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Robust H_∞ Triggered Control of Linear Systems

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ABSTRACT— ***In this paper, we study the impact of triggered control strategies for a class of uncertain linear systems. The event condition is proposed based on the relative error between the current state and the state at last sample time. We introduce robust H_∞ theory into self-triggered sample strategy, and achieve both resource utilization and disturbance attenuation properties. We investigate the full-information feedback H_∞ control for the perturbed linear system and develop a linear matrix inequality (LMI)-based sufficient condition guaranteeing the robust asymptotic stability of the closed-loop system. Employing a self-triggered control, we provide a method of designing the longest sampling period for the relevant H_∞ controller when the system is stabilized and robust. A cart and pendulum example is given to show the efficiency of the theoretical result.***

Keywords— Event-triggered control, Self-triggered control, Robust H_∞ control, Cart and pendulum system event-triggered control, Self-triggered control, Robust H_∞ control, Cart and pendulum system

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1. INTRODUCTION

The Event-triggered control has received considerable attention in recent years. It has the apparent benefits of reducing the resource utilization, including communication load and processor, and providing improved system robustness. It requires however persistent information of the system states to determine at which time the control input should be updated and transmitted to the plant. Self-triggered scheme was then proposed to eliminate this disadvantage, however on the expense of sacrificing the system robustness.

For systems with sensors, actuators and controllers distributed over networks, the premise of lossless data communication frequently adopted in conventional control design theory is no longer valid. This is due to the uncertainties such as data packet loss, corruption by noise, delay etc. associated with the communication line. Some of these communication networks used nowadays are Ethernet, FireWire and DeviceNet [1]. Control design for systems over networks, therefore has gained a lot of research interest in the recent years [2]. In the literature, a common approach to this problem is to transmit the control signal periodically, since this simplifies the design. However for this approach, lack of time synchronization causes problems in systems with uncertain delays, sampling jitter and multiple sampling rates etc. Further, the transmission line is not used efficiently in this technique, as the periodic transmission of the signal is independent of the status of the transmission link.

To alleviate the unnecessary waste of communication and computation resources, an alternative to periodic control, namely, event-triggered control has been proposed [3]-[6]. Effectively, event-triggered control is introduced for the possibility of reducing resources usage (that is, sampling rate, CPU time, and network access frequency) while preserving systems stability in networked control systems (NCSs) and embedded control systems [7]. The triggering mechanisms are referring to the situation in which the control signals are kept constant until the violation of a condition on certain signals of the plant triggers the re-computation of the control signals.

In this regard, several issues in event-based control signal transmission have been considered. These include level crossing sampling, amplitude sampling or magnitude driven sampling and Lebesgue sampling are a few names used to describe this scheme in literature. The event triggered data communication utilizes the transmission medium very efficiently. The control signals are sent only in the case of an occurrence of an event contrary to the periodic execution approach reducing the load on the communication line significantly. This approach has also been adopted in industry for some time, since it is easy to implement and efficient in performance. Complementary to the wide applications of this scheme little literature is available. The reason for little work on this subject is due to its complicated nature, since it incorporates a hybrid dynamical system (continuous-time system dynamics and discrete-time control strategy together). Most of the work done though does not emphasis the analysis of the discrete time part. Some recent works have presented few fundamental results for the even-triggered control methods [8]-[9], [10]-[11].

The advantages of event-triggered control are generally well-motivated and some practical applications show its potential. However, there are still some problems that need to be addressed before event-triggered control can be fruitfully applied in networked-control systems (NCs). Looking at the literature on event-triggered control, it turns out that most of the work considers static state feedback controllers, which assumes that full plants state can be measured. As in many control applications the full state information may not be available for measurement, it is important to study stability and performance of event-triggered control systems with static and dynamical output feedback based controllers. However, there are not many theoretical results on this problem in the literature. It must be emphasized that the fact that the controller is based on output feedback instead of state feedback does not allow for straightforward extensions of existing event-triggering mechanisms if a minimum-time between two subsequent events has to be guaranteed.

