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Global Chaos Identical and Nonidentical Synchronization of a New 3-D Chaotic Systems Using Linear Active Control

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ABSTRACT— ***Synchronization of chaotic systems is a strategy wherein two chaotic oscillators adjust a given property of their motion to a periodic behavior due to their mutual coupling or forcing. This paper has studied and investigated the Chaos Synchronization problem of unified 3-D chaotic systems and two different 3-D chaotic systems using the Active Control Technique. Based on Lyapunov Stability Theory and Ruth-Hurwitz Criterion and using Active Control Algorithm, it has been shown that the proposed schemes have outstanding transient performances and that analytically as well as graphically, synchronization is globally exponentially stable. Numerical simulations and graphs are imparted to show the efficiency and effectiveness of the proposed schemes.***

Keywords— Synchronization, Lyapunov Stability Theory, Hurwitz Criterion, Active Control

1. INTRODUCTION

The aim of chaos control is to stabilize a previously chosen unstable periodic orbit by means of small parameter perturbations applied to the system, so the chaotic dynamics is substituted by a periodic one, chosen at will among the several available [1]. This makes chaotic systems very interesting because they allow different uses, without performing structural changes, and employing a minimal external input [2].

There is another important application of controlling chaos, known as the “synchronization of chaotic systems ". Synchronization of chaos as a process wherein two chaotic systems (either identical or nonidentical) regulate a given property of their motion to a similar behavior, due to pairing or forcing which ranges from complete agreement of trajectories to interlocking of phases [3]. If one consider two identical chaotic systems starting from different initial conditions, then the critical sensitivity to initial conditions (Butterfly Effect) implies that their differences sprout exponentially in time, and that they will blossom in an unsynchronized manner. However these differences can be reduced to zero by supplying right feedback signal from one system to another system and force the two systems into a synchronized behavior such that after developing the chaotic motion, two systems are in stepwise during the course of time [4].

After the pioneering work of Peccora and Carroll on synchronization of identical chaotic systems, several algorithms [5] have been developed and applied to synchronize two (identical/non-identical) chaotic systems. Among them, Linear Active Control algorithm is an effective approach to synchronize two identical (nearly identical) and non-identical chaotic systems.

Recently, the synchronization problem via Active Control Techniques has attracted great interests among the researchers and have been widely accepted as one of the powerful technique used to synchronize two identical as well as nonidentical chaotic systems[6-8].

Chaos synchronization using Active control was proposed by E. W. Bai, et. al., [9] and further developed by A. N.Njah & U.E.Vincent [10] . Active Control Technique has recently been accepted as one of the most efficient technique for synchronizing both identical and nonidentical chaotic systems because of its implementation to practical systems such as Windmi and Coullet systems, Nonlinear Circuit, Chemical Reactors etc. [11-13]. An Active Controller can be easily designed according to the given conditions of the chaotic system to accomplish synchronization asymptotically if the nonlinearity of the system is known. There are no derivatives in the controller to execute the controller and this characteristic gives an advantage to Active Control Technique over other synchronization approaches.

Recently, there has been spiraling interest developed theoretically as well as experimentally to precipitate 3-D chaotic oscillators. The importance of the 3-D differential equations, is that relatively simple 3-D systems could reveal a very complex or more specifically chaotic behavior. The 3-dimensional chaotic systems have broad band potential applications in different scientific fields [5]. Looking into the wide range of applications of 3-D chaotic systems, various 3-D chaotic systems have been generated and applied to many practical systems and have shown some useful results [14-16].

In ref [17], the authors studied a new 3-D Autonomous Chaotic System which is based on a quadratic cross product term and a quadratic exponential nonlinear term. In ref [17], the authors proposed a novel 3-D chaotic system which is topologically different from the Lorenz System. The two-scroll attractor from the new system exhibits multiplex chaotic dynamics. The new system shows some typical characteristics of a dynamical system such as Route to Chaos, Stable Fixed Points, Period-doubling Bifurcation and quasi-periodic loops etc. In ref [17], the nonlinear dynamical properties of the new chaotic system such as Equilibrium Points, Phase Portraits, Lyapunov Exponents, Bifurcation diagram, and Poincare mapping etc have been extensively studied.

