Images Reconstruction Algorithm Based on DCT and Compressive Sensing

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ABSTRACT— ***In order to improve the quality of reconstructed images, a method combining DCT algorithm and compressive sensing theory is presented. Firstly, the DCT transform is applied to change an image to sparse domain, and then the high-frequency coefficients of the sparse domain is measured by using a Gaussian random, finally, the algorithm of OMP is used to reconstruct the image. Compared with the direct compressive sensing algorithm, simulation results demonstrated that the presented algorithm improved the quality of the reconstructed image significantly. For the same number of sampled data sets, the PSNR of the presented algorithm was improved about 5 dB.***

Keywords— Compressive Sensing, Images Reconstruction, Orthogonal Matching Pursuit, DCT transform.

1. INTRODUCTION

In order to avoid losing information, when a digital system capture a signal or an image, one must sample at least two times faster than the signal bandwidth according to Nyquist sampling theorem. In many applications, including digital image and video cameras, the Nyquist rate is so high that too many samples result, making compression a necessity prior to storage or transmission. In other applications, such as radars, increasing the sampling rate is very expensive or impossible [1]. Compressive sensing, a recent years developed new theory[1,2], make representing signals at a rate significantly below the Nyquist rate to be possible. Compressive sensing theory, being presented since 2006, has been widely concerned by the international academic community, and obtained preliminary application in a single pixel image, analog information conversion, medical imaging, remote sensing and many other aspects. Some institutions in china have carried out the research work of the CS, CS theory and method is becoming one of the researching hot spot of modern information theory and practice [3,4,5]. In paper [6], an image compressed sensing algorithm based on wavelet tree structure and iterative shrinkage was proposed, and the algorithm combined wavelet theory and CS theory together achieved good compression effect. Discrete cosine transform (DCT) is the most close to the K-L transform algorithm in the aspect of removing correlation component, and because it has a lot of fast algorithm, easy to realize, so it is often used in signal processing and lossy image compression. This paper addresses compressive sensing to show how DCT sparse processing of an image can be added to before measurement procedure. The use of the DCT sparse processing is shown to have advantages in terms of PSNR.

The remaining of the paper is organized as follows. Section II briefly introduces the basic concepts of compressive sensing. Following, section III presents the idea of adding DCT sparse transformation to the image before measurement procedure and the image reconstruction method of combining DCT transformation and compressive sensing theory. The corresponding reconstruction computer simulation procedure and results we obtained by applying this algorithm to the Lena image are described in section IV. Finally, section V presents our conclusions.

2. THE COMPRESSIVE SENSING PROBLEM

Suppose a signal in RN can be represented in terms of a orthogonal basis of N×1 vectors $\{\psi_i\}_{i=1}^{N}$. Using the N×N basis matrix $\Psi = \lfloor \psi \quad \psi_2 \quad \dots \quad \psi_N \rfloor$ with the vectors $\{\psi_i\}$ as columns, a signal X can be expressed as

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$$X = \sum_{i=1}^{N} s_i \psi_i = \Psi S \tag{1}$$

where S is the N×1 column vector of weighting coefficients. Obviously, X and S are equivalent representations of the signal, with X in the time or space domain and S in the Ψ domain. The signal X is K-sparse if it is a linear combination of only K basis vectors; that is, only K of the si coefficients in equation (1) are nonzero and (N−K) are zero. The case of interest is when K<N. The signal x is compressible if the representation (1) has just a few large coefficients and many small coefficients. According to compressive sensing theory[1,2], random linear measurements are generally valid for the reconstruction of signals that are sparse in some domain. If Φ is a M×N (M<N) random matrix with normal independent identically distributed entries, then the linear measurements defined by

$$\mathrm{Y} = [y_1 \quad y_2 \quad \cdots \quad y_M]^T = \Phi X = \Phi \Psi S = \Theta \cdot \mathrm{s} \tag{2}$$

where Y are M dimensional measurements.

The problem remains on how to reconstruct the original samples from the random measurements Y. It is an optimization problem defined as

$$\hat{X} = \arg\min \|X\|_0 \qquad st. \qquad \Phi X = Y \tag{3}$$

where $\|X\|_0$ means the 0-norm, that is the number of non-zero entries in s. Unfortunately, a direct approach to (3) is both numerically unstable and NP problem. Normally, an approximate solution is to use the 1-norm instead of the 0-norm, so the reconstruction problem in this case is

$$\hat{X} = \arg\min \|X\|_1 \qquad st. \qquad \Phi X = Y \tag{4}$$

Normally used approach to equation (4) is linear program known as basis pursuit, matching pursuit, orthogonal matching pursuit (OMP) algorithm [7], and so on. In this paper, the OMP algorithm is exploited to reconstruct images.

3. COMPRESSIVE SENSING METHODS BASED ON DCT TRASFORMATION

One of the application bases for compressive sensing theory is signal sparse. Classic sparse transformations include discrete cosine transform (DCT), discrete Fourier transformation (DFT) and discrete wavelet transformation (DWT). In paper [3] and [4], the wavelet transformation is used to change the image to sparse signals. Actually DCT is most close to the K-L transformation of optimal transformation performance. So it is often used in signal and image processing. Paper [5] studied on a method of partial two-dimensional DCT combined CS used in the image coding & decoding systems. DCT has the property of centralizing energy to low frequency. For example, the DCT coefficients for 7×7 pixels of an image and its pixels are shown separately in fig.1 (b) and fig.1 (a). The elements near zero, that is sparse components, are in lower right corner. The original image is decomposed into high frequency band and low frequency band after DCT transform, high frequency band can be considered to be sparse, as illustrating in Fig.1b, but the low frequency band coefficients are not sparse. If all the coefficients together are compressed by equation (2), done as paper [5], the correlation between low frequency approximation coefficients may be destroyed, and leads to reconstruction error.