In [20]–[31], several different event-triggering mechanisms and control strategies are proposed. For instance, in [20] and [21], an impulsive control action is applied to the system that resets the state to the origin every time the state of the plant exceeds a certain threshold. The analysis is performed for first-order stochastic systems, as analysis of larger-dimensional systems is difficult, and it is shown that the variance of the state is smaller when compared to a sampled-data controller, while having approximately the same number of control updates. Another interesting approach to event-triggered control is presented in [22]-[24], in which the system is controlled in open loop, using an input generator that uses a prediction of the states of the plant to produce a control signal. These predicted states are only corrected in case the true plant state deviates too much from its predicted value. Such a deviation can be caused by disturbances, [22]-[23], or by the fact that the plant model is incorrect [24].

In order to obtain a persistent state information to detect the event condition, particular hardware is required in the event-triggered control system. Such a hardware is not often available. Self-triggered paradigm was then proposed to eliminate this disadvantage. The rationale behind self-trigger is to merge the advantage of time-triggered and event triggered implementation: reduce the usage of the system resource without relying on additional hardware. Looked at in this light, the state of the plant are measured (or estimated) to compute the next sampling time in different methods. It turns out that they have one thing in common; the robustness of the system is sacrificed, as mentioned in [11].

Therefore, in this paper, we investigate triggered control strategies for a wide-class of linear systems subject to norm-bounded parametric uncertainties. The event condition is proposed based on the relative error between the current state and the state at last sample-time. We seek to design a robust H_∞ controller in a self-triggered paradigm. Toward our goal, we examine a full-information feedback H_∞ control based on the event-triggered scheme for the perturbed linear system in a very general form. We then establish a sufficient condition which guarantees the robust asymptotic stability of the system. The developed condition is expressed in the form of linear matrix inequality (LMI). We proceed by deriving a self-triggered control system and provide an estimation of the lower bound of the control execution time for the relevant H_∞ controller. A cart and pendulum system is considered for computer simulation to illustrate the effectiveness of the theoretical results.

Fact 1 For any real matrices $\sum_1$, $\sum_2$, and $\sum_3$ with appropriate dimensions and $\sum_3^t \sum_3 \le I$, it follows that

$$\sum_1 \sum_3 \sum_2 + \sum_2^t \sum_3^t \sum_1^t \le \alpha \sum_1 \sum_1^t + \alpha^{-1} \sum_2^t \sum_2 , \quad \forall \alpha > 0$$

2. PROBLEM STATEMENT

Consider the linear uncertain continuous-time system

$$\dot{x} = [A + \Delta A]x(t) + [B + \Delta B]u(t) + \Gamma w(t)$$
$$z = [C + \Delta C]x(t) + [D + \Delta D]u(t) + \Phi w(t) \tag{1}$$

where $x \in \Re^n$ represents the system state, $x(0) \in x_0$is the initial state, $w \in \Re^p$, $|w| \le W|x|$, $\forall t > 0$, $W > 0$ denotes the exogenous bounded, $u \in \Re^m$, and $z \in \Re^q$ are the control input and output signals, respectively. Matrices $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $C \in \Re^{q\times n}$, $D \in \Re^{q\times m}$, $\Gamma \in \Re^{n\times p}$, $\Phi \in \Re^{q\times p}$, with (A, B) controllable and (A, C) observable. The uncertain matrices $\Delta A, \cdots \Delta D$ are represented by

$$[\Delta A \;\; \Delta B] = M_1 \Delta [N_1 \;\; N_2], [\Delta C \;\; \Delta D] = M_2 \Delta [N_1 \;\; N_2], \Delta\Delta^t \le I \tag{2}$$

In the sequel, we consider that this system is working in closed-loop configuration with a state feedback controller

$$u(t) = Kx(t) \tag{3}$$

which is only computed and transmitted on discrete time instants t_j. After one control input update, $u(t) = [K + \Delta K]x(t_j)$, the closed-loop system is described by:

$$\begin{aligned}
\dot{x}(t) &= [A_K + \Delta A_K]x(t) + [BK + \Delta B_K]e(t) + \Gamma w(t) \\
z \quad &= [C_K + \Delta C_K]x(t) + [DK + \Delta D_K]e(t) + \Phi w(t) \\
A_K &= A + BK, \Delta A_K = (\Delta A + BK), \Delta B_K = \Delta BK \\
C_K &= C + DK, \Delta C_K = (\Delta C + \Delta DK), \Delta D_K = \Delta DK \\
e(t) &= x(t_j) - x(t), t \in [t_j + t_{j+1})
\end{aligned} \tag{4}$$

where $e(t)$ is the measurement error. Motivated by [10], the robust event detection condition can be expressed as:

$$\|e_k(t)\|^2 < \|x(t)\|, \sigma > 0, \forall \Delta: \Delta\Delta^t \leq I \tag{5}$$

which indicates that if and only if this condition is violated for all admissible uncertainties, the feedback loop is closed. In this regard, the problem of interest is to determine a suitable control gain K so that the system (4) is robustly asymptotically stable for all perturbations such that $\Delta\Delta^t \leq I$ and satisfies the disturbance attenuation property

$$\|z(t)\|^2 < \gamma^2 \|x(t)\|, \gamma > 0 \tag{6}$$

Meanwhile the parameter σ in (5) which decides the sampling frequency is as large as possible. To enhance the tractability of our approach, we consider that the following proposition holds:

Proposition 1: There exists a gain matrix $K \in \Re^{m\times n}$ such that uncertain closed-loop system $\dot{x}(t) = [A_K + \Delta A_K]x(t)$ is asymptotically stable for all uncertainties satisfying $\Delta\Delta^t \leq I$. This implies that there exist $P > 0, H > 0$ the LMI

$$P(A + \Delta A_K) + (A + \Delta A_K)^t P + H < 0 \tag{7}$$

has a feasible solution for all uncertainties satisfying $\Delta\Delta^t \leq I$.

Remark 1: The rationale behind Proposition 1 is to ensure that the state-feedback design is a nominal task executed before the implementation of the event-triggering task. In turn, it will pave that way to the subsequent design stages.

3. MAIN RESULTS

Define the Lyapunov function $V(x) = x^t(t)Px(t), P > 0$. In the sequel, we treat robust triggered control methods for the linear uncertain system (4).

3.1 Robust event-triggered control

The following theorems establish the main results:

Theorem 1: Consider the system (4) and let matrix gain K satisfies inequality (7). If there exist a positive-definite matrix $P \in \Re^{n\times n}$ and a positive scalar $\sigma > 0$, satisfying the following inequality

$$\begin{bmatrix} P(A_K + \Delta A_K) + (A_K + \Delta A_K)^t P + \sigma I & P(BK + \Delta B_K) & P\Gamma & (C_K + \Delta C_k)^t \\ \bullet & -I & 0 & (DK + \Delta D_k)^t \\ \bullet & \bullet & -\gamma^2 I & \Phi^t \\ \bullet & \bullet & \bullet & -I \end{bmatrix} < 0 \tag{8}$$

Then system (4) under the event condition (5) is robustly asymptotically stable for all uncertainties satisfying $\Delta\Delta^t \leq I$ and the transfer function from disturbance w to output z, denoted by $T_{zw}(s)$, satisfies $||T_{zw}(s)||_\infty < \gamma$.

Proof: The time-derivative of V (x) along the solution of (4) with $w \equiv 0$ is:

$$\begin{aligned}
\dot{V}(x) \quad &= 2x^t P[(A_K + \Delta A_K)x(t) + (BK + \Delta B_K)e(t) \\
&= x^t P[(A_K + \Delta A_K) + (A_K + \Delta A_K)^t P]x + 2x^T P(BK + \Delta B_K)e(t) \\
&\leq x^t P[(A_K + \Delta A_K) + (A_K + \Delta A_K)^t P + P(BK + \Delta B_K)(BK + \Delta B_K)^t P]x \\
&\quad + e^t(t)e(t)
\end{aligned} \tag{9}$$

for all admissible uncertainties such that $\Delta\Delta^t \leq I$. In view of (5), we can express $\dot{V}(x)$ as

$$\dot{V}(x) \leq x^t P[(A_K + \Delta A_K) + (A_K + \Delta A_K)^t P + P(BK + \Delta B_K)(BK + \Delta B_K)^t P + \sigma I]x \quad (10)$$

From (8) and Proposition 1, it follows by the Schur complements that

$$\begin{bmatrix} P(A_K + \Delta A_K) + (A_K + \Delta A_K)^t P + \sigma I & P(BK + \Delta B_K) \\ \bullet & -I \end{bmatrix} < 0 \quad (11)$$