Motivated from above, the main objective of this paper is to utilize the Linear Active Control Technique to study and investigate the new results for global chaos synchronization of identical and nonidentical chaotic systems introduced in references [17]. Based on Ruth-Hurwitz criterion [18] and Lyapunov Stability Theory [19], a class of Active Control Schemes will be designed to achieve the synchronization globally exponentially. Numerical simulations and graphs will be imparted to show the performance and effectiveness of the proposed approaches.

To the best of our knowledge, the same study has not done before. The rest of the paper is organized as follows: Unit 2, discusses the problem statement and the proposed methodology. In unit 3, chaos synchronization of identical and nonidentical new 3-D chaotic system will be investigated. In unit 4, numerical simulations are furnished to show the effectiveness of the proposed methodology. In unit 5, the concluding remarks will be given.

2. DESIGNING OF A LINEAR ACTIVE CONTROLLER

The problem of chaos synchronization is to design a coupling between the two chaotic systems such that the chaotic time evaluation becomes perfect. The output of the slave (response) system asymptotically follows the output of the master (drive) system.

Consider a master system described by the following differential equation,

$$\dot{x} = A_1 x + F_1(x) \tag{2.1}$$

and a slave system is described as,

$$\dot{y} = A_2 y + F_2(y) + \psi(t) \tag{2.2}$$

Where **x, y** $\in R^n$ are the state vectors, $F_1(x), F_2(y) : R^n \to R^n$ are the nonlinear functions and $A_{1,} A_2 \in R^{n\times n}$ are constant system matrices of the corresponding master-slave systems respectively, and '$\psi(t) \in R^{n\times 1}$' is the control input injected to the slave system.

From systems of equations (3.1.2) and (3.1.3), the error dynamics can be described as:

$$\dot{e} = \dot{y} - \dot{x}$$

$$\Rightarrow \qquad \dot{e} = A_2 y + F_2(y) + \psi(t) - A_1 x - F_1(x)$$

$$\Rightarrow \qquad \dot{e} = Be + G(x, y) + \psi(t) \qquad (2.3)$$

Where, $\dot{e}_i = y_i - x_i$, $\mathrm{i} = 1, 2, ..., n$, $B = \bar{A}_2 - \bar{A}_1$ is the common parts of the system matrices in master-slave systems and $\mathrm{G}(x, y) = F_2(y) - F_1(x) + A_2 y - A_1 x$ contains the nonlinear functions and non-common terms and $\psi(t) = [\psi_1(t), \psi_2(t), ..., \psi_n(t)]^T \in R^{n\times 1}$ is the Active control input .

If $F_1(\square) = F_2(\square)$ *and / or* $A_1 = A_2$ then **x** and **y** are the states of two unified chaotic systems and if

$F_1(\square) \neq F_2(\square)$ *and / or* $A_1 \neq A_2$, then **x** and **y** are the states of two nonidentical chaotic systems.

An appropriate Active feedback controller '$\psi(t)$' that satisfies the error system converges to equilibrium point (zero),

i.e, $\quad \lim_{t\to\infty} e_i = \lim_{t\to\infty} \left| y_i(t) - x_i(t) \right| = 0$, $\forall x, y, e \in R^n$,

then the systems (2.1) and (2.2) are said to be synchronized [8].

Thus the basic problem in synchronizing two unified/different chaotic systems is to design a proper Active Feedback Controller that eliminates nonlinear terms and non-common parts and to contain another linear part to achieve asymptotically stability [10].To achieve this goal, let us assume the following theorem.

Theorem 1. The trajectories of the two (identical or nonidentical) chaotic systems (2.1) and (2.2) for any initial conditions, $\left(x_{1m}(0), x_{2m}(0), ..., x_{nm}(0) \neq y_{1s}(0), y_{2s}(0), ..., y_{ns}(0)\right)$ will be synchronized asymptotically globally with suitable Active Feedback Controller, $\psi(t) = [\psi_1(t), \psi_2(t), ..., \psi_n(t)]^T \in R^{n\times 1}$.