The idea of compressive sensing method based on DCT is as follows. Firstly, pixels of an image are change into

214	212	212	211	211	211	211
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214	212	212	211	210	210	209
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213	212	212	211	210	210	209
212	212	212	211	210	210	209

(a) 7×7 Image Pixels

1478.0	8.900	0.8000	0.9000	0.4000	2.0000	0.3000
1.4000	-0.0000	1.2000	0.7000	0.6000	0.2000	0.4000
0.4000	-1.2000	0.0000	-0.4000	-0.2000	-0.3000	0.1000
0.7000	-0.7000	0.4000	0.0000	0.1000	-0.1000	0.2000
0.5000	-0.6000	0.2000	-0.1000	0.0000	-0.1000	0.1000
0.5000	-0.2000	0.3000	0.1000	0.1000	-0.0000	0.1000
0.0000	-0.4000	-0.1000	-0.20000	-00.1000	-0.1000	0.0000

(b) Coefficients of DCT for the Image Block

Fig. 1: DCT's Property of Centralizing Energy to Low Frequency

frequency domain by two-dimensional DCT transform, and then construct the measurement matrix. Because of the low frequency component contains the main information of the image, while the high frequency compared with it, will be less important. Random measurement is applied only to the high frequency components to achieve the purpose of compression. The concrete procedure of the algorithm can now be stated as follows.

Step 1, Compute the coefficients of an image of N×N using two-dimensional DCT transform, and partition the matrix of these coefficients into four sub-band {LH1, HL1, HH1, LL1}, which is similar to the results of wavelet decomposition.

Step 2, Select the appropriate M value, construct M×N/2 size random measurement matrix Φ ,where the matrix elements $\phi_{i,j}$ is independent and identically distributed random variables from a Gaussian probability density function with mean zero and variance 1/N. LH1, HL1and HH1 is measured separately using equation (2), and obtain 3 sub-band measurement values, but the LL1 of low frequency sub-band coefficients remain unchanged.

Step 3, Reconstruct the 3High frequency coefficient matrixes, $\widetilde{LH}1$, $\widetilde{HL}1$, $\widetilde{HH}1$, using OMP algorithm[7], and obtain the recovered image using 2D IDCT transform for $\widetilde{LH}1$, $\widetilde{HL}1$, $\widetilde{HH}1$ and $LL1$.

4. EXPERIMENTAL RESULTS

Take the 8bit Lena image of 256×256 size as example to evaluate the effectiveness of the presented method. Firstly, we directly use compressive sensing theory for Lena image, that is, the random measurement is directly applied to Lena image rather than the DCT coefficients of the image. When compressing rate (here refers to the number of samples of compressed sensing image and the original sample size ratio) is set to 0.7, the recovered image is shown in figure 2 (b), while original Lena is shown in fig. 2 (b). One can see that the image distortion is obvious. The recovered image is shown in figure 2 (c), when the algorithm proposed in this paper is used also with the same compressing rate. Obviously,

(a) Original Image (b) Recovering Image Using Direct CS （c）Recovering Image Using the Proposed Method
Fig.2: Lena images

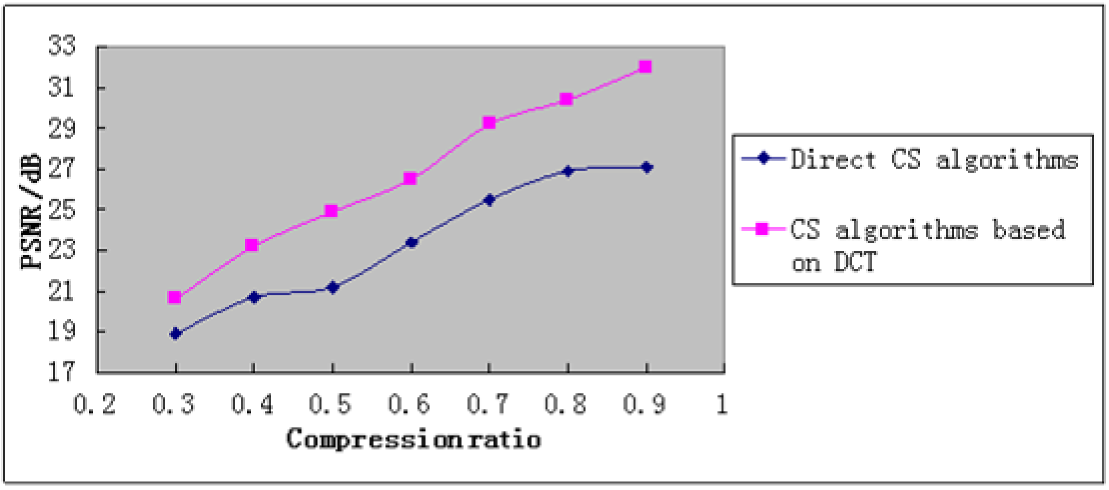

Fig.3: The PSNR values at different Compressing rate for the two algorithms

the quality of the recovering image in fig. 2 (c) is much better than in fig. 2 (b).

As illustrated in Fig. 3, the PSNR of recovering images based on proposed method is about 5 dB higher than the direct compressive sensing methods at different compressing rate.

5. CONCLUSIONS

An inherent shortcoming of digital images is the huge amount of data which conflict with the transmission capacity of the internet. So the digital image compression and recovery is a very hot research domain. Through the study of the proposed image compression and recovert based on DCT algorithm, one can get the conclusions that compared to the direct image compressive sensing, use of compressive sensing technology combined with DCT algorithm can achieve bigger image compression rate at the same PSNR.

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