

Optimal Two-Part Tariff Licensing under Returns to Scale

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ABSTRACT--- *In this study we investigate a two-part tariff licensing contract that enables an incumbent innovator to license a cost reducing innovation in a Cournot duopoly characterized by non-constant returns to scale. We identify the optimal two-part tariff licensing contract based on the cost reduction imposed by the use of the new technology and the market parameter.*

Keywords—Patent Licensing; Royalty; Ad-valorem; two tariff, Cournot Duopoly; Game Theory

1. INTRODUCTION

A patent is defined as the exclusive right provided by the law to inventors in order to make use of, and to exploit, their inventions for a limited period of time. Therefore, the aim of a patent is twofold, firstly, acts as incentive of innovation and secondly, provides the inventors with diffusion control of their inventions.

Patent licensing is the main tool used for disseminating the effects of innovation. The owner of a patent charges some payments for using the new technology to the authorized firms obtaining in this way a return on his investment in research and development. The profits of the patentee will depend on the structure adopted by the licensing agreement. In the literature the licensing policies are categorized into the following main categories, (a) up front fee, which is usually a fixed fee that entitles the licensee to use the new technology to produce as many units as he wishes; (b) per unit royalty, that requires the licensee to pay a certain amount per unit of production; (c) value based (ad valorem) royalty, which in its simplest form consists of a uniform percentage (the rate) of the value of the products sold by the licensee; and (d) a two-part tariff, that is, a hybrid license consisting of a fixed fee and either a per-unit or an ad valorem royalty. The technology transfer by means of one of these type of contracts can be done in different oligopoly models, as Cournot, Bertrand or Stackelberg model.

After its initialization by Arrow (1962), the patent licensing analysis has developed along three main research directions.

The research work belonging to the first direction seeks to find the optimal licensing scheme under the assumption that there exists an external patentee whose sole objective is to sell the patent to other firms. Following this line, Kamien and Tauman (1986), in their seminal work, considered an external patentee and a homogeneous Cournot oligopoly and shown that licensing by means of up front fees dominates licensing by a per unit royalty. Similar results can be found in Katz and Shapiro (1986) and Kamien et al., (1992). Bousquet et al., (1998) consider an external innovator trying to identify the optimal licensing policy (ad valorem or per unit) under uncertainty. See also Kamien (1992) for a survey.

The second direction on the other hand, investigates the optimal licensing scheme by assuming that the patentee is an internal inventor. Considering the case of an internal innovator, Wang (1998) found that in a Cournot duopoly a per-unit royalty licensing can be superior to an up front fee licensing if the innovation is non-drastic. Kamien and Tauman (2002) consider the Wang's model in the case of any number of competitors, their findings shows that when the number of firms in the industry exceeds a certain threshold number, the patentee's profit is optimized when licensing by means of a royalty rather than by auctioning fixed fee licenses. Filippini (2005) modeled the competition as a Stackelberg duopoly with the innovator as the Stackelberg leader, he proved that the optimal policy has only royalty and no upfront fee. The works of Wang (2002) and Wang and Yang (1999) are concerned with the superiority of up front fee under Bertrand and Cournot competition in the presence of product differentiation. While San Martin and Saracho (2010) have shown that in the classic homogenous good Cournot duopoly an internal patentee will always prefer the ad valorem royalty to a per unit royalty.

Finally in the third research direction the optimal licensing scheme is identified by assuming that the patentee is either an internal or an external innovator. In this spirit Fauli-Oller and Sandoni (2004) have stated that the optimal licensing always includes a positive royalty. Sen and Tauman (2007) examined a Cournot oligopoly under the existence of both an internal and an external innovator, their findings indicate that when the number of firms is not too small, licensing of an innovations involves a positive royalty.