Proof. Let us suppose that parameters of the master and slave systems are known and the states of both systems (2.1) and (2.2) are measurable. An appropriate refinement of the Active Feedback Controller deduct the unstable eigenvalue to a stable location. The feedback control signal $\psi(t)$ is constructed in two segments. The first part eradicates the nonlinear terms from (2.3) and the second part $u(t)$ acts as an external impute to stabilize the error dynamics (2.3).

i.e., $$\psi(t) = -G(x, y) + u(t)$$

Where $u(t) = -Ce = -C(\mathrm{y}_i - x_i)$ is a linear controller and $C \in R^{n\times n}$ is a feedback constant gain matrix. Thus the error dynamics (2.1.3) becomes,

$$\dot{e} = Be + u(t)$$

$$\Rightarrow \qquad \dot{e} = Be - Ce = (B - C)e$$

$$\Rightarrow \qquad \dot{e} = De \qquad (2.4)$$

Where, $D = (B - C)$ is $n \times n$ matrix.

From equation (2.4), if the error dynamics (2.4) is a linear system of the form, $\dot{e} = De$ and if the system matrix $\boldsymbol{D}$ is Hurwitz [18], i.e., if all the eigenvalues of the system symmetric matrix $\boldsymbol{D}$ are negative, then by the Linear Control Theory [18], the error signal will be asymptotically stable.

2.1 Criteria for Globally Exponentially Stability

The active control design uses Lyapunov Stability Theory for establishing globally exponentially synchronization between the two (identical or nonidentical) chaotic systems.

Let if we choose a candidate Lyapunov Error Function as:

$$V(t) = e^T Pe$$

where $P = dig(p_1, p_2, ...p_n) \in \mathrm{R}^{n\times n}$ is a positive definite matrix, where $V : R^n \to R^n$ is a positive definite function by construction [11].

If an Active feedback controller $\psi(t) = [\psi_1(t), \psi_2(t), ..., \psi_n(t)]^T \in R^{n\times 1}$ is designed such that,

$$\dot{V}(e) = -e^T Qe\,,$$

then, $\dot{V} : R^n \to R^n$ is a negative definite function [19] and hence the two systems (2.1) and (2.2) will be a globally exponentially stable by Lyapunov Stability Theory[19].

3. IDENTICAL SYNCHRONIZATION OF A NEW 3D CHAOTIC SYSTEM [16] VIA ACTIVE CONTROL

System Description: In reference [17], the authors proposed and investigated a new 3D autonomous chaotic system. The differential equations for the new chaotic systems is given as,

$$\left.\begin{aligned} \dot{x} &= a_1(y - x) \\ \dot{y} &= a_2 x - a_3 xz \\ \dot{z} &= e^{xy} - a_4 z \end{aligned}\right\} \qquad (3.1.1)$$

where $x, y, z \in R^n$ are the state variables and a_1, a_2, a_3 *and* a_4 are the parameters of the new system The new system [17] exhibits a chaotic attractor for, $a_1 = 10, a_2 = 40, a_3 = 2$ *and* $a_4 = 3$ with initial conditions are (2.2, 2.4, 28) and (4.4, 4.8, 40).

For the dynamical properties such as Equilibrium points, Phase portraits, Bifurcation diagram and Lyapunov Exponents etc for the system (3.1.1), please study reference [17].

In this section, the aim of the study is to achieve stable synchronization between two identical chaotic systems [17] using Active Controller. To clinch this goal, let us consider the master-slave systems arrangement for the identical synchronization of a new system [17] which is described as:

$$\left.\begin{aligned} \dot{x}_1 &= a_1(y_1 - x_1) \\ \dot{y}_1 &= a_2 x_1 - a_3 x_1 z_1 \\ \dot{z}_1 &= e^{x_1 y_1} - a_4 z_1 \end{aligned}\right\} \qquad \text{(Master system)} \qquad (3.1.2)$$

and

$$\left.\begin{aligned} \dot{x}_2 &= a_1(y_2 - x_2) + \psi_1 \\ \dot{y}_2 &= a_2 x_2 - a_3 x_2 z_2 + \psi_2 \\ \dot{z}_2 &= e^{x_2 y_2} - a_4 z_2 + \psi_3 \end{aligned}\right\} \qquad \text{(Slave system)} \qquad (3.1.3)$$

where $x_1, y_1, z_1 \in R^n$ are the state variables of the drive system with a_1, a_2, a_3 *and* a_4 are the system parameters and $x_2, y_2, z_2 \in R^n$ are the state variables of the corresponding response system and $\psi(t) = [\psi_1(t), \psi_2(t), \psi_3(t)]^T \in R^{n\times 1}$ are the Active Feedback Controller.

