Diophantine Equation $f(n)=n$

Saowaros Srisuk

Department of Mathematics, Faculty of Science, Burapha University
Chonburi, Thailand

Email: saowaros.srisuk [AT] gmail.com

ABSTRACT— ***The purpose of this paper is to define the function*** $f(n)$ ***and find a positive integer*** n ***with digits more than*** 1 ***such that*** $f(n)=n$**.**

Keywords— Diophantine Equation.

---

1. INTRODUCTION

In 2015, Brian Miceli proposed the problem by giving positive integer n with three digits, define $f(n)$ to be the sum of three digits of n, their three products in pairs and the product all three digits.. Find all positive integers n such that $f(n)=n$.

In 2016, Sanitta M., Hari K., and Megha R. have extended the above problem by considering the Diophantine equation $f(n)=n$ where n is a positive integer with digits more than 1 and the results have been shown for the number of digits of n as 2, 3, 4, 5 and 6.

In this paper, we define a different function $f(n)$ and find positive integer n with digits more than 1 such that $f(n)=n$.

2. MAIN RESULTS

2.1 The first form

For positive integer n with two digits. Let the tenth digits be a and unit digit be b of n. Thus

$$n = ab = 10a + b$$

and define

$$f(n) = 2a + b + ab$$

so,

$$2a + b + ab = 10a + b$$

or

$$a(b-8) = 0$$

Hence, $a=0$ or $b=8$. But $a=0$ is not possible because in this case n has only one digit. Hence, $b=8$ and the tenth digit a can be any value from 1 to 9. Therefore, the value of n are 8, 28, 38, 48, 58, 68, 78, 88 and 98.

For positive integer n with three digits. Let the hundredth digit be a, the tenth digit be b and unit digit be c of n. Thus

$$n = abc = 100a + 10b + c$$

and define

$$f(n) = 4a + 2b + c + 2ab + 2ac + bc + abc$$

so,

$$4a + 2b + c + 2ab + 2ac + bc + abc = 100a + 10b + c$$

or

$$(2 + c)(2a + b + ab) = 100a + 10b$$

Let $c = 8$, we get

$$2a + b + ab = 10a + b$$

Hence, $a = 0$ or $b = 8$. But $a = 0$ is not possible because in this case n has only two digits. Hence, $b = 8$ and the hundredth digit a can be any value from 1 to 9. Therefore, the value of n are 188, 288, 388, 488, 588, 688, 788, 888 and 988.

For positive integer n with four digits. Let the thousandth digit be a, the hundredth digit be b, the tenth digit be c and unit digit be d of n. Thus

$$n = abcd = 1000a + 100b + 10c + d$$

and define

$$f(n) = 8a + 4b + 2c + d + 4ab + 4ac + 4ad + 2bc + 2bd + cd + 2abc + 2abd + 2acd + bcd + abcd$$

so,

$$8a + 4b + 2c + d + 4ab + 4ac + 4ad + 2bc + 2bd + cd + 2abc + 2abd + 2acd + bcd + abcd = 1000a + 100b + 10c + d$$

or

$$(2 + d)(4a + 2b + c + 2ab + 2ac + bc + abc) = 1000a + 100b + 10c.$$

Let $d = 8$, we get

$$4a + 2b + c + 2ab + 2ac + bc + abc = 100a + 10b + c$$

or

$$(2 + c)(2a + b + ab) = 100a + 10b$$

Let $c = 8$, we get

$$2a + b + ab = 10a + b$$

Hence, $a = 0$ or $b = 8$. But $a = 0$ is not possible because in this case n has only three digits. Hence, $b = 8$ and the thousandth digit a can be any value from 1 to 9. Therefore, the value of n are 1888, 2888, 3888, 4888, 5888, 6888, 7888, 8888 and 9888.