In turn, inequality (11) is equivalent to

$$P(A_K + \Delta A_K) + (A_K + \Delta A_K)^t P + P(BK + \Delta B_K)(BK + \Delta B_K)^t P + \sigma I < 0 \quad (12)$$

which ensures that $\dot{V}(x) < 0$ for all uncertainties satisfying $\Delta\Delta^t \leq I$. Therefore system (4) is internally robustly stable. From (6), we define the performance measure:

$$\begin{aligned}
J_T &= \int_0^T [\|z(t)\|^2 - \gamma^2 \|x(t)\|^2]\, dt \\
&= \int_0^T \left[z^t(t)z(t) - \gamma^2 w^t(t)w(t) + \dot{V}(x)\right] dt - V\big(x(T)\big) \\
&= \int_0^T [z^t(t)z(t) - \gamma^2 w^t(t)w(t) \\
&\quad + 2x^t P[(A_K + \Delta A_K)x(t) + (BK + \Delta B_K)e(t) + \Gamma w(t)]]dt - V(x(T)) \\
&= \int_0^T \begin{bmatrix} x \\ e \\ w \end{bmatrix}^t \left\{ \begin{bmatrix} (C_K + \Delta C_K)^t \\ (DK + \Delta D_K)^t \\ \Phi^t \end{bmatrix} [(C_K + \Delta C_K) \quad (DK + \Delta D_K) \quad \Phi] \right. \\
&\quad \left. + \begin{bmatrix} P(A_K + \Delta A_K) + (A_K + \Delta A_K)^t P & P(BK + \Delta B_K) & P\Gamma \\ \bullet & -I & 0 \\ \bullet & \bullet & -\gamma^2 I \end{bmatrix} \right\} \begin{bmatrix} x \\ e \\ w \end{bmatrix} dt \\
&\quad + \sigma x^t(t)x(t) - e^t(t)e(t) - V\big(x(T)\big) \\
&= \int_0^T \begin{bmatrix} x \\ e \\ w \end{bmatrix}^t \left\{ \begin{bmatrix} (C_K + \Delta C_K)^t \\ (DK + \Delta D_K)^t \\ \Phi^t \end{bmatrix} [(C_K + \Delta C_K) \quad (DK + \Delta D_K) \quad \Phi] \right. \\
&\quad \left. + \begin{bmatrix} P(A_K + \Delta A_K) + (A_K + \Delta A_K)^t P & P(BK + \Delta B_K) & P\Gamma \\ \bullet & -I & 0 \\ \bullet & \bullet & -\gamma^2 I \end{bmatrix} \right\} \begin{bmatrix} x \\ e \\ w \end{bmatrix} dt \\
&\quad - V\big(x(T)\big)
\end{aligned} \quad (13)$$

From (8), it follows by the Schur complements that

$$\begin{bmatrix} (C_K + \Delta C_K)^t \\ (DK + \Delta D_K)^t \\ \Phi^t \end{bmatrix} [(C_K + \Delta C_K) \quad (DK + \Delta D_K) \quad \Phi] + \begin{bmatrix} P(A_K + \Delta A_K) + (A_K + \Delta A_K)^t P & P(BK + \Delta B_K) & P\Gamma \\ \bullet & -I & 0 \\ \bullet & \bullet & -\gamma^2 I \end{bmatrix} < 0 \quad (14)$$

Which guarantees that $\int_0^T [z^t(t)z(t) - \gamma^2 w^t(t)w(t) + \dot{V}(x)]dt < 0$. Since the system is robustly asymptotically stable, we let $T \to \infty$, to get $\|z\|_2^2 < \|w\|_2^2$, or equivalently $\|T_{zw}(s)\|_\infty < \gamma$. ■

Remark 2: It is readily seen that Theorem 1 1 provides a sufficient condition guaranteeing the asymptotic stability and robustness of system (4). Observe that the 'best' event triggering level σ^* , corresponding to the longest sampling time, can be readily determined by solving the maximization problem

$$\max\ \sigma \text{ subject to } LMI(8)$$

for all uncertainties satisfying $\Delta\Delta^t \leq I$. In the developed event-triggered control strategy, the control execution is usually triggered by the event condition (5), which recalls for some particular hardware to detect the information of the system states and determine at which time the control task should be executed.

The next theorem provides a bounding expression for the stabilization of system (4).

Theorem 2: Consider the system (4) and given gain matrix K. If there exist a positive definite matrix P and positive scalars σ, α ,γ satisfying the following inequality

$$\begin{bmatrix} PA_K + A_K^t P & PBK & P\Gamma & C_K^t & PM_1 & \alpha(N_1+N_2K)^t \\ \bullet & -I & 0 & K^tD^t & 0 & \alpha K^t N_2^t \\ \bullet & \bullet & -\gamma^2 I & \Phi^t & 0 & 0 \\ \bullet & \bullet & \bullet & -I & M_2 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\alpha I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\alpha I \end{bmatrix} < 0 \tag{15}$$

Then system (4) under the event condition (5) is asymptotically stable with disturbance attenuation level γ.