From systems of equations (3.1.2) and (3.1.3), the error dynamics can be described as:

$$\left.\begin{aligned} \dot{e}_1 &= a_1(e_2 - e_1) + \psi_1 \\ \dot{e}_2 &= a_2 e_1 + a_3 x_1 z_1 - a_3 x_2 z_2 + \psi_2 \\ \dot{e}_3 &= -a_4 e_3 + e^{x_2 y_2} - e^{x_1 y_1} + \psi_3 \end{aligned}\right\} \qquad (3.1.4)$$

Where $e_1 = x_2 - x_1, e_2 = y_2 - y_1$ and $e_3 = z_2 - z_1$

The control signal $\psi(t) \in \mathrm{R}^{n\times 1}$ will be constructed in two phases. In the first phase, the controller will vanish the nonlinear terms and in the second phase, it will acts as an external impute $u(t)$ to stabilize the error signal. Thus to arrive at asymptotically globally synchronization using Active Control, let us assume the following theorem.

Theorem 2. The two controlled Chaotic Systems (3.1.2) and (3.1.3) will achieve asymptotically, globally synchronization for initial conditions $(x_m(0), y_m(0), z_m(0)) \neq (x_s(0), y_s(0), z_s(0))$ with following Active Control law:

$$\left.\begin{aligned} \psi_1(t) &= -a_1 e_2 + u_1(t) \\ \psi_2(t) &= -a_2 e_2 - a_2 e_1 - a_3(x_1 z_1 - x_2 z_2) + u_2(t) \\ \psi_3(t) &= e^{x_1 y_1} - e^{x_2 y_2} + u_3(t) \end{aligned}\right\} \qquad (3.1.5)$$

Proof. Let us assumed that that the states of both systems (3.1.2) and (3.1.3) are measurable and parameters of the master and slave systems are known. Substituting equation (3.1.5) in equation (3.1.4), we have,

$$\left.\begin{aligned} \dot{e}_1 &= -a_1 e_1 + u_1(t) \\ \dot{e}_2 &= -a_2 e_2 + u_2(t) \\ \dot{e}_3 &= -a_4 e_3 + u_3(t) \end{aligned}\right\} \qquad (3.1.6)$$

Where,

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = -\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \qquad (3.1.7)$$

The error system (3.1.6) to be controlled is a linear system with a control input u_1, u_2 *and* u_3 as functions of the error states e_1, e_2 *and* e_3 respectively where the constants $d_{ij's}$ are known as feedback gains. As long as these feedbacks stabilize the error system then e_1, e_2 *and* e_3 converge to zero as time **'t'** tends to infinity [8]. This implies that the two unified chaotic systems (3.1.2) and (3.1.3) are synchronized asymptotically.

Replacing equation (3.1.7) in equation (3.1.6), we have,

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \begin{pmatrix} -a_1 & 0 & 0 \\ 0 & -a_2 & 0 \\ 0 & 0 & -a_4 \end{pmatrix} - \begin{pmatrix} d_{11} & d_{d12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} -a_1 - d_{11} & -d_{12} & -d_{13} \\ -d_{21} & -a_2 - d_{22} & -d_{23} \\ -d_{31} & -d_{32} & -a_4 - d_{33} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \qquad (3.1.8)$$

There are number of choices available for the controller coefficient $d_{ij's}$ and the choosing of a (3×3) gain matrix, $D = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$ should be such that the closed loop system (3.1.6) must have all the eigenvalues with negative real parts so that the error dynamics (3.1.6) converges to zero as time ***t*** tends to infinity [8].

The rate of convergence of the error system is controlled by the numerical value of the coefficients of the feedback gains. For the following particular choice of feedback gain matrix and considering, $a_1 = 10, a_2 = 40 \ and \ a_4 = 3$,

$$D_1 = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} = \begin{pmatrix} -8 & 0 & 0 \\ 0 & -38 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

the error system (3.1.8) becomes,

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \qquad (3.1.9)$$

From equation (3.1.9), It is clear that the error system (3.1.9) is a linear system of the form, $\dot{e} = Be$. Thus by linear control theory, the system matrix D_1 is Hurwitz [18], and so all the eigenvalues of the system matrix D_1 are negative (-2, -2, -2).