2.2 The second form

For positive integer n with two digits. Let the tenth digits be a and unit digit be b of n. Thus

$$n = ab = 10a + b$$

and define

$$f(n) = \frac{m^2}{m}a + \frac{m^2}{m^2}b + \frac{m^2}{m \cdot m}ab = ma + b + ab$$

so,

$$ma + b + ab = 10a + b$$

or

$$a(b-10+m)=0$$

Hence, $a=0$ or $b=10-m$. But $a=0$ is not possible because in this case n has only one digit. Hence, $b=10-m$ and the tenth digit a can be any value from 1 to 9. Therefore, the value of n are $1b, 2b, 3b, 4b, 5b, 6b, 7b, 8b$ and $9b$.

For positive integer n with three digits. Let the hundredth digit be a, the tenth digit be b and unit digit be c of n. Thus

$$n=abc=100a+10b+c$$

and define

$$f(n)=\frac{m^3}{m}a+\frac{m^3}{m^2}b+\frac{m^3}{m^3}c+\frac{m^3}{m\cdot m}ab+\frac{m^3}{m\cdot m}ac+\frac{m^3}{m^2\cdot m}bc+\frac{m^3}{m\cdot m\cdot m}abc$$
$$=m^2a+mb+c+mab+mac+bc+abc$$

so,

$$m^2a+mb+c+mab+mac+bc+abc=100a+10b+c$$

or

$$(m+c)(ma+b+ab)=100a+10b$$

Let $c=10-m$, we get

$$ma+b+ab=10a+b$$

Hence, $a=0$ or $b=10-m$. But $a=0$ is not possible because in this case n has only two digits. Hence, $c=b=10-m$ and the hundredth digit a can be any value from 1 to 9. Therefore, the value of n are $1bc, 2bc, 3bc, 4bc, 5bc, 6bc, 7bc, 8bc$ and $9bc$.

For positive integer n with four digits. Let the thousandth digit be a, the hundredth digit be b, the tenth digit be c and unit digit be d of n. Thus

$$n=abcd=1000a+100b+10c+d$$

and define

$$f(n)=\frac{m^4}{m}a+\frac{m^4}{m^2}b+\frac{m^4}{m^3}c+\frac{m^4}{m^4}d+\frac{m^4}{m\cdot m}ab+\frac{m^4}{m\cdot m}ac+\frac{m^4}{m\cdot m}ad$$
$$+\frac{m^4}{m^2\cdot m}bc+\frac{m^4}{m^2\cdot m}bd+\frac{m^4}{m^3\cdot m}cd+\frac{m^4}{m\cdot m\cdot m}abc+\frac{m^4}{m\cdot m\cdot m}abd$$
$$+\frac{m^4}{m\cdot m\cdot m}acd+\frac{m^4}{m^2\cdot m\cdot m}bcd+\frac{m^4}{m\cdot m\cdot m\cdot m}abcd$$
$$=m^3a+m^2b+mc+d+m^2ab+m^2ac+m^2ad+mbc+mbd+cd$$
$$+mabc+mabd+macd+bcd+abcd$$

so,

$$m^3a+m^2b+mc+d+m^2ab+m^2ac+m^2ad+mbc+mbd+cd+mabc+mabd+macd+bcd+abcd=1000a+100b+10c+d$$

or

$$(m+d)(m^2a+mb+c+mab+mac+bc+abc)=1000a+100b+10c.$$

Let $d = 10 - m$, we get

$$m^2 a + mb + c + mab + mac + bc + abc = 100a + 10b + c$$

Or

$$(m + c)(ma + b + ab) = 100a + 10b.$$

Let $c = 10 - m$, we get

$$ma + b + ab = 10a + b.$$

Hence, $a = 0$ or $b = 10 - m$. But $a = 0$ is not possible because in this case n has only three digits. Hence, $d = c = b = 10 - m$ and the thousandth digit a can be any value from 1 to 9. Therefore, the value of n are $1bcd$, $2bcd$, $3bcd$, $4bcd$, $5bcd$, $6bcd$, $7bcd$, $8bcd$ and $9bcd$.

Remark: If $m = 2$ then the result is the same as the first form.