Proof: We decompose the matrix in (8) into nominal and uncertain parts and manipulating using **Fact 1** to yield:

$$\begin{bmatrix} PA_K + A_K^t P + \sigma I & PBK & P\Gamma & C_K^t \\ \bullet & -I & 0 & K^tD^t \\ \bullet & \bullet & -\gamma^2 I & \Phi^t \\ \bullet & \bullet & \bullet & -I \end{bmatrix} + \begin{bmatrix} PM_1 \\ 0 \\ 0 \\ M_2 \end{bmatrix} \Delta \begin{bmatrix} (N_1+N_2K)^t \\ K^tN_2^t \\ 0 \\ 0 \end{bmatrix}^t +$$

$$\begin{bmatrix} (N_1+N_2K)^t \\ K^tN_2^t \\ 0 \\ 0 \end{bmatrix} \Delta^t \begin{bmatrix} PM_1 \\ 0 \\ 0 \\ M_2 \end{bmatrix}^t \leq$$

$$\begin{bmatrix} PA_K + A_K^t P & PBK & P\Gamma & C_K^t & PM_1 & \alpha(N_1+N_2K)^t \\ \bullet & -I & 0 & K^tD^t & 0 & \alpha K^t N_2^t \\ \bullet & \bullet & -\gamma^2 I & \Phi^t & 0 & 0 \\ \bullet & \bullet & \bullet & -I & M_2 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\alpha I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\alpha I \end{bmatrix} \tag{16}$$

which proves the theorem in view of (15). ■

Remark 3: It is readily seen from Theorem 2 that the 'best' event triggering level σ^*, corresponding to the longest sampling time, can be readily determined by solving the maximization problem

$$\max \sigma \text{ subject to LMI (15)} \tag{17}$$

3.2 Robust self-triggered control

In the sequel, our objective is to overcome the disadvantage arising from event-triggered implementation that additional hardware devices required to test the event condition. There is an extra task by computing a uniform sampling time offline and then execute the control task in a periodic time-triggered scheme. In [9], a self-triggered concept was proposed. Recall the event condition (5) can be equivalently expressed as

$$\frac{\|e(t)\|}{\|x(t)\|} < \sqrt{\sigma}, \forall\Delta: \Delta\Delta^t \leq I \tag{18}$$

It can be easily seen that this event condition is given based on the relative error between the current state and the state at the last sample time instant. More importantly, whenever the current state is sampled, hence $x(t_j) = x(t)$, the relative error is set to be

$$\frac{\|e(t)\|}{\|x(t)\|} = \frac{\|x(t_j) - x(t)\|}{\|x(t)\|}, \forall\Delta: \Delta\Delta^t \leq I \tag{19}$$

The time-interval during which $\|e(t)\|/\|x(t)\|$ grows from 0 to $\sqrt{\sigma}$ identifies the time when the control task should be executed next. During this period, we have from (4) that

$$\dot{x}(t) = [A_K + \Delta A_K]x(t) + [BK + \Delta B_K]e(t) + \Gamma w(t)$$
$$\dot{e}(t) = -\dot{x}(t) \quad (20)$$

Motivated by [27], we proceed to estimate the quantity $d/dt\|e(t)\|/\|x(t)\|$.

$$\frac{d}{dt}\frac{\|e(t)\|}{\|x(t)\|} = \frac{d}{dt}\frac{[e^T(t)e(t)]^{\frac{1}{2}}}{[x^T(t)x(t)]^{\frac{1}{2}}}$$
$$= \left[\frac{e_k^T(t)KC\dot{x}(t)}{\|e_k(t)\|\|x(t)\|} - \frac{x^T(t)\dot{x}(t)\|e_k(t)\|}{\|x(t)\|^3}\right]$$
$$= \frac{e^T\dot{e}}{\|e(t)\|\|x(t)\|} - \frac{x^T(t)\dot{x}}{\|x(t)\|^2}\frac{\|e(t)\|}{\|x(t)\|}$$
$$\leq \frac{\|\dot{x}\|}{\|x(t)\|} + \frac{\|\dot{x}\|}{\|x(t)\|}\frac{\|e(t)\|}{\|x(t)\|} = \frac{\|\dot{x}\|}{\|x(t)\|}\left(1 + \frac{\|e(t)\|}{\|x(t)\|}\right)$$
$$\leq \frac{1}{\|x(t)\|}(\|[A_K + \Delta A_K]\|\|x(t)\| + \|[BK + \Delta B_K]\|\|e(t)\| + \|\Gamma\|\|w(t)\|)$$
$$\times\left(1 + \frac{\|e(t)\|}{\|x(t)\|}\right)$$
$$:= L_o + L_1\frac{\|e(t)\|}{\|x(t)\|} + L_2\frac{\|e(t)\|^2}{\|x(t)\|} \quad (21)$$