To check globally exponentially stability, let us construct a Lyapunov Error Function Candidate as;

$$V(e) = \frac{1}{2}(e_1^{\ 2} + e_2^{\ 2} + e_3^{\ 2})$$

$$\dot{V}(e) = (e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3) = -(a_1 + d_{11})e_1^{\ 2} - (a_2 + d_{22})e_2^{\ 2} - (a_4 + d_{33})e_1^{\ 2}$$

$$\dot{V}(e) = -2e_1^{\ 2} - 2_2^{\ 2} - 2e_1^{\ 2} = -e^T Qe$$

Where $Q = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ which is also a positive definite matrix.

Hence the above system (3.1.9) is globally exponentially stable, which implies that the two identical chaotic systems (3.1.2) and (3.1.3) are synchronized globally exponentially.

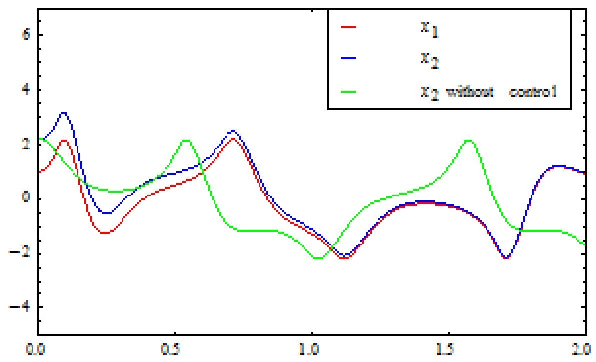

Figure 1: Time series of x_1 & x_2 (Identical systems [17])

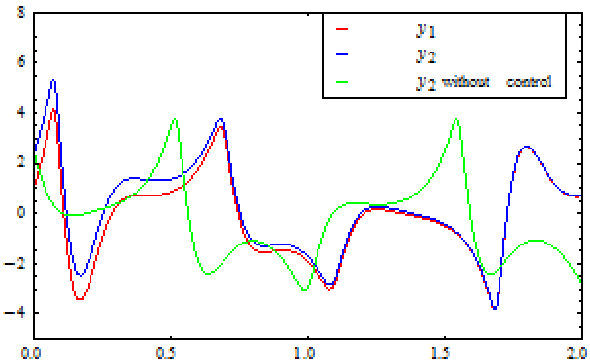

Figure 2: Time series of y_1 & y_2 (Identical systems [17])

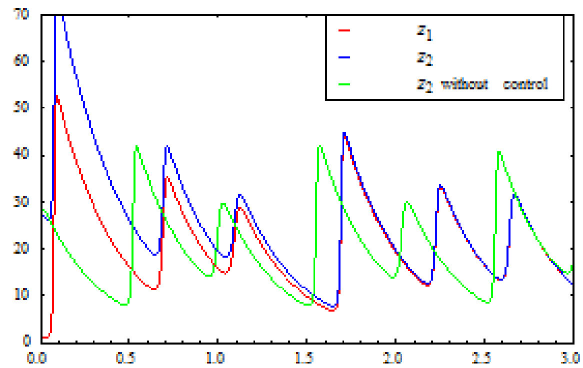

Figure 3: Time series of z_1 & z_2 (Identical systems [17])

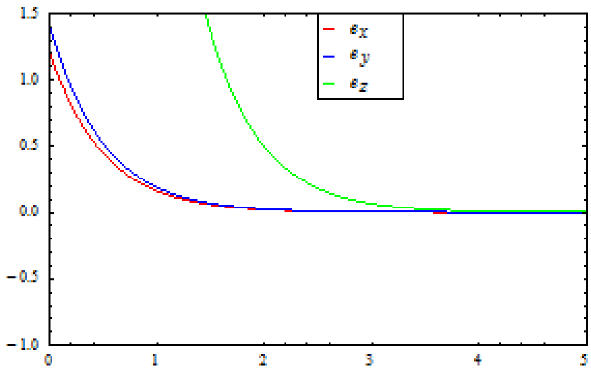

Figure 4: Time series of errors (Identical systems [17])

