Ad-valorem and Royalty Licensing under Decreasing Returns to Scale

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ABSTRACT— In this paper we study the licensing of a cost reducing innovation by an internal patentee. The analysis considers a Cournot duopoly characterized by non-constant returns to scale. The patentee chooses strategically whether to charge a per-unit or an ad-valorem royalty to the other firm in the industry. Our results indicate that an internal innovator may generate different incentives for his licensing contract depending on the cost reduction imposed by the use of the new technology.

Keywords—Patent Licensing; Royalty; Ad-valorem; Cournot Duopoly; Game Theory

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1. INTRODUCTION

Patents provide inventors with diffusion control of their inventions during a period of time. Thus, a patent acts as incentive for innovation. The licensing policies are categorized in up frond fee, per unit royalty and ad valorem royalty. In this paper we study the impact of scale economies on a tree stage Cournot duopoly game of patent licensing.

The patent licensing analysis was initiated by Arrow [1], who studies licensing to a perfectly competitive industry and to a monopoly using a royalty. Most of the theoretical studies in licensing focused in the comparison of up frond fee and per-unit royalty under two assumptions: (a) the existence of an external innovator and (b) the competition of both the licenser and the licensees in the product market. In their seminal work, Kamien and Tauman [6] considered an external patentee and a homogeneous Cournot oligopoly and shown that licensing by means of up frond fees dominates licensing by a per unit. Similar results can be found in Katz and Shapiro [8] and Kamien et al. [4].

Considering the case of an internal innovator, Wang [12] found that per-unit royalty licensing can be superior to up frond fee licensing for an internal patentee and a non drastic innovation in a Cournot duopoly. Kamien and Tauman [7] consider the Wang's model in the case of any number of competitors, their findings shows that when the number of firms in the industry exceeds a certain threshold number, the patentee's profit is optimized when licensing by means of a royalty rather than by auctioning fixed fee licenses. Filippini [15] modeled the competition as a Stackelberg duopoly with the innovator as the Stackelberg leader, he proved that has shown that the optimal policy has only royalty and no upfront fee.The works of Wang and Yang [14] and Wang [13] are concerned with the superiority of up frond fee under Bertrand and Cournot competition in the presence of product differentiation. In the same spirit Fauli-Oller and Sandoni [3] have stated that the optimal licensing always includes a positive royalty.

Sen and Tauman [10] examined a Cournot oligopoly under the existence of both an internal and an external innovator; their findings indicate that when the number of firms is not too small, licensing of an innovation involves a positive royalty.

While the literature is generally focused on per unit and fixed fees or a combination of these two, apart from the next two notable exceptions, no paper has investigated the ad valorem royalties. Bousquet et al., [2] consider an external

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innovator trying to identify the optimal licensing policy (ad valorem or per unit) under uncertainty. While San Martin and Saracho [9] have shown that in the classic homogenous good Cournot duopoly an internal patentee will always prefer the ad valorem royalty to a per unit royalty.

Although abundant, the existing literature is constrained by the assumption of constant returns to scale, failing to capture an important aspect of production, the existence of returns to scale. One exception is the seminal work of Sen and Stamatopoulos [11] who extended Sen and Tauman's model by examine the problem of licensing of cost reducing innovation under returns to scale. Their main results state that incidence of positive royalties and diffusion of innovation are both inversely related to returns to scale.

In the current paper we analyze the relevance of the ad valorem royalty in licensing under returns to scale. This paper provides an extension of the model of royalty licensing proposed by San Martin and Saracho [9]. We construct a three stage non cooperative model under returns to scale and an internal patentee in the classic homogeneous Cournot duopoly. At the first stage the patent-holding duopolist acts by setting an ad-valorem or a per-unit royalty rate. At the second stage, the other firm decides the conditions which allow it to accept or not the offer from the patent holder. In the third stage, both firms engage in a non cooperative game in quantities.