where

$$L_o = \|[A_K + \Delta A_K]\| + \|\Gamma\|\frac{\|w(t)\|}{\|x(t)\|}, L_1 = \|[A_K + \Delta A_K]\| + \|\Gamma\|\frac{\|w(t)\|}{\|x(t)\|}\|x(t)\| + \|[BK + \Delta B_K]\|$$
$$L_2 = \|[BK + \Delta B_K]\|, \forall\Delta: \Delta\Delta^t \leq I \quad (22)$$

It is readily seen that $L_1 = L_0 + L_2$. Letting $\zeta(t) = \|e(t)\|/\|x(t)\|$ then we recast (21) in the standard form of quadratic differential equation:

$$\dot{\zeta}(t) = L_0 + L_1\zeta(t) + L_2\zeta^2(t), \zeta(0) = 0 \quad (23)$$

whose solution is given by

$$\zeta(t) = \frac{1}{2L_2}\left[-L_1 + \sqrt{D}tan\left(\frac{t}{2} + arctan\left[\frac{L_1}{\sqrt{D}}\right]\right)\right]$$
$$D = 4L_0L_2 - L_1^2 \quad (24)$$

It follows that

$$\frac{\|e(t)\|}{\|x(t)\|} \leq \zeta(t,0)$$

which gives an upper bound for the error to state ratio. Simple mathematical manipulations show that

$$\zeta(t) = \frac{-1}{2L_2}\left[L_0 + L_2 - \omega tan\left(\frac{1}{2}\omega t + \xi\right)\right]$$
$$\xi = arctan\left(\frac{L_2}{\omega}\right), \omega = |L_0 - L_2| \quad (25)$$

Finally, by solving the equation

$$\zeta(T_f) = \sqrt{\sigma}$$

the lower bound of the desired execution time can be computed as

$$T_f = \frac{2}{\omega}\left[arctan\left(\frac{L_0+L_2+2\sqrt{\sigma}+L_2}{\omega}\right) - \xi\right] \quad (26)$$

Remark 4: From the mathematical nature of expression (26), it is readily seen that as L_o and/or L_2 increases the estimated execution time T_f will be getting smaller. In view of the bounding condition on the disturbance $|w| \leq W|x|$, we should consider $L_0 = \|[A_K + \Delta A_K]\| + \|\Gamma\|W$ when computing the value of T_f. It is significant to observe from (26), that the sampling time T_f is completely decided by σ given the system parameters. In turn, this defines the event condition. The crucial issue here is that the co-selection of σ and the robust performance of the system under consideration is achieved by solving a scalar maximization problem over LMIs. This is a fundamental contribution of our work.

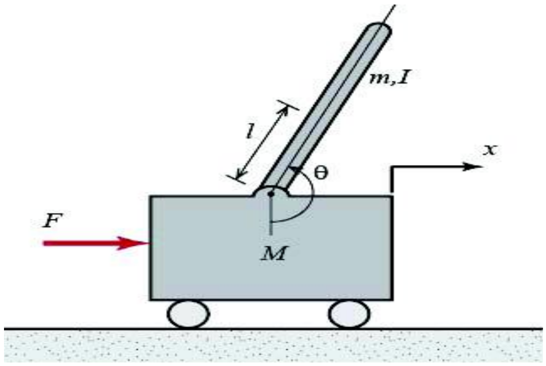

Figure 1: Schematic diagram of cart and inverted pendulum

4. SIMULATION RESULTS

In what follows, the developed theory is tested by simulation using a typical system example. A cart and inverted pendulum system, shown in Fig. 1, is considered for simulation studies, where x_d is the position of the cart, θ is the angular position of the pendulum, and u is the input force. The state variables are chosen as $[x_d, \dot{x}_d, \theta, \dot{\theta}_d]^T$. The system parameters are: $M = 0.5kg$, $m = 0.2kg$, $L = 0.3m$, b(coefficient of friction for cart) = $0.1N/m/sec$, I(*mass moment of inertia of the pendulum*) = $0.006kg.m^2$. The linearized model has the state space matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.1818 & 2.6727 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.4545 & 31.1818 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1.8182 \\ 0 \\ 4.5455 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Let the perturbation matrices be given by in the following cases:
Case I:

$$M_1 = \begin{bmatrix} 1.2 \\ 0.6 \\ 0.8 \\ 0.7 \end{bmatrix}, M_2 = \begin{bmatrix} 0.2 \\ 0.8 \\ 0.5 \\ 1.3 \end{bmatrix}, N_1^t = \begin{bmatrix} 2.1 \\ 1.3 \\ 0.6 \\ 0.9 \end{bmatrix}, N_2 = 1.3$$

Case II:

$$M_1 = \begin{bmatrix} 0.4 \\ 0.5 \\ 0 \\ 0.1 \end{bmatrix}, M_2 = \begin{bmatrix} 0.2 \\ 0 \\ 0.6 \\ 0 \end{bmatrix}, N_1^t = \begin{bmatrix} 0 \\ 0.2 \\ 0.6 \\ 0 \end{bmatrix}, N_2 = 0.4$$

Observe that cases I and II correspond to large and small parameter perturbations, respectively. Initially, using computer simulation with step 0.01s the nominal open-loop response is depicted in Fig. 2, from which it is seen that that open-loop is unstable. This is easily verified since the nominal system matrix A has eigenvalues of 0,−5.6041,−0.4128, 5.5651.

Next, we choose the feedback gain $K = [-1.0000, -1.6567, 18.6854, 3.4594]$ to stabilize the system. We also choose the external disturbance $\omega(t)$ of the form

$$\omega(t) = \frac{1}{1 + e^t}, t \in [0, \infty]$$

The feasible solution of the maximization problem (17) for case I is given by

$$P = \begin{bmatrix} 1.2145 & -1.0205 & -0.0466 & -0.0132 \\ \bullet & 1.3357 & 0.07566 & 0.2832 \\ \bullet & \bullet & 1.2875 & 0.1694 \\ \bullet & \bullet & \bullet & 2.6788 \end{bmatrix}, \gamma = 1.41, \alpha = 8.54, \sigma_{max} = 0.0604$$

From (22), we get L_o = 3.494 and L_2 = 11.363. Computing (26) yields T_f = 0.0304. Once again, using a digital simulation with step 0.01*s*, the closed-loop dynamic behavior is depicted in Fig. 3.

On the other hand, the feasible solution of the maximization problem (17) for case I is given by

$$P = \begin{bmatrix} 0.3370 & -0.0304 & -0.0497 & -0.0042 \\ \bullet & 0.2630 & 0.0579 & 0.2771 \\ \bullet & \bullet & 0.1261 & 0.0734 \\ \bullet & \bullet & \bullet & 0.5911 \end{bmatrix}, \gamma = 0.32, \alpha = 4.67, \sigma_{max} = 0.0305$$

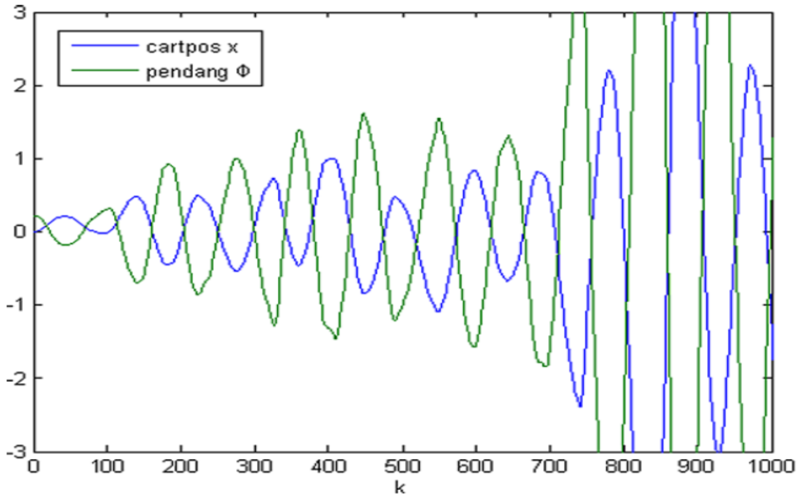

Figure 2: Open-loop response of cart position and angular position of the pendulum