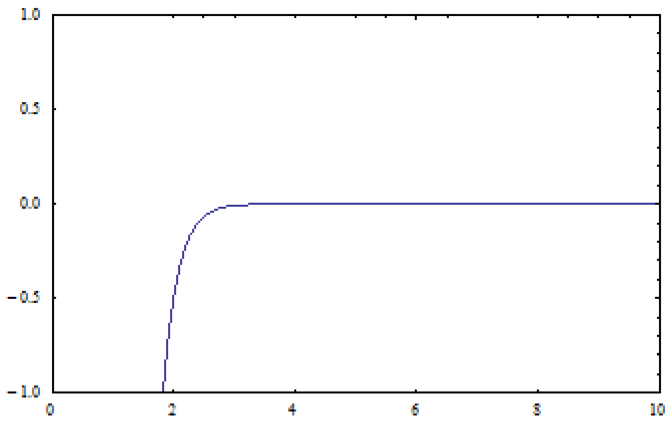

Figure 5: Derivative of Lyapunov Error Fynction (Identical systems [17])

3.2 Nonidentical Synchronization Between New [17] and Li Chaotic Systems

To achieve nonidentical synchronization for the new chaotic system [17] using Active Control Strategy, it is assumed that the new chaotic system [17] drives the Li Chaotic system [20]. Therefore, the master and slave systems configuration is given as;

$$\left.\begin{aligned} \dot{x}_1 &= a_1(y_1 - x_1) \\ \dot{y}_1 &= a_2x_1 - a_3x_1z_1 \\ \dot{z}_1 &= e^{x_1y_1} - a_4z_1 \end{aligned}\right\} \quad \text{(Master system)} \qquad (3.2.1)$$

and

$$\left.\begin{aligned} \dot{x}_2 &= p(y_2 - x_2) + \psi_1 \\ \dot{y}_2 &= -y_2 + x_2z_2 + \psi_2 \\ \dot{z}_2 &= q - x_2y_2 - rz_2 + \psi_3 \end{aligned}\right\} \quad \text{(Slave system)} \qquad (3.2.2)$$

where $x_1, y_1, z_1 \in R^n$ and $x_2, y_2, z_2 \in R^n$ are the corresponding state vectors of drive and response systems respectively, a_1, a_2, a_3 *and* a_4 are the system parameters of the master system and *p*, *q* and *r* are the system parameters of the slave system and $\psi(t) = [\psi_1(t), \psi_2(t), \psi_3(t)]^T \in R^{n\times 1}$ are the Active Feedback Controller that is yet to be designed.

The Li system describes a chaotic behavior with the parameters; $p = 5, q = 16$ and $r = 1$.

From equation (3.2.1) and (3.2.2), the error dynamics can be described as;

$$\left.\begin{aligned} \dot{e}_1 &= -pe_1 + (p - a_1)x_1 + a_1y_1 + py_2 + \psi_1 \\ \dot{e}_2 &= -e_2 - a_2x_1 - y_1 + x_2z_2 + a_3x_1z_1 + \psi_2 \\ \dot{e}_3 &= -a_4e_3 + (a_4 - r)z_2 + q - x_2y_2 - e^{x_1y_1} + \psi_3 \end{aligned}\right\} \qquad (3.2.3)$$

To achieve asymptotically globally synchronization using Active Control, re-defining the controller $\psi(t) = [\psi_1(t), \psi_2(t), \psi_3(t)]^T$ as,

$$\left.\begin{aligned} \psi_1(t) &= (a_1 - p)x_1 - a_1y_1 - py_2 + u_1(t) \\ \psi_2(t) &= a_2x_1 + y_1 - x_2z_2 - a_3x_1z_1 + u_2(t) \\ \psi_3(t) &= (r - a_4)z_2 - q + x_2y_2 + e^{x_1y_1} + u_3(t) \end{aligned}\right\} \qquad (3.2.4)$$