The majority of the optimal licensing contracts proposed in the literature is based either on a pure fixed fee mechanism or a pure per unit royalty mechanism. However as empirical studies shown, two-part tariffs are most often observed in practice, for example Rostoker (1984) found out that 46% of the licensing cases he studied in USA included a two tariff mechanism based on a fixed fee plus a per unit royalty. Additionally, Bousquet et al., (1998) concluded that that 78% of contracts they studied in French, included royalties (alone or together with a fixed fee) and, more importantly, 96% were ad valorem royalties. Yet, a little research work is devoted to licensing mechanisms by means of a two-part tariff when the innovator is an internal producer. Besides the works of Fauli-Oller and Sandoni (2004) and Sen and Tauman (2007) mentioned before, recently Kitagawa et al., (2014) investigated a two-part tariff licensing by an incumbent innovator who competes with a potential rival who may self-develop the technology in a differentiated Cournot duopoly. Their main results suggest that the optimal contract always involves a positive royalty. While the two tariff licensing mechanism in a differentiated Cournotduopoly proposed by Martin and Saracho (2014) suggests that the patentee would prefer to use ad valorem royalties rather than per-unit royalties depending on the type of substitution and the degree of differentiation. Colombo and Filippini (2014) analyze an optimal two-part licensing scheme based on ad valorem royalties within a differentiated Bertrand duopoly. Their main conclusion states that under the assumption of the Bertrand competition the internal patentee will prefer a per unit royalty based two tariff contract.

Although abundant, the existing literature is constrained by the assumption of constant returns to scale, failing to capture an important aspect of production, the existence of returns to scale. Two exception are Sen and Stamatopoulos (2009) and Karakitsiou and Mavrommati . (2013). In their seminal work Sen and Stamatopoulos (2009) extended Sen and Tauman's model by examine the problem of licensing of cost reducing innovation under returns to scale. Their main results state that incidence of positive royalties and diffusion of innovation are both inversely related to returns to scale. Karakitsiou and Mavrommati (2013) considered a Cournot duopoly characterized by non-constant returns to scale. The patentee chooses strategically whether to charge a per-unit or an ad-valorem royalty to the other firm in the industry. Their results indicate that an internal innovator may prefer different licensing contract depending on the cost reduction imposed by the use of the new technology.

In the current paper we extend the analysis of the relevance of the ad valorem royalty in licensing a non-drastring innovation. In particular, we study optimal two tariff licensing schemes of an internal patentee in a homogeneous Cournot duopolistic industry where the production is characterized by the existence of returns to scale. We construct a three stage non cooperative model. At the first stage the patent-holding duopolist acts by setting an ad-valorem plus a fixed fee or a per-unit royalty rate plus a fixed fee. At the second stage, the other firm decides the conditions which allow it to accept or not the offer from the patent holder. In the third stage, both firms engage in a non-cooperative game in quantities.

Our main results show that under the underlying assumptions the optimal two tariff contract offered by the patentee is affected by both the reduction to the marginal cost induced by the innovation and the market parameter. Particularly, the optimal policy of the patentee is to set a contract involving a pure per unit royalty when either the market parameter is high or the cost reduction caused by the innovation is rather small. In contrast when the cost reduction is high and the market parameter is small then a two-part tariff involving an ad valorem royalty plus a fixed fee is preferred. A two-part tariff involving a per unit royalty plus a fixed fee is the optimal licensing mechanism when the cost reduction is intermediate and the market parameter small.

The rest of of the paper is organized as follows. Section 2 presents the model of the patent licensing. Section 3 analyzes the proposed royalty licensing schemes. Section 4 compares the schemes, while Section 5 concludes with some final remarks.

2. THE MODEL

Consider a Cournot duopolistic industry that produces a homogeneous good. The inverse demand function is given by $p = a - Q$ for $a \geq Q$ and $p = 0$ for $a < Q$, where $Q = \sum_{i=1}^2 q_i$, p denotes the market price and q_i the output of the firm i , ($i = 1, 2$). We call parameter a as market parameter.