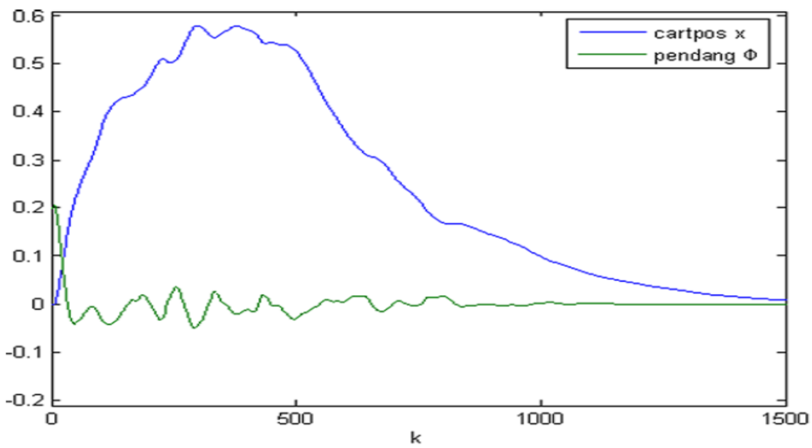

Figure 3: Closed-loop dynamic behavior of cart position and angular position of the pendulum- Case I

From (22), we get L_o = 6.835 and L_2 = 7.625. Computing (26) yields T_f = 0.0215. The ensuing closed-loop dynamic behavior is depicted in Fig. 4.

On considering Fig. 3 versus Fig. 4 in the closed-loop case versus Fig. 2 in the open-loop case, it is readily evident that

- The main system states (cart position and angular position of the pendulum) converge to the equilibrium point as desired. In case of small perturbation, the conversion was quite rapid.

- The execution time T_f is affected by the magnitude of parameter perturbations.
- The disturbance causes very little effect on the system.
- The controlled system is effectively robust no matter the size of perturbation.

Thus the simulation results conform with the theoretical developments and emphasize that the developed H_∞ controller perform well in regulating the dynamic behavior of the cart and inverted pendulum system.

5. CONCLUSIONS

In this paper, we have investigated robust H_∞ theory into event- and self-triggered strategies for a class of continuous-time systems with norm-bounded parametric uncertainties. The event condition has been presented based on the relative error between the current state and the state at last sample time. Using full-information feedback, we have developed a sufficient condition which guarantees the robust asymptotic stability of the closed-loop system. The derived condition is expressed in the form of linear matrix inequality (LMI). We have extended the analysis to the case of self-triggered control mechanism to overcome the hardware drawback that event-triggered control mechanism. An estimate of the lower bound of the control execution time for the developed H_∞ controller given that the system is robustly stabilized. The effectiveness of the developed results is illustrated on the control of a cart and pendulum problem.

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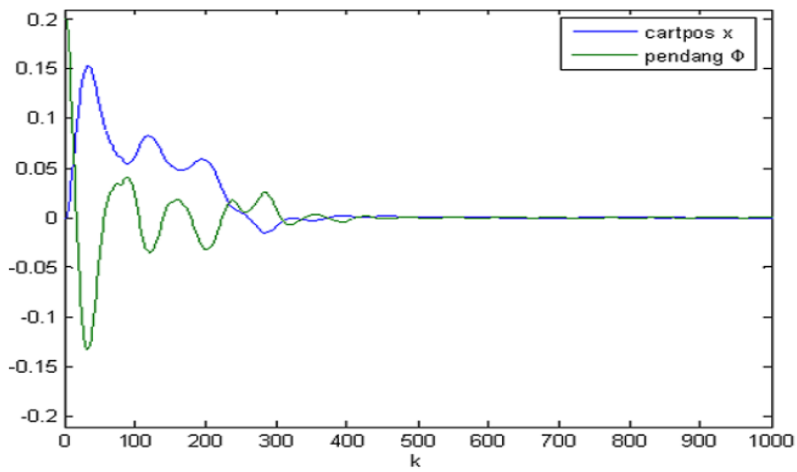

Figure 4: Closed-loop dynamic behavior of cart position and angular position of the pendulum- Case II.

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