2.3 The third form

For positive integer n with two digits. Let the tenth digits be a and unit digit be b of n. Thus

$$n = ab = 10a + b$$

and define

$$f(n) = \frac{2!}{1!}a + \frac{2!}{2!}b + \frac{2!}{1! \cdot 2}ab = 2a + b + ab$$

so,

$$2a + b + ab = 10a + b$$

Hence, $a = 0$ or $b = 8$. But $a = 0$ is not possible because in this case n has only one digit. Hence, $b = 8$ and the tenth digit a can be any value from 1 to 9. Therefore, the value of n are 18, 28, 38, 48, 58, 68, 78, 88 and 98.

For positive integer n with three digits. Let the hundredth digit be a, the tenth digit be b and unit digit be c of n. Thus

$$n = abc = 100a + 10b + c$$

and define

$$f(n) = \frac{3!}{1!}a + \frac{3!}{2!}b + \frac{3!}{3!}c + \frac{3!}{1! \cdot 2}ab + \frac{3!}{1! \cdot 3}ac + \frac{3!}{2! \cdot 3}bc + \frac{3!}{1! \cdot 2 \cdot 3}abc$$

$$= 6a + 3b + c + 3ab + 2ac + bc + abc$$

so,

$$6a + 3b + c + 3ab + 2ac + bc + abc = 100a + 10b + c$$

or

$$(3 + c)(2a + b + ab) = 100a + 10b$$

Let $c = 7$, we get

$$2a + b + ab = 10a + b$$

Hence, $a = 0$ or $b = 8$. But $a = 0$ is not possible because in this case n has only two digits. Hence, $b = 8$ and the hundredth digit a can be any value from 1 to 9. Therefore, the value of n are 187, 287, 387, 487, 587, 687, 787, 887 and 987.

For positive integer n with four digits. Let the thousandth digit be a, the hundredth digit be b, the tenth digit be c and unit digit be d of n. Thus

$$n = abcd = 1000a + 100b + 10c + d$$

and define

$$\begin{aligned} f(n) &= \frac{4!}{1!}a + \frac{4!}{2!}b + \frac{4!}{3!}c + \frac{4!}{4!}d + \frac{4!}{1!\cdot 2}ab + \frac{4!}{1!\cdot 3}ac + \frac{4!}{1!\cdot 4}ad \\ &\quad + \frac{4!}{2!\cdot 3}bc + \frac{4!}{2!\cdot 4}bd + \frac{4!}{3!\cdot 4}cd + \frac{4!}{1!\cdot 2\cdot 3}abc + \frac{4!}{1!\cdot 2\cdot 4}abd \\ &\quad + \frac{4!}{1!\cdot 3\cdot 4}acd + \frac{4!}{2!\cdot 3\cdot 4}bcd + \frac{4!}{1!\cdot 2\cdot 3\cdot 4}abcd \\ &= 24a + 12b + 4c + d + 12ab + 8ac + 6ad + 4bc + 3bd + cd + 4abc + 3abd + 2acd + bcd + abcd \end{aligned}$$

so,

$$24a + 12b + 4c + d + 12ab + 8ac + 6ad + 4bc + 3bd + cd + 4abc + 3abd + 2acd + bcd + abcd = 1000a + 100b + 10c + d$$

or

$$(4+d)(6a + 3b + c + 3ab + 2ac + bc + abc) = 1000a + 100b + 10c.$$

Let $d = 6$, we get

$$6a + 3b + c + 3ab + 2ac + bc + abc = 100a + 10b + c$$

Or

$$(3+c)(2a + b + ab) = 100a + 10b.$$

Let $c = 7$, we get

$$2a + b + ab = 10a + b.$$

Hence, $a = 0$ or $b = 8$. But $a = 0$ is not possible because in this case n has only three digits. Hence, $b = 8$ and the thousandth digit a can be any value from 1 to 9. Therefore, the value of n are 1876, 2876, 3876, 4876, 5876, 6876, 7876, 8876 and 9876.

3. CONCLUSION

The results of n for the Diophantine equation $f(n) = n$ have been shown for $n = 2, 3$ and 4 digits by defining difference $f(n)$.

4. REFERENCES

[1] Miceli, B., Problem of week No. 2, 2015
[2] Sarita, M., Hari, K., and Megha, R., "On the Diophantine Equation $f(n) = n$." European Journal of Mathematics and Computer Science, vol. 3, no. 1, pp. 49-52, 2016.