A Componential Reduction to the Euler-Lagrange Equation Using Energy Structure Theory

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ABSTRACT---- ***The Euler-Lagrange equation can be used for a variety of the thermal processes from microscopic and macroscopic points of view [1-3]. In this case, the main challenge is calculating the potential energy using the Lagrangian density. Since the energy structure equation has the effects of the second law of thermodynamics as its base, in this paper, this equation is used as potential energy for the Euler-Lagrange equation. Since the energy structure equation has been presented based on the energy components as well as independent and dependent energy components concepts, therefore, a componential reduction to the Euler-Lagrange equation will be extracted. The resultant equation will be satisfied for all independent components activated in the performed process. Also, the resultant equation can be used to investigate different paths whenever the same amount of energy is applied to the system in different conditions. Also, a quasi-static path is used as a reference path.***

Keywords: *Euler-Lagrange equation; potential energy; energy structure equation; energy components; independent components; quasi-static path*

1. INTRODUCTION

The Euler-Lagrange equation can be developed and extracts for a variety of the problems involved thermal effects, from microscopic and macroscopic points of view [1-3]. Ref [1]. Presents a mathematically analysis to non-local gradient operators, and established the relevant effects between the non-local and classical gradient operators. A great investigation was done to the fundamental of the Euler-Lagrange equation in [2-3]. This study provides a variety domains to develop the Euler-Lagrange equation for microscopic applications. Also, the relation between the bond and state-based for relevant heat conduction models were extracted in [2-3].

From the perspective of the heat conduction theory of solid mechanics, the state-based dynamics have been known as a theory that has good numerical features [4]. The applications of the bond-based dynamic theory is limited by the physical properties of materials and required models model. Also, the state-based dynamic theory has the classical continuum theory as its base [5].

Energy structure theory and some of its scientific applications have been developed in 2020-2021 [6-12]. The first and second laws of thermodynamics are the basis of the energy structure theory and its relevant equations [6]. This theory introduces and develops some new physical quantities including activated and non-activated energy components, energy structure equation, reversible and irreversible components, irreversibility structure, irreversibility components, etc.

The energy structure equation is known as one of the most important equations of the energy structure theory. This equation has been extracted based on the activated energy components as well as independent and dependent components of energy [7]. Due to this, energy structure theory can be used to analyze a wide scope of thermal and physical applications including irreversibility, viscoelasticity problems, probability concept in general physics, etc. [6-8,12].

In this paper, the energy structure equation is used as potential energy in the Euler-Lagrange equation. By this, a componential reduction to the Euler-Lagrange equation will be extracted, and also the resultant equation is investigated in different paths using a quasi-static path as a reference path.

2. EULER-LAGRANGE EQUATION

Using the Lagrangian formalism, the Euler-Lagrange equation is as follows:

$$\frac{d}{dt}\left(\frac{\partial £}{\partial \dot{T}}\right) - \frac{\partial £}{\partial T} = 0 \quad (1)$$

Where:

$$£ = \int_V LdV \quad (2)$$

That L is the Lagrangian density and T is temperature.

Equation (1) must be satisfied in thermal problems. This equation takes Lagrangian density as a function of temperature. The Lagrangian density of a material point can be calculated as a function of the location vector of the material point as well as relevant quantities [2,3,4].

3. ENERGY STRUCTURE EQUATION

When a physical process is performed, in fact, some of its energy components are activated [8]. In a particular process, activated energy components can be independent or dependent on the condition of the energy applying.

Independent components are those that will be activated in all feasible different conditions of energy applying. When the performed process is quasi-static, only independent components will be activated. Therefore, this path is used as a reference path [8].

Equation (3) is the energy structure equation that is presented by energy structure theory [6,8,11]:

$$U_T = (u_1 + u_2 + \cdots + u_m) + [g_1 + \cdots + g_k] + [h_1 + \cdots + h_n] + U_{T_0} \quad (3)$$

Where:

$$g_j = g_j(u_1, u_2, \ldots, u_m) \quad (4)$$

$$h_p = h_p(\dot{u}_1, \ldots, \dot{u}_m) \quad (5)$$

Where u_i is the independent energy component that has been activated in the performed process. Also, g_j and h_p are dependent components, and the term U_{T_0} is non-changed energy in the performed process.