Our main results show that, under decreasing returns to scale, a patentee will always prefer a contract based on an ad valorem royalty when the rate charged is rather high. On the other hand, when the ad valorem rate is rather small his preference is affected by the reduction in marginal cost induced by the use of the new technology. More precisely, when the reduction is low he will prefer an ad valorem based contract instead of a per unit one and vice versa. That is, when the reduction of marginal cost is low the negative impact of decreasing returns to scale and the low production drive the patentee in an ad valorem charge increasing thus his licensing revenues. On the contrary, when this reduction is high, the patentee prefers to be more aggressive, to produce more and is more profitable for him to charge a per unit royalty to the other firm. In conclusion, the patentee will prefer ad valorem royalty since it guarantees payoff advantages.

The rest of of the paper is organized as follows. Section 2 presents the model of the patent licensing and analyzes the two royalty licensing schemes considered. Section 3 compares the schemes, while Section 4 concludes with some final remarks.

2. LICENSING IN A HOMOGENEOUS PRODUCT COURNOT DUOPOLY UNDER RETURNS TO SCALE

In this section we examine the effect of returns to scale on the optimal licensing policies. Consider a Cournot duopoly with two firms 1 and 2. Let p denotes the market price and q_i the output of the firm i. The inverse demand function is given by$p = a - Q$ for $a \geq Q$ and $p = 0$ for $a < Q$, where $Q = \sum_{i=1}^{2} q_i$. Prior to innovation there exists a specific level of technology identical to both firms. In order to capture both increasing and decreasing returns to scale, we assume that the existing technology could be represented by the total cost function, $TC = (bq_i + c)q_i$. The constant b determines the nature of the technology. When $b > 0$ the function $bq_i + c$ is increasing, therefore the technology exhibits decreasing returns to scale. When $b = 0$ the marginal function is equal to c implying the presence of constant returns to scale. When $b < 0$ the function $bq_i + c$ is decreasing if $q_i = -c/b$ so the technology exhibits increasing returns to scale until this level of production, and stays zero beyond it.

Without loss of generality, we assume that firm 1 owns a patent for a new non drastic cost reducing technology. More precisely we assume that the new technology results in cost reduction on the parameter c. Hence, the total cost function under the new technology is $TC^{NT} = (bq_i)q_i$ where NT denotes the use of the new technology. When $b \leq 0$, $TC^{NT} < 0, \forall q_i > 0$. Therefore, our analysis is focused only in the case of decreasing returns to scale ($b > 0$).

According to Arrow [1] an innovation is drastic if the monopoly price under the new technology does not exceed the unit production cost of the old technology otherwise it is non drastic. In our case since the innovation is non drastic, it must be that $c < \underline{c}$, where $\underline{c} = \frac{a(b+1)}{b+2}$.

The interaction between the internal patentee and the licensee is modeled as a three stage non-cooperative game. We call this game R. At the first stage, the owner of the patent, sets a royalty per unit or an ad-valorem rate. At the second stage firm 2, reacts and decides to accept or not the proposed rate. In the final stage both firms participate in a non-cooperative quantity competition game.

2.1 The Cournot equilibrium with no transaction between the firms

We will start our analysis by presenting the Cournot game of the duopolists in the case where firm 2 is not willing to buy firm's 1 patent. Results of this model will be used to make inferences about the alternative licensing schemes studied later.

The profit functions of the firms are given by the equations

$$\overline{\Pi}_1{}^{NI} = (a - q_1 - q_2)q_1 - bq_1^2 \tag{1}$$

$$\overline{\Pi}_2{}^{NI} = (a - q_1 - q_2)q_2 - (bq_2 + c)q_2 \tag{2}$$

Firm 1 will choose such that to maximizes its profit, which leads to the firm's 1 reaction function:

$$\bar{q}_1 = \frac{a - q_2}{2(b+1)} \tag{3}$$

Similarly the maximization of firm's 2 profit function yields to the firm's 2 reaction function:

$$\bar{q}_2 = \frac{a - q_1 - c}{2(b+1)} \tag{4}$$

Then, optimal quantity produced by each firm is given by the intersection of these two reaction functions:

$$\bar{q}_1^{NI^\star} = \frac{a(2b+1) + c}{(2b+3)(2b+1)}$$

$$\bar{q}_2^{NI^\star} = \frac{2b(a-c) + a - 2c}{(2b+3)(2b+1)} \tag{5}$$

and the firms' equilibrium profits are

$$\overline{\Pi}_1^{NI^\star} = \frac{(2ba + c + a)^2(1+b)}{(2b+3)^2(2b+1)^2} \tag{6}$$

$$\Pi_2^{NI^\star} = \frac{(2ba - 2c - a - 2bc)^2(1+b)}{(2b+3)^2(2b+1)^2} \tag{7}$$