Substituting equations (3.2.4) in equation (3.2.3), we have,

$$\begin{aligned} \dot{e}_1 &= -pe_1 + u_1(t) \\ \dot{e}_2 &= -e_2 + u_2(t) \\ \dot{e}_3 &= -a_4e_3 + u_3(t) \end{aligned} \qquad (3.2.5)$$

where, $$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = -\begin{pmatrix} d_{11} & d_{12} & d_{12} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \qquad (3.2.6)$$

Replacing equation (3.2.6) in (3.2.5), we have,

$$\begin{pmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\end{pmatrix}=\begin{pmatrix}-p & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -a_4\end{pmatrix}\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix}-\begin{pmatrix}d_{11} & d_{12} & d_{13}\\ d_{21} & d_{22} & d_{23}\\ d_{31} & d_{32} & d_{33}\end{pmatrix}\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix}$$

i.e.,

$$\begin{pmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\end{pmatrix}=\begin{pmatrix}-p-d_{11} & -d_{12} & -d_{13}\\ -d_{21} & -1-d_{22} & -d_{23}\\ -d_{31} & -d_{32} & -a_4-d_{33}\end{pmatrix}\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix} \tag{3.2.7}$$

For the specific choice of feedback gains;

$$D_2=\begin{pmatrix}d_{11} & d_{12} & d_{13}\\ d_{21} & d_{22} & d_{23}\\ d_{31} & d_{32} & d_{33}\end{pmatrix}=\begin{pmatrix}-4 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix}$$

With this particular choice, the error system (2.2.8) becomes,

$$\begin{pmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\end{pmatrix}=\begin{pmatrix}-1 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -1\end{pmatrix}\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix} \tag{3.2.8}$$

From equation (3.2.8), It can be seen that the error system (3.2.8) is a linear system of the form, $\dot{e}=De$. Thus by linear control theory, the system matrix D_2 is Hurwitz [18] and so all the eigenvalues of the system matrix D_2 are negative (-1, -2, -1).

Now let us construct a Lyapunov Error Function Candidate as:

$$V(e)=\frac{1}{2}(e_1{}^2+e_2{}^2+e_3{}^2)$$

then

$$\dot{V}(e)=-(p+d_{11})e_1{}^2-2e_2{}^2-(a_4+d_{33})e_3{}^2=-e^T\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{pmatrix}e\prec 0$$

Where

$$Q=\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{pmatrix}$$

Hence the above system (3.2.8) is globally exponentially stable, which implies that the two nonidentical chaotic systems (3.1.2) and (3.1.2) are synchronized globally exponentially.

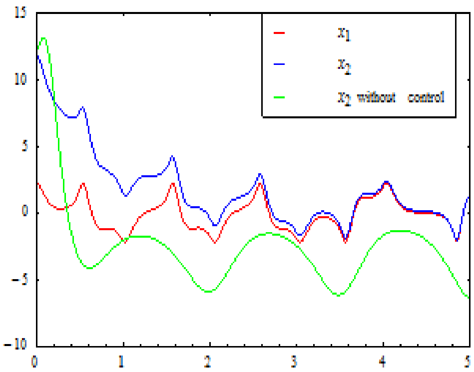

Figure 6: Time series of x_1 & x_2 (Nonidentical system [17] and Li system)

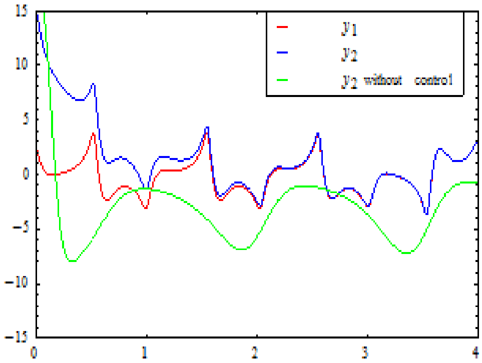

Figure 7: Time series of y_1 & y_2 (Nonidentical system [17] and Li system)

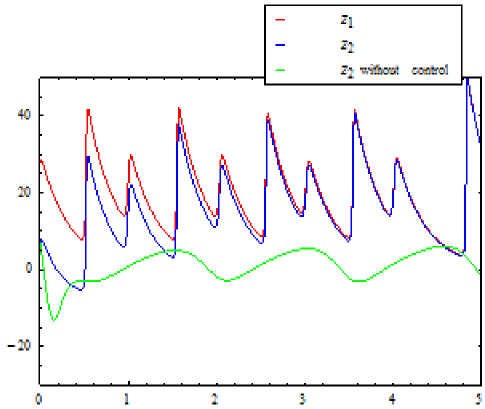

Figure 8: Time series of z_1 and z_2 (Nonidentical system [17] & Li system)