4. COMPONENTIAL REDUCTION TO THE EULER-LAGRANGE EQUATION

The energy structure equation also can be used as a potential equation [6,9,12]. In this part, using the energy structure equation as potential energy, the Euler-Lagrange equation is investigated for which the same amount of the energy is applied to the system in different conditions. Also, a quasi-static path is used as a reference path.

Therefore, two paths are considered, as shown in figure 1:

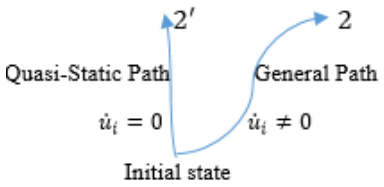

Figure 1. Different feasible paths for applying the same amount of energy

Quasi-static path $1 - 2'$: $£ = u_1 + u_2 + \cdots + u_m$ (6)

General path $1 - 2$: $£ = (u_1 + u_2 + \cdots + u_m) + [g_1 + \cdots + g_k] + [h_1 + \cdots + h_n]$ (7)

Therefore, for path $1 - 2'$, equation (1) can be reduced as follows:

$$\frac{d}{dt}\left(\frac{\partial(\sum_m u_i)}{\partial \dot{T}}\right) - \frac{\partial(\sum_m u_i)}{\partial T} = 0 \quad (8)$$

Or:

$$\sum_m \frac{d}{dt}\left(\frac{\partial u_i}{\partial \dot{T}}\right) - \sum_m \frac{\partial u_i}{\partial T} = 0 \quad (9)$$

Equation (9) is a componential equation, and in fact, must be satisfied for all independent components activated in path $1-2'$.

Because of the basis of the kinematic theory of dissipated energy, in thermal problems, function g_j does not be activated [8,13]. Therefore, in path $1-2$, equation (1) can be reduced as follows:

$$\frac{d}{dt}\left(\frac{\partial(\sum_m u_i+\sum_n h_p)}{\partial \dot{T}}\right) - \frac{\partial(\sum_m u_i+\sum_n h_p)}{\partial T} = 0 \quad (10)$$

Or:

$$\sum_m \frac{d}{dt}\left(\frac{\partial u_i}{\partial \dot{T}}\right) - \sum_m \frac{\partial u_i}{\partial T} = \sum_n \frac{d}{dt}\left(\frac{\partial h_p}{\partial \dot{T}}\right) - \sum_n \frac{\partial h_p}{\partial T} \quad (11)$$

Equation (11) is a componential equation, and in fact, must be satisfied for all independent components activated in path $1-2$.

Term $\sum_n \frac{d}{dt}\left(\frac{\partial h_p}{\partial \dot{T}}\right) - \sum_n \frac{\partial h_p}{\partial T}$ is the difference between equations (9) and (11). In fact, this term takes the effects of the condition of energy applying on the response of the considered problem. Using the energy structure theory, it can be concluded that terms $(\frac{\partial h_p}{\partial \dot{T}})$ and $(\frac{\partial h_p}{\partial T})$ have a positive sign [7-8], but the sign of term $(\sum_n \frac{d}{dt}\left(\frac{\partial h_p}{\partial \dot{T}}\right) - \sum_n \frac{\partial h_p}{\partial T})$ also is depending on the materials property.

5. CONCLUSIONS

The Euler-Lagrange equation can be used for a wide scope of the thermal problems. In this case, calculating the Lagrangian density is the main challenge. In fact, this density depends on the size as well as material properties in the considered problems.

The energy structure equation has the second law of thermodynamics as its base. In fact, this equation directly can consider the condition of energy applying to the system. The energy structure equation is extracted based on the energy component of the system and therefore, can be known as a componential equation. By studying this equation in different paths, the effects of the second law on the final state for the same amount of energy applying can be investigated. The energy structure equation also can be used as potential energy for relevant applications.

Using the energy structure equation as potential energy for the Euler-Lagrange equation, a componential reduction to the Euler-Lagrange equation is extracted. Equations (9) and (11) take reduced equations for quasi-static path and general path. By the assumption that the same amount of energy is applied to the system in different paths as well as considering quasi-static path as a reference path, term $\sum_n \frac{d}{dt}\left(\frac{\partial h_p}{\partial \dot{T}}\right) - \sum_n \frac{\partial h_p}{\partial T}$ takes the effects on the conditions of the energy applying on the response of the considered problem.

Since Equations (9) and (11) are componential reduction to the Euler-Lagrange equation, therefore, these two equations also must be satisfied for all independent energy components that have been activated in the performed process.

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