Asian Journal of Business and Management (ISSN: 2321 - 2802)
Volume 12 – Issue 1, December 2024

Model Selection for the Reliability of a Machine

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ABSTRACT— ***Model selection is an important factor in the non-homogeneous poisson process used to determine the reliability of a machine. In this article, the most commonly used power law model and log-linear model in the non-homogeneous poisson process are compared. For the reliability of a bank's ATM machine, the reliability values and expected failure numbers were calculated according to both models by using the times between failures. The obtained values were compared and it was decided which model was the better choice.***

Keywords— Distribution, maximum likelihood method, log-linear model, non-homogeneous poisson process, power law model.

1. INTRODUCTION

The usefulness of the system depends on its performance and functionality. The main feature of the systems is that they are reliable. The reliability of the systems is an important issue in many areas of engineering, health and social.
In reliability sources, systems are generally examined in two groups as non-repairable systems and repairable systems. When non-repairable systems are broken, they cannot be reused. These systems break down only once and become unusable. The distribution in failure times of such systems can be modeled as the life model of the Weibull distribution. Repairable systems are systems that can be reused after a process or an intervention after they break down. Modeling of these systems is handled with their failure times. Generally, the models used for the failure time of the systems that can be repaired are the non-homogeneous poisson process (NHPP) and the renewal process. While the renewal process is preferred if the deterioration frequency is constant, the Poisson process is preferred for unstable failures. Failure rates are an important issue in the Poisson process.
Zahedi [1], Crowder *et. al.* [2], Saldanha [3], Ascher and Hansen [4] are some of the people who have worked on systems that can be repaired. In this study, our aim is to model repairable systems using NHPP.
Reliability, it can be defined as the probability that a system will adequately perform its intended function under certain conditions. In other words, reliability is the analysis of the failure rate of parts or units of a system.
According to Elsayed [5], reliability is the probability of the product or service working without deterioration in a certain period of time, under working conditions considered suitable for the product. In other words, reliability can be used as a measure of the success of a properly functioning system.
The reliability criteria of the system are, "How long will the product work without deterioration?", "At what rate will the products deteriorate before the warranty expires?", "How long a warranty period should be given for a new product?" is to answer questions like.
It can be said that a product created can have an average working time of 100 hours, 200 hours, or 2000 hours. So, life time varies from product to product. When examining these changes, probabilities, distributions, mean and variability measures should be taken into account.
Developments in science and technology in recent years have made it inevitable for systems to be established in an even more reliable structure. The concept of reliability is extremely important in processes covering stages such as design, construction, distribution and operation in technology. Failure or damage that may occur in a system can cause important social consequences.

2. RELIABILITY FUNCTION

Let the lifetime of a system be a continuous N non-negative random variables. Accordingly, the probability of system failure before t time is

$$F(t) = P(N \le t) = \int_{-\infty}^{t} f(t)dt \,. \tag{1}$$

Accordingly,

$$R(t) = 1 - F(t) = P(N > t), t > 0 \tag{2}$$

is the probability that a system will work after time *t* if it is known that a system works until time *t* [6,7]. $R(t)$ function is called "Life Function". When the distribution function $F(t)$ is an increasing function, $R(t)$ is a decreasing function of the reliability function. The sudden failure or deterioration rate of a system known not to fail after time *t* is as follows.

$$h(t) = \lim_{\Delta t \to 0} \frac{P(t < N \le t + \Delta t | N > t}{\Delta t} = \frac{f(t)}{1 - F(t)} \tag{3}$$

This function is called the hazard function. The reliability function expresses the probability of success while the hazard function expresses to failure.

In the reliability analysis of the system, after the system design is made, the reliability should be evaluated and compared with the acceptable level of reliability. If the desired is not met, the design should be reviewed and a new design should be made and reliability should be recalculated. This design process continues until it meets the desired performance and reliability level.

In reliability analysis, it is important to observe whether the reliability of the systems is meaningful and whether the system contains trends. Increasing reliability of the system is observed if there is a significant and appropriately increasing trend between Time Between Failures (TBF). Here it is observed that if there is a decreasing trend between consecutive TBF, the system has a decreasing reliability.

In such cases, the NHPP model is used to create the data model for TBF. However, if there is no trend observed in TBF data and does not provide the assumption of independence, the consecutive Poisson Process model is selected and applied.

Generally, the following five-step path is followed when choosing models for reliability analysis [8-12].

First, TBF that follow each other is determined.

In the second stage, the appropriate probability model for the data is selected, similar distribution and independence assumptions are checked.

If the assumptions are met, the failure data are modeled with an appropriate model (Normal, Weibull, Exponential, Lognormal, etc.).

Where model assumptions are not valid, failure data is used with an unstable model.

As a result, testing is done for the preferred model.

