and the same numerator strategy to solve the problems, but they perform poor on the use of transitive strategy and residual strategy flexibly. According to Cramer et al (2002), they found that students can develop the same numerator, the same denominator, transitive, and residual strategy when comparing fractional size after experimental teaching. This implies that appropriate teaching focuses on the use of theses strategies can efficiently promote students' development on these strategies. Therefore, we strongly recommend that the transitive strategy and residual strategy should be edited into mathematics textbooks and teaching in the school mathematics classrooms in Taiwan, which can promote students' thinking when comparing fractional size.

Research studies show that many students lack the use of residual strategy to solve the problems because they are used to finding "the common denominator" when comparing fractional size. They did not think the relation between number and fraction so they cannot find another way to solve the problems naturally, which restrained them from developing number sense. Therefore, in order to develop students' effective learning strategy, we should avoid them learning written computation too early (Cramer, Post, & delMas, 2002; Yang et al., 2009; Yang, Li, & Lin, 2008). The shortcomings of the students are result of teaching and learning strategies, which should promote the use of transitive and residual strategy. This is highly related with teacher training and the future teachers must meet these aspects. Furthermore, the developers of curricula must be alerted to these difficulties of students so that the textbooks must be adjusted to mathematics teaching and learning. Finally, the potential influence of learning computation in use of the strategies referred should be a possible study to develop in the future.

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