We assume that there is an existing production technology under which both firms have zero fixed cost. The existing technology could be represented by the total cost function, $TC_i = (bq_i + c)q_i$. The constant b determines the nature of the technology. When $b > 0$ the function $(bq_i + c)$ is increasing, therefore the technology exhibits decreasing returns to scale. When $b = 0$, $(bq_i + c) = c$ implying the presence of constant returns to scale, while when $b < 0$ the function $(bq_i + c)$ is decreasing if $q_i \leq -c/b$ so the technology exhibits increasing returns to scale until this level of production, and stays zero beyond it.

We further assume that firm 1 owns a non-drastic cost reducing innovation which creates a new technology that lowers its marginal cost by the amount of c .

According to Arrow (1962) an innovation is drastic if the monopoly price under the new technology does not exceed the unit production cost of the old technology otherwise it is non-drastic. In our case it is easy to verify that the assumption of the non-drastic innovation holds only if $c < c_1$, where $c_1 = \frac{a(2b+1)}{2(b+1)}$.

Hence, the total cost function under the new technology is $TC' = bq_1^2$. However, when $b \leq 0, TC' < 0, \forall q_i > 0$. Therefore, our analysis is focused only in the case of decreasing returns to scale ($b > 0$).

The interaction between the internal patentee and the licensee is modeled as a three stage non-cooperative game. We call this game R . In the first stage, the patent-holding duopolist acts by setting a per-unit royalty plus fixed fee or an ad valorem royalty plus fixed fee. In the second stage, the other firm decides the conditions which allow it to accept or not the offer from the patent holder. In the third stage, both firms engage in a non-cooperative game in quantities.

2.1 The Cournot Equilibrium with no Transaction between the Firms

We will start our analysis by presenting the Cournot game of the duopolists in the case where firm 2 is not willing to buy firm's 1 patent. Results of this model will be used to make inferences about the alternative licensing schemes studied later.

The profit functions of the firms are given by the equations

$$\Pi_1^{NI} = (a - q_1 - q_2)q_1 - bq_1^2 \quad (1)$$

$$\Pi_2^{NI} = (a - q_1 - q_2)q_2 - (bq_2 + c)q_2 \quad (2)$$

Firm 1 will choose q_1 such that to maximize its profit, which leads to the firm's 1 reaction function:

$$q_1 = \frac{a - q_2}{2(b + 1)} \quad (3)$$

Similarly the maximization of firm's 2 profit function yields to the firm's 2 reaction function:

$$q_2 = \frac{a - q_1 - c}{2(b + 1)} \quad (4)$$

Then, optimal quantity produced by each firm is given by the intersection of these two reaction functions:

$$q_1^{NI} = \frac{a(2b + 1) + c}{(2b + 3)(2b + 1)} \quad (5)$$

$$q_2^{NI} = \frac{2b(a - c) + a - 2c}{(2b + 3)(2b + 1)}$$

and the firms' equilibrium profits are

$$\Pi_1^{NI} = \frac{(2ba + c + a)^2(1 + b)}{(2b + 3)^2(2b + 1)^2} \quad (6)$$

$$\Pi_2^{NI} = \frac{(2ba - 2c - a - 2bc)^2(1 + b)}{(2b + 3)^2(2b + 1)^2} \quad (7)$$

3. TWO TARIFF MECHANISMS

3.1 Per Unit Royalty plus a Fixed Fee Licensing

In this section we present the solution of the game under the assumption that the licensee charges a per unit royalty rate r and a fixed fee F for the use of his innovation by the other firm.