3. NON-HOMOGENOUS POISSON PROCESS AND MODELING

The most important step in reliability is determining the appropriate model to be used to model failure data. Therefore, the first step that needs to be done is to examine whether the data are distributed independently and similarly. If the data is distributed independently and identically, the failure process is generally modeled by renewal processes such as Weibull and normal. However, if the data is not distributed identically and there is a trend in the data, the NHPP which accepts that the TBF changes depending on time should be used and the failure process should be modeled with the NHPP. The reliability function of the Poisson distribution is as follows.

$$R(k) = \sum_{x=0}^{k} \frac{(\lambda t)^x e^{-\lambda t}}{x!} \tag{4}$$

There are two main applications of reliability in the Poisson process. The first is to define the number of failure in a time interval. The second is to use it as an approximation for the binomial distribution when the binomial parameter is small.

The Non-Homogeneous Poisson Process is the most preferred model for reliability. NHPP models offer an analytical solution to describe the behavior of failure conditions that occur during testing. Its main feature is that it estimates the mean value function of the cumulative sum of failures that are likely to be observed until a certain time. NHPP is used for non-time dependent failures.

For example, NHPP is used as a frequently used process in modelling gas turbines [13],hard drives [14], machine tool breakdown [15] and for modelling software reliability [16].

In the NHPP model, the rate of occurrence of failures is usually expressed by the intensity function $w(t)$ [17]. $w(t)$ is also expressed as a function of the mean value.

$N(t)$ is an NHPP with the intensity function $w(t)$ if the following conditions are met to give the number of failures that occur in the $t \ge 0$ time interval.

$N(0) = 0$

$N(t)$ has a Poisson distribution with $\int_0^t w(t)dt$ parameters for $t > 0$.

$N(t_1), N(t_2) - N(t_1), \ldots, N(t_m) - N(t_{m-1})$ are independent random variables for each $0 \le t_1 \le t_2 \le \ldots \le t_m$.

If $N(t)$ has a poisson distribution with $\int_0^t w(t)dt$ parameters, the following equations can be written.

$$E[N(t)] = \int_0^t w(t)dt = W(t), \qquad Var[N(t)] = \int_0^t w(t)dt = W(t) \tag{5}$$

The number of faults in any interval $(t_1, t_2]$ in NHPP has a $\int_{t_1}^{t_2} w(t)dt$ mean poisson distribution. Therefore, the probability of *k* failures in this interval is defined as follows [18].

$$P[N(t_2) - N(t_1) = k] = \frac{1}{k!}(\int_{t_1}^{t_2} w(t)dt)^k \exp(-\int_{t_1}^{t_2} w(t))dt \tag{6}$$

In this article will be based on the power law model and log-linear model commonly used in the NHPP. The main reason for their widespread use is that the rate of occurrence of failures which failures occur is in the same form as the hazard ratio of the Weibull distribution.

3.1. LOG-LINEAR MODEL AND PARAMETER ESTIMATION

We have data from m independent systems managed by NHPP with the same density function. Here, the *j th* system was observed in the time interval $(S_j, T_j]$ among the events observed in $t_{1j}, t_{2j}, \ldots, t_{nj}$ times. The maximum likelihood function for these data is expressed by Meeker and Escobar [19] as follows.

$$L = \prod_{i=1}^{m} \{\prod_{j=1}^{n_j} w(t_{ij})\} \exp[-(W(T_j) - W(S_j)] \tag{7}$$

Maximum likelihood function of $w_1(t)$ is given by

$$L = \prod_{i=1}^{n} w_1(t) \exp\left[N(0, t_0\right] \tag{8}$$

Log-linear model was discussed by Cox and Lewis in 1966 [20], and given by

$$w_1(t) = \exp(\alpha_0 + \alpha_1 t) \tag{9}$$

When the value of $w_1(t)$ is substituted in Eq. 7, log-likelihood function

$$l_1 = n\alpha_0 + \alpha_1 \sum_{i=1}^{n} t_i - \frac{\exp(\alpha_0)\exp(\alpha_1 t_0 - 1)}{\alpha_1} \tag{10}$$

and the maximum likelihood estimator of α_1 can be obtined by solving the equation:

$$l_1 = \sum_{i=1}^{n} t_i + \frac{n}{\alpha_1} + \frac{nt_0}{1 - \exp(-\alpha_1 t)} = 0 \tag{11}$$

After obtaining $\hat{\alpha}_1$, one has

$$\hat{\alpha}_0 = \ln\left(\frac{n\hat{\alpha}_1}{\exp(\hat{\alpha}_1 t_0) - 1}\right) \tag{12}$$

3.2. POWER LAW MODEL AND PARAMETER ESTIMATION

The second model is based on the Weibull distribution and is referred to as the power law model [21]. It is given by

$$w_2(t) = \lambda\beta t^{\beta-1}, \quad \lambda, \beta > 0 \text{ and } t \geq 0 \tag{13}$$

The log-likelihood function for the $w_2(t)$ power law model is given by

$$l_2 = \sum_{i=1}^{n} \ln\lambda + \ln\beta + (\beta-1)\ln t_i] - \lambda t_0^{\lambda} \tag{14}$$

The obtained maximum likelihood estimators are,

$$\hat{\lambda} = n / t_0^{\beta} \quad \text{and} \quad \hat{\beta} = \frac{n}{n\ln t_0 - \sum_{i=1}^{n}\ln t_i} \tag{15}$$

4. APPLICATION

In this study, in order to test the reliability of a bank's ATM machine, TBF was obtained for 70 data of 2019. Analyzes were made with Minitab 14 package program.