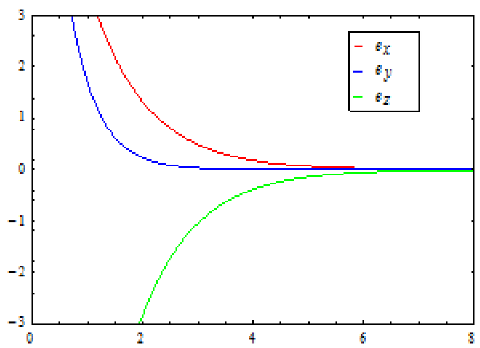

Figure 9: Time series of Errors (Nonidentical system [17] and Li system)

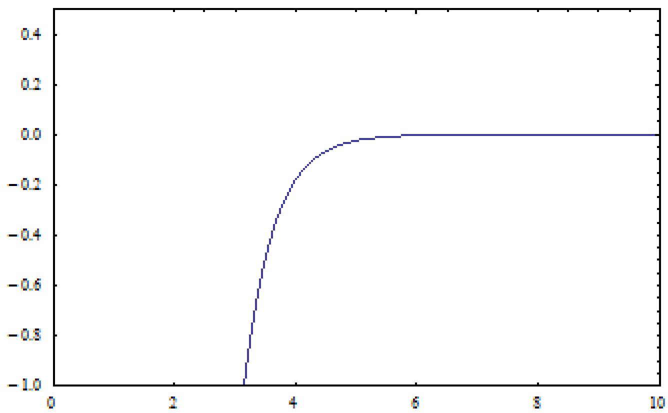

Figure 10: Derivative of Lyapunov Error Function (Nonidentical system [17] and Li system)

4. NUMERICAL SIMULATIONS

Numerical simulations are furnished to validate the advantages and potency of our proposed method. The parameters for new chaotic system [17] are taken as, $a_1 = 10, a_2 = 40, a_3 = 2$ and $a_4 = 3$ where the initial conditions are (2.2, 2.4, 28) and (4.4, 4.8, 46).

For the Li Chaotic System the parameters are selected as ; $p = 5, q = 16$ and $r = 1$ where the initial conditions are taken as; $x_1(0) = 12$, $y_1(0) = 15$, $z_1(0) = 7$, $x_2(0) = -5$, $y_2(0) = 0$, $z_2(0) = -10$.

5. CONCLUSION

In this paper, global chaos synchronization of an identical and nonidentical new 3-D chaotic system has been investigated. Based on Lyapunov Stability Theory and Ruth-Hurwitz Criterion and using the Active Control Algorithm, a class of active controllers are designed to achieve the global stability of the error dynamics. Since the Lyapunov exponents are not required for numerical calculations, Active Control Algorithm is an efficient technique to synchronize two identical as well as nonidentical chaotic systems.

In this study, using the Active Control Algorithm, it has been shown that the proposed schemes have excellent transient performances and that analytically as well as graphically the synchronization is globally exponentially stable. Results are also presented in graphical forms with time history (figures 1- 10).

Figures 4 and 9 show the synchronization errors of two identical chaotic systems [17] and nonidentical chaotic systems [17, 20] respectively. For the two different chaotic systems ([17] and Li system), that contain parameters mismatch and different structures , the controllers were used to synchronize the states of master and slave systems globally exponentially which shows that the proposed controllers are efficient with enough transient speed.

Figures 5 and 10 show the derivative of Lyapunov Error Functions of identical and nonidentical chaotic systems [17, 20] respectively. It has been shown that the error signals converge to the origin very smoothly with minimum rate of decay which shows that the error systems (figures 4 and 10) are feedback stabilized and the investigated controllers are more robust to accidental mismatch in the transmitter and receiver.

Further research on synchronization of this new system [17] can be beneficial to many fields such as, Information sciences, Communication, Electricity and Medical sciences etc.

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