We are interested for sub-game perfect Nash equilibrium of the game. The game must be solved by backward induction. At the third stage each firm chooses the production quantity that maximizes his profit given the per unit production royalty r and the fixed fee F which are set in the first stage of the game. Thus, firm 1 solves the problem

$$\max_{q_1} \Pi_1^{rf} = (a - q_1 - q_2)q_1 - bq_1^2 + rq_2 + F, \quad (8)$$

while firm 2 solves the problem

$$\max_{q_1} \Pi_2^{rf} = (a - q_1 - q_2)q_2 - bq_2^2 - rq_2 - F. \quad (9)$$

The solution of the problems yields to the equilibrium production level of each firm:

$$q_1^{rf} = \frac{a(2b + 1) + r}{(2b + 3)(2b + 1)} \quad (10)$$

$$\bar{q}_2^{rf} = \frac{a(1 + 2b) - 2r(b + 1)}{(2b + 3)(2b + 1)} \quad (11)$$

At second stage, firm 2 will not buy the new technology if its marginal cost with innovation is greater than this without it and will not buy the innovation if its profits with the patent are at least as high as those without innovation. In other words, firm 2 is willing to buy the innovation if the following holds

$$2bq_2 + r \leq 2bq_2 + c \Rightarrow r \leq c, \quad (12)$$

and

$$F \leq \Pi_2^{rf} - \Pi_2^{NI} \quad (13)$$

At the first stage, firm 1 chooses the r and F which maximizes its profit while taking into account the restrictions imposed by the previously discussed stages.

So, the problem solved by firm 1 is the following

$$\max_{r,F} (a - q_1 - q_2)q_1 - bq_1^2 + rq_2 + F \quad (14)$$

$$\text{s.t (10), (11), (12), (13)} \quad (15)$$

The solution of the problem leads us to the following findings:

First Case

$$F = \frac{2(b + 1)^2(c - r)[(2b + 1)a - b(c + r) - r - c]}{(2b + 3)^2(1 + 2b)^2}$$

$$r < c$$

$$\begin{aligned} \Pi_1^{rf} &= \frac{(b + 1)(1 + 2b)^2 a^2}{(2b + 3)^2(1 + 2b)^2} + \frac{(1 + 2b)[(2r + 2c)b^2 + (6r + 2c)b + 2c + 3r]a}{(2b + 3)^2(1 + 2b)^2} + \\ &+ \frac{(2c^2 - 6r^2)b^3(-18r^4 - 6c^2)b^2 + (-15r^2 - 6c^2)b - 3r^2 - 2c^2}{(2b + 3)^2(1 + 2b)^2} + \\ &+ \frac{(-15r^2 - 6c^2)b - 3r^2 - 2c^2}{(2b + 3)^2(1 + 2b)^2} \end{aligned}$$

Second Case

$$F = 0$$

$$r = c$$

$$\Pi_1^{rf} = \frac{(1 + 5b + 8b^2 + 4b^3)a^2 + (5c + 24b^2c + 8b^3c + 20bc)a + (-5 - 21b - 8b^3 - 24b^2)c^2}{(2b + 3)^2(2b + 1)^2}$$

Comparing the two cases we can conclude that if the market parameter a of the duopoly is quite small then the patentee prefers to charge a two-part tariff (r, F) regardless of the reduction to the cost caused by the use of the new technology. On the other hand, if the market parameter takes high values then it is the value of the cost reduction which affects the optimal policy of the patentee. More specific, if the parameter c takes quite high values then the optimal policy of the patentee involves a two tariff mechanism (r, F) . If parameter c is rather small then his optimal policy involves a pure per unit royalty r . That is,

If $a > a_1$ then

$$\Pi_1^i > \Pi_1^{rf} \text{ for } 0 < c < c_3$$

$$\Pi_1^{rf} > \Pi_1^i \text{ for } c_3 < c < c_1$$

If $0 < a < a_1$ and $0 < c < c_1$ then $\Pi_1^{rf} > \Pi_1^f$

These results lead us to the following proposition,

Proposition 1 In a duopolistic industry that operates under decreasing returns to scale and produces a homogeneous product an internal patentee will prefer licensing a non-drastic innovation by means of a combination of a per unit royalty plus a fixed fee than by means of a per unit royalty if and only if:

The market parameter is quite small ($0 < a < a_1$) or

The market parameter is quite high ($a > a_1$) and the cost parameter takes large values ($c_3 < c < c_1$).