The first step is to establish a suitable probabilistic model for the given TBF data. In order to make a model selection, it is necessary to investigate whether there is a trend for the cumulative failure numbers (CFN) and the cumulative time between failures (CTBF) according to the data obtained. It is clear from Figure 1 that the occurrence of faults is not linear and time dependent. Since no linearity is observed for TBF data, models such as NHPP are used.

Solving equations (11) and (12) for $w_1(t)$, $\hat{\alpha}_0 = -6.017$ and $\hat{\alpha}_1 = 0.000234$ are obtained. Thus the log-linear model is obtained as $w_1(t) = \exp(-6.017 + 0.000234t)$.

Solving equations (15) for $w_2(t)$ $\hat{\lambda} = 0.0042$ and $\hat{\beta} = 1.0710$ are obtained. Thus the power law model is obtained as $w_2(t) = (0.0042)(1.0710)t^{-0.071}$.

Accordingly, the expected failure numbers at certain time intervals are given in Table 1.

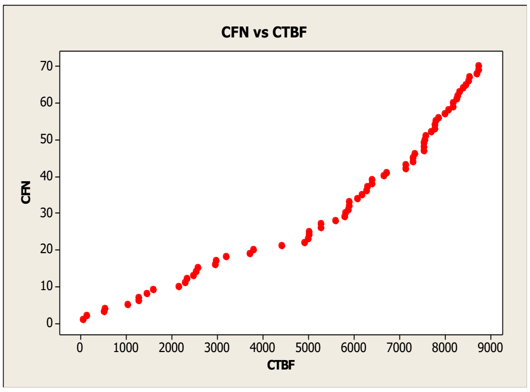

Figure 1. Trend Graphic

In addition, the probability of *k* faults occurring in the $(t_1, t_2]$ interval can also be calculated. For example, the probability of occurrence of *3* failures in (100,500] time interval is calculated using the power law density function, which is a model of the NHPP process.

Table 1. Expected failure numbers

Time (h)	Power-Law Model	Log-linear Model
10	0.05	0.024
50	0.29	0.122
500	3.49	1.29
1000	7.34	2.7455
2000	15.43	6.2150

$$m(500)-m(100)=\int_{100}^{500} w(t)dt = 2.87$$

and from eq. (6),

$$PN(500)-N(100)=3]=0.1472.$$

The estimated value of the next failure time is calculated by the following formula.

$$t_{n+1}=(t_n^{\hat{\beta}}+\frac{1}{\hat{\lambda}})^{\frac{1}{\hat{\beta}}}$$

Accordingly, the next failure occurs at $t_{n+1}=8842.07$ time.

Similar values can be found for the log-linear model. NHPP assumes that the number of failure in any $(t_1, t_2]$ interval has a Poisson distribution with a mean of $\int_{t_1}^{t_2} w(t)dt$. In this case, the reliability function is

$$R(t_1,t_2)=P[N(t_2)-N(t_1)=0]=e^{-\int_{t_1}^{t_2} w(t)dt}=e^{-[m(t_2)-m(t_1)]}.$$

Accordingly, the reliabilities calculated for various time intervals are given in Table 1.

Table 2. Reliability values for specific time intervals

Time (h)	Power-Law Model $R(t_1,t_2)$	Log-linear Model $R(t_1,t_2)$
0-10	0.9512	0.975
20-50	0.8352	0.928
100-250	0.3534	0.6833
500-1000	0.0212	0.233

As seen in Table 2, reliability levels decrease over time. It is seen here that a decreasing trend of TBF data occurs and this system deteriorates over time.

5. CONCLUSION

In this study, a bank's ATM machine was chosen as the application subject. As a result of the analysis, it was observed that the TBF data contained a decreasing trend, thus the reliability of the ATM machine decreased. According to ATM data, the values obtained for both models were compared. Looking at Table 1, the expected failure numbers calculated for the log-linear model are lower than the values calculated for the power law model. Looking at Table 2, the reliability is higher in the log-linear model for certain time intervals. Accordingly, it has been determined that the log-linear model is a better choice than the power law model. It should not be ignored that some of the failure records of the said ATM are caused by external factors (such as user error, dust, abrasion). It is also important that the periodic maintenance of the machine in question is done on time.

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