2.2 Per unit royalty licensing

Under this licensing scheme the patentee will charge to licensee a fixed royalty rate r for the use of its innovation. Accordingly, the licensee pays a certain amount per unit of output produced with the patented technology. We are interested for sub-game perfect Nash equilibrium of the game. The game must be solved by backward induction. At the third stage each firm chooses the production quantity that maximizes his profit given the per unit production royalty r which is set in the first stage of the game,

Thus, firm 1 solves the problem

$$\max_{q_1} \overline{\Pi}_1^{ro} = (a - q_1 - q_2)q_1 - bq_1^2 + rq_2, \tag{8}$$

while firm 2 solves the problem

$$\max_{q_1} \overline{\Pi}_1^{ro} = (a - q_1 - q_2)q_2 - bq_2^2 - rq_2 \tag{9}$$

The solution of the problems yields to the equilibrium production level of each firm:

$$q_1^{ro^\star} = \frac{a(2b+1) + r}{(2b+3)(2b+1)} \tag{10}$$

$$\bar{q}_2^{ro^\star} = \frac{a(1+2b) - 2r(b+1)}{(2b+3)(2b+1)} \tag{11}$$

At second stage, firm 2 will not buy the new technology if its marginal cost with innovation is greater than this without it. In other words, firm 2 is willing to buy the innovation if the following holds:

$$bq_2 + r \leq bq_2 + c \Rightarrow r \leq c. \tag{12}$$

At the first stage, firm 1 chooses the r which maximizes its profit while taking into account the restrictions imposed by the previously discussed stages.

So, the problem solved by firm 1 is the following

$$\max_r (a - q_1 - q_2)q_1 - bq_1^2 + rq_2 \tag{13}$$

$$s.t \;\; (10), (11), (12) \tag{14}$$

The solution of the problem leads us to the following findings:

a) The patentee will set a per unit royalty equal to reduction in the marginal cost of the production induced by the innovation i.e., $r = c$

b) The production of the industry is $\bar{Q}^{ro^\star} = \frac{2a-c}{2b+3}$ and

c) the patentee's profit is

$$\bar{\Pi}_1^{ro^\star} = \frac{(1 + 5b + 8b^2 + 4b^3)a^2 + (5c + 24b^2c + 8b^3c + 20bc)a + (5 - 21b - 8b^3 - 24b^2)c^2}{(2b + 3)^2(2b + 1)^2}$$

2.3 Ad valorem royalty licensing

In this subsection, we suppose that licensor make use of a license contract with an ad valorem royalty, denoted by d. When the licensing contract is based on a ad valorem royalty, the compensation paid by the licensee is proportional to the sales revenues which he collects from the output produced with the patented technology.

Firm 1 wants to determine the production quantity that maximizes its own profit which is the sum of the profits from its production and the profits from licensing.

Firm's 1 problem is then:

$$\max_{q_1} \; \bar{\Pi}_1^{ad} = (a - q_1 - q_2)q_1 - bq_1^2 + d(a - q_1 - q_2)q_2 \tag{15}$$

One the other hand, firm 2 seeks the production quantity that maximizes its profit. Firm's 2 profits is given by the difference between the profits from its production and the ad valorem royalty it will pay to the patentee. Hence, the firm's 2 problem is

$$\max_{q_2} \; \bar{\Pi}_2^{ad} = (a - q_1 - q_2)q_2 - bq_2^2 - d(a - q_1 - q_2)q_2 \tag{16}$$

Solving these maximization problems yields to the corresponding optimal production levels

$$\bar{q}_1^{ad^\star} = \frac{a(1 + 2b) - da(2 - d)}{(2b + 3 - d)(2b + 1 - d)} \tag{17}$$

$$\bar{q}_2^{ad^\star} = \frac{a(1 + 2b)(1 - d)}{(2b + 3 - d)(2b + 1 - d)} \tag{18}$$