Proof: It is resulting by comparing the patentee profits under per unit royalty mechanism and per unit royalty plus fixed fee mechanism. \square

3.2 Ad valorem royalty plus a fixed fee mechanism

In this subsection, we suppose that licensor make use of a license contract which combines an ad valorem royalty, denoted by d and a fixed fee F . When the licensing contract is based on a ad valorem royalty, the compensation paid by the licensee is proportional to the sales revenues which he collects from the output produced with the patented technology.

Firm 1 wants to determine the production quantity that maximizes its own profit which is the sum of the profits from its production and the profits from licensing. Firm's 1 problem is then:

$$\max_{q_1} \Pi_1^{adf} = (a - q_1 - q_2)q_1 - bq_1^2 + d(a - q_1 - q_2)q_2 + F \quad (16)$$

On the other hand, firm 2 seeks the production quantity that maximizes its profit. Firm's 2 profit is given by the difference between the profits from its production and the ad valorem royalty it will pay to the patentee. Hence, the firm's 2 problem is

$$\max_{q_2} \Pi_2^{adf} = (a - q_1 - q_2)q_2 - bq_2^2 - d(a - q_1 - q_2)q_2 - F \quad (17)$$

Solving these maximization problems yields to the corresponding optimal production levels

$$q_1^{adf} = \frac{a(1 + 2b) - da(2 - d)}{(2b + 3 - d)(2b + 1 - d)} \quad (18)$$

$$q_2^{adf} = \frac{a(1 + 2b)(1 - d)}{(2b + 3 - d)(2b + 1 - d)} \quad (19)$$

At the second stage, firm 2 will decide to buy the innovation from the patentee if and only if its profits with the license are equal to or higher than those without the license. The profits of firm 2 without licensing is given by Eq.(7), thus the following inequality must hold:

$$\bar{\Pi}_2^{NI^*} \leq (1 - d)(a - q_1 - q_2)q_2 - F, \quad (20)$$

and

$$0 < d < 1. \quad (21)$$

At the first stage the patentee will set the ad-valorem royalty d , which maximizes his total profits subject to the restrictions imposed by the second and third stages of the game. So, he solves the following problem:

$$\max_{d,F} \Pi_1^{adf} = (a - q_1 - q_2)q_1 - bq_1^2 + d(a - q_1 - q_2)q_2 + F \quad (22)$$

$$\text{s.t (18),(19),(20),(21)} \quad (23)$$

The solution of this problem yields to the following results:

If $c_2 < c < c_1$ and $d < 1$

$$F = \frac{a[da(2b + 1) - c(2b - d + 3)](d - 1)(2b + 1)}{(2b + 1 - d)(2b + 3 - d)^2}$$

$$\Pi_1^{adf} = \frac{a[-bad^4 + (2b(a - c) - (a + c))d^3 + (a(2b + 3) + c(8b^2 + 14b + 5))d^2}{(2b + 3 - d)^2(1 + 2b - d)^2} +$$

$$+ \frac{(-1((1 + 2b)(4bc(b + 3) + a(2b + 3) + 7c)d + (1 + 2b)^2(a(b + 1) + c(2b + 3)))}{(2b + 3 - d)^2(1 + 2b - d)^2}$$

It should be noticed that when the effect of the new technology on the production cost is strong then the patentee is interested in including a positive fixed fee in his contract independently of the ad valorem rate he will choose and the following holds:

$$\Pi_1^{adf} > \Pi_1^{ad} \quad \forall c \in (c_2, c_1), \text{ and } d < 1$$

This remark lead to the following proposition,

Proposition 2 In a duopolistic industry that operates under decreasing returns to scale and produces a homogeneous product an internal patentee will prefer licensing a non-drastric innovation by means of a combination of an ad valorem royalty plus a fixed fee (d, F) than by means of an ad valorem royalty d if and only if:

The cost parameter c takes large values ($c_2 < c < c_1$).

Proof: It is resulting by comparing the patentee profits under a combination of an ad valorem royalty plus a fixed fee mechanism and an ad valorem royalty mechanism.

□

4. COMPARISON OF THE MECHANISMS

In this section we compare the patentee's profit under the two-part tariff we studied. This comparison will allow us to set the values of the parameters for which the optimal contract of the patentee involves either a (r, F) or a (d, F)contract. We distinguish the the following cases:

If $a > a_2$ the validate interval is $c_3 < c < c_1$ and

$$\Pi_1^{rf} > \Pi_1^{adf}$$

If $a_1 < a < a_2$ the validate interval is $c_2 < c < c_1$ and

$$\Pi_1^{rf} > \Pi_1^{adf}$$

If $0 < a < a_1$ the validate interval is $c_2 < c < c_1$ and

$$\Pi_1^{adf} > \Pi_1^{rf}$$

Therefore, we can establish the following proposition,

Proposition 3 In a duopolistic industry that operates under decreasing returns to scale and produces a homogeneous product an internal patentee will prefer licensing a non-drastric innovation by means of a combination of an ad valorem royalty plus a fixed fee than by means of per unit royalty plus a fixed fee if and only if:

The market parameter take small values ($0 < a < a_1$).

Proof: It is resulting by comparing the patentee profits under a combination of an ad valorem royalty plus a fixed fee mechanism and a combination of a per unit royalty plus a fixed fee mechanism. □

Obviously, as the patentee's profit functions under both mechanisms are continuous, for the rest of the values either the patentee will prefer to license the new technology by means of a combination of a per-unit royalty and a fixed fee or he will prefer or he will prefer a licensing scheme based on pure royalties. Results obtained after the solution of the R game, which are presented in Tables (1)-(3) confirm this statement.

Table 1: Structure of profit advantage in licensing mechanisms for $a > a_2$

$0 < c < c_2$	$c_2 < c < c_3$	$c_3 < c < c_1$
$\Pi_1^r > \Pi_1^{rf}$	$\Pi_1^r > \Pi_1^{rf}$ and $\Pi_1^r > \Pi_1^{adf}$	$\Pi_1^{rf} > \Pi_1^r$ and $\Pi_1^{rf} > \Pi_1^{adf}$

Table 2: Structure of profit advantage in licensing mechanisms for $a_1 < a < a_2$

$0 < c < c_3$	$c_2 < c < c_3$	$c_3 < c < c_1$
$\Pi_1^r > \Pi_1^{rf}$	$\Pi_1^{rf} > \Pi_1^r$	$\Pi_1^{rf} > \Pi_1^r$ and $\Pi_1^{rf} > \Pi_1^{adf}$

Table 3: Structure of profit advantage in licensing mechanisms for $0 < a < a_1$

$0 < c < c_2$	$c_2 < c < c_1$
$\Pi_1^{rf} > \Pi_1^r$	$\Pi_1^{rf} > \Pi_1^r$ and $\Pi_1^{adf} > \Pi_1^{rf}$

5. CONCLUSIONS

This article considers the optimal two-part licensing scheme in a Cournot duopoly where the production is characterized by the existence of return to scale. We investigate all the parameters affecting the optimal licensing policy of an internal patentee and we identify the optimal two-part tariff licensing contract based on the cost reduction imposed by the use of the new technology and the market parameter. We show that a two tariff mechanism prevails when the patentee's innovation provides a quite high cost reduction. However, the type of the two-part tariff that the patentee's contract involve is affected the market parameter, (i) if it takes large values then the optimum patentee policy is a per unit based two-part tariff mechanism and; (ii) if it takes small values the optimum policy of the patentee is to set a two-tariff mechanism which includes a positive ad valorem royalty plus a fixed fee.

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