At the second stage, firm 2 will decide to buy the innovation from the patentee if and only if its profits with the license are equal to or higher than those without the license. The profits of firm 2 without licensing are given by Eq. (7), thus the following inequality must hold:

$$\bar{\Pi}_2^{NI^\star} \leq \Pi_2^{ad} \tag{19}$$

At the first stage the patentee will set the ad-valorem royalty, which maximizes his total profits subject to the restrictions imposed by the second and third stages of the game. So, he solves the following problem:

$$\max_d \; \Pi_1^{ad} - (a - q_1 - q_2)q_1 - bq_1^2 + d(a - q_1 - q_2)q_2 \tag{20}$$

$$\text{s.t } (17),(18),(19) \tag{21}$$

The patentee's profit, under the ad valorem royalty mechanism is:

$$\overline{\Pi}_1^{ad\star} = \frac{\left((-8d^2+8d+4)b^3+(4d^3-16d^2+8d+8)b^2+(-d^4+6d^3-8d^2-2d+5)b+1+2d+d^2\right)a^2}{(2b+3-d)^2(2b+1)^2}$$

Remark 1 *The validation of the game R discussed in the case of the existence of decreasing returns to scale requires that the ad valorem rate must be less than 1.* $(d < 1)$

Proposition 1 *The solution of the problem (19), given (17) and (18) and* $b > 0$ *ensures that the following hold:*

(a) $\overline{\Pi}_2^{ad} - \overline{\Pi}_2^{ad} \begin{cases} \leq 0 \; if \; 0 < c < \check{c} \\ > 0 \; if \; \check{c} < c < \underline{c} \end{cases} \quad \forall \, 0 < d < \underline{d}$

(b) $\overline{\Pi}_2^{ad} - \overline{\Pi}_2^{ad} \leq 0 \quad \forall \; \underline{d} < d < 1$

where $\underline{d}$ *depends on* a *and* b *i.e.,* $\underline{d}(a,b)$ *and* $\check{c}$ *depends on* a, b *and* d *i.e.,* $\check{c}(a,b,d)$.

The incentive of firm 2 to acquire the innovation by an ad valorem rate is determined to a large extent by the magnitude of the reduction in the marginal cost of production. The negative impact of the decreasing returns to scale on the cost advantage is higher when this reduction is low.

Proof: We examine the difference $w_2 = \Pi_2^{ad} - \Pi_2^{NI^\star}$, for $b > 0, 0 < c < \underline{c}$ and $d < 1$. $w_2 = 0$ for $c = \check{c}$[2] and $c = \check{\check{c}}$. For $0 < d < \underline{d}$ the function $\check{\check{c}} > \underline{c}$ and the $\check{c} < \underline{c}$. The $\underline{d}$ is the only real root of a quartic function $\check{c} - \underline{c} = 0$ for the interval we are interested. Hence, only the $\check{c}$ affects the sign of w_2.

- For $0 < c < \check{c}$ the sign of w_2 is negative, consequently $\Pi_2^{NI^\star} > \Pi_2^{ad}$.
- For $\check{c} < c < \underline{c}$ the sing of w_2 is positive consequently $\Pi_2^{ad} > \Pi_2^{NI^\star}$.

For $\underline{d} < d < 1$, the $\check{c} > \underline{c}$ and the $\check{\check{c}} > \underline{c}$ therefore is only negative in the examined internal which means that $\Pi_2^{NI^\star} > \overline{\Pi}_2^{ad}$. □

3. COMPARISON OF THE MECHANISMS

We next examine the issue of ad valorem versus per unit royalty of patentee's profitability.

Proposition 2 *Consider the game R . For* $b > 0$ *and* $0 < d < \underline{d}$ *and* $\check{c} < c < \underline{c}$ *there exist* $\bar{\bar{d}} < \underline{d}$ *and* $\check{c} < \check{\check{c}} < \underline{c}$ *such that the following hold:*

(a) If $0 < d < \bar{\bar{d}}$ *then*

(i) $\overline{\Pi}_1^{ad\star} > \overline{\Pi}_1^{ro^\star}$ *for* $\check{c} < c < \check{\check{c}}$

(ii) $\overline{\Pi}_1^{ro^\star} > \overline{\Pi}_1^{ad^\star}$ *for* $\check{\check{c}} < c < \underline{c}$

(b) if $\bar{\bar{d}} < d < \underline{d}$, *then* $\overline{\Pi}_1^{ad^\star} > \overline{\Pi}_1^{ro^\star}$

where $\bar{\bar{d}}$ *depends on* a *and* b *i.e.,* $\bar{\bar{d}}(a,b)$ *and* $\check{\check{c}}$ *depends on* a, b *and* d *i.e.,* $\check{\check{c}}(a,b,d)$.

Proof: We are interested in the intervals $b > 0, 0 < d < \underline{d}$ and $\check{c} < c < \underline{c}$. Let w_3 be the difference between the patentee's profit function under the ad valorem and per unit royalty, i.e., $w_3 = \overline{\Pi}_1^{ad^\star} - \overline{\Pi}_1^{ro^\star}$. $w_3 = 0$ for $c = \check{\check{c}}$[3] and

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[2] $\check{c} = \frac{\left((b+1)+(2b+3-d)(2b+1-d)-\sqrt{(d-1)^2(b+1)(2b+3)^2(1+2b)^2(b-d+1)}\right)(1+2b)a}{2(b+1)^2(2b+3-d)(2b+1-d)}$

[3] $\check{\check{c}} = (\frac{-\left(4b^2+10b+5)(2b+3-d)(2b+1-d+\sqrt{(2b+3)^2\Delta}\right)(1+2b)a}{2(b+1)(8b^2+16b+5)(2b+3-d)(2b+1-d)}$ $\Delta = (-256d+64+256d^2)b^6+(-128d^{3n}+384-1408b+1280d^2)b^5+(2752d^2-576d^3+32d^4+896-3040d)b^4+(1040+3200d^2+96d^4-3328d-$

$c = \check{c}$.

For $0 < d < \bar{\bar{d}}$ the function $\check{c} > \underline{c}$ and $\check{c} < \check{\check{c}} < \underline{c}$. The $\bar{\bar{d}}$is the only real root of a quartic function $\check{\check{c}} - \underline{c} = 0$ for the interval we are interested in. Hence, $\check{\check{c}}$ the affects the sign of w_3.

- For $\check{c} < c < \check{\check{c}}$ the sign of w_3 is positive implying that $\bar{\Pi}_1^{ad^\star} > \Pi_1^{ro^\star}$.
- For $\check{\check{c}} < c < \underline{c}$ the sign of w_3 is negative implying that $\Pi_1^{ro^\star} > \bar{\Pi}_1^{ad^\star}$.

For $\bar{\bar{d}} < d < \underline{d}$ the $\check{\check{c}} < \check{c}$ and $\check{c} > \underline{c}$. Therefore the sign of w_3 is only positive in the examined interval which means that $\bar{\Pi}_1^{ad^\star} > \Pi_1^{ro^\star}$. □

In the case of decreasing returns to scale we have the superiority of the ad valorem charge instead of per unit rate in licensing if and only if the ad valorem rate is high. When the ad valorem rate is low the reduction in marginal cost of production plays an important role in the patentee's preference. More specifically, if this reduction is low, the patentee prefers an ad valorem royalty, while if the reduction is high, he prefers a royalty per unit. The superiority of the ad valorem royalty can be explained by the inclination of the patentee to be less aggressive. The revenues from licensing become increasing in the price output. The patentee commits to produce less however his total profit increases as the revenues from licensing increases.

4. CONCLUSION

In this paper we have studied and compared licensing by means of a per unit royalty and licensing by means of an ad valorem royalty in a Cournot duopoly model under returns to scale. The main conclusion of our analysis is that the presence of ad valorem and per unit royalty is both related to decreasing returns of scale. The superiority of an ad valorem royalty is strongly related to the marginal cost of production. The patentee is strategically less aggressive because his revenues from licensing become increasing. The reason is that the reduction in marginal cost of production is low and the other firm allows him to control this cost in his arrival. In the presence of decreasing returns to scale, an ad valorem royalty is more likely to occur.

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