

# On the Irreversibility in Mechanical Systems Using a New Macroscopic Energy Structure Modeling

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**ABSTRACT**---- This paper presents a macroscopic applied innovate modeling to study the performance effects of the second law of thermodynamics on the mechanical systems. To investigate the irreversibility in mechanical systems, the energy structure of the system can be studied. Some energy components relate to the reversible processes and remaining relate to the irreversible process. Existing models are based on the studying sub structures and therefore, need a large volume of the calculations. In this paper, at first, using a macroscopic quasi-statistical approach, a new energy structure equation is extracted and by examining it's variation in the different paths, the irreversible components as well as their structures are studied. Using the kinematic theories of dissipated energy, it can be concluded that the extracted equations have the same base as the different formulations of the second law of thermodynamics. Finally, as a mechanical system example with the possibility of irreversibility in the possible performed processes, the extracted equations are developed for viscoelasticity problems. And also the matching of the results with expected results is shown.

**Keywords**---- Mechanical Systems; Energy Structure equation; Irreversibility; work bound; Viscoelasticity

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## 1. INTRODUCTION

Irreversibility in one of the most important concepts from the perspective of all branches of science that has been extracted in the different formulations of the second law of thermodynamics [1,2]. Entropy is the most important quantity used to formulating of the second law and base on this law, entropy will be generated in all performed processes [2].

Equations (1) and (2) are two main formulations of the second law:

$$\oint \frac{\delta Q}{T} \leq 0 \quad (1)$$

$$ds_{gen} \geq 0 \quad (2)$$

That  $\delta Q$  is the heat exchanged at temperature  $T$  and  $s_{gen}$  is the generated entropy.

In fact, second law states that always some of the useful energy exchanges to the thermal energy.

Based on the second law of thermodynamics, when some energy is applied to the system, the work done by system will has upper bound. Carnot has studied the upper bound in thermodynamic cycles [1]. Carnot's equation depends on the hot and cold temperature in considered Carnot's cycle only [2].

Also, investigating the effects of the second law on the performance of the mechanical systems may be very important in their design. Due to the second law, it can be concluded that mechanical systems (or dynamically loaded bodies) have upper bound to the work done by them [3]. The work bound for these systems must be studied using the relevant deformation theories and models. Also, exiting as well as value of the upper work bound in systems for multichannel queues has been investigated [4].

Also, from the other point of view work bound in mechanical systems can be studied. One of these approach is investigation from the perspective of the free energy of the system and considered performed process [5]. Free energy is a part of the energy that can be used in the performed processes. The upper and lower bonds for free energy change can be using the second law formulation.

Discussion on the using of the second law to determine the work bound can be expanded by other examples. For example, work bounds cyclically loaded creeping structures can be presented [6]. In this case as well as similar cases, the sub-

structures of the system are investigated [7-10]. Studying at the molecular level need using the molecular structure modeling as well as deformation theories.

One of the most important mechanical properties of materials that can be effective on the loss of the useful energy, is viscoelasticity. At non-zero energy applying rate, these materials, always experience irreversible process because of the dissipated energy [11]. Therefore, due to the second law, these mechanical systems have the work bound. Dissipation energy process can have different bases from the perspective of the sub-structures [12-15]. In fact, their molecular structures have the main role in the relevant analysis.

The dissipation energy can be studied using studying the dynamical on the sub-structures [16-19]. Some of the most important reasons of the irreversibility in physical systems are: friction of solids; imperfect fluidity of fluids; imperfect elasticity of solids; inequalities of temperature and consequent conduction of heat produced by stresses in solids and fluids; imperfect magnetic retentiveness; residual electric polarization of dielectrics; generation of heat by electric currents induced by motion; diffusion of fluids, solution of solids in fluids and other chemical changes and absorption of radiant heat and light.

In this paper, at first, using a new macroscopic quasi-statistical approach to the physical processes, the effects of the second law on the energy structure are studied. Then, based on the macroscopic energy components of the system as well as using the independent and dependent components concept, an energy structure is extracted. Also, using the examining of the variation of the energy structure equation in different paths, the irreversibility is studied and also, based on the energy components of the system is modeled. Finally, as a mechanical system example, viscoelasticity problems are investigated and extracted equations are developed for these problems.

## 2. MACROSCOPIC QUASI-STATISTICAL MODELING TO THE MECHANICAL PROCESSES

If the same amount of energy is applied to the system in different conditions, different energy components may be activated [16-18]. When energy is applied to the system in a quasi-static path (Which is a path that the rate of applying energy can be considered zero to it), the performed process is known as a reversible process [19], and also, the activated components can be known as reversible components. These components (that can be named as independent components), also will be activated in other conditions of energy applying to the system (from the point of view of the energy applying rate). Finally, among activated components, some of them will be dependent on the independent components.

In general (non-zero applying energy rate), some secondary components may be activated that are dependent on the applying energy rate (or rate of the independent components) [16]. Therefore, it can be concluded that equation (3) can be considered as the energy structure equation of the system:

$$U_T = (u_1 + u_2 + \dots + u_m) + [g_1 + \dots + g_k] + [h_1 + \dots + h_n] + U_{T_0} \quad (3)$$

Where:

$$g_j = g_j(u_1, u_2, \dots, u_m) \quad (4)$$

$$h_p = h_p(\dot{u}_1, \dots, \dot{u}_m) \quad (5)$$

The functions  $g_j$  represent the value-dependent components of the independent components, while the functions  $h_p$  show the components which are dependent on the rate of the independent components. Also  $U_{T_0}$  represents the sum of all the components that did not participate in the performed process (in fact, this term can be not considered in equation (3), because it does not change in performed process).

Equation (3) has the same base as the second law of thermodynamic. Based on the different formulations of the second law, always some of the available work will be converted to thermal energy. As we know that the thermal energy is dependent on the kinematic of sub-structures, while other forms of energy depending on the place and displacement of the sub-structures [20].

## 3. IRREVERSIBILITY AND ITS STRUCTURE

To investigate the irreversibility, it is assumed that the same amount of energy is applied to the system in different conditions. The quasi-static (zero rates of energy applying) path is considered as a reference path and the variation between this path and general paths (non-zero rate of energy applying) is calculated.

Therefore, using the energy structure equation when the same energy is applied to the system, equation (6) can be written:

$$\sum_{i=1}^m \left[ \left( 1 + \sum_{j=1}^k \left( \frac{\partial g_j}{\partial u_i} \right) \right) (\delta u_i - \delta u'_i) \right] = - \sum_{i=1}^m \left[ \left( \sum_{p=1}^n \left( \frac{\partial h_p}{\partial \dot{u}_i} \right) \right) \delta \dot{u}_i \right] \quad (6)$$

Where  $\delta u_i$  is the variation of the  $u_i$  when  $\dot{u}_i \neq 0$  as the value of energy  $\delta U_T$  is given to the system and  $\delta u'_i$  is the variation of the  $u_i$  when  $\dot{u}_i \cong 0$  as the value of energy  $\delta U_T$  is given to the system.

Equation (6) also can be rewritten as follows:

$$\left( 1 + \sum_{j=1}^k \left( \frac{\partial g_j}{\partial u_i} \right) \right) (\delta u_i - \delta u'_i) = - \left( \sum_{p=1}^n \left( \frac{\partial h_p}{\partial \dot{u}_i} \right) \right) \delta \dot{u}_i \quad (7)$$

Therefore, the variation between general and quasi-static paths is equal to  $[- \left( \sum_{p=1}^n \left( \frac{\partial h_p}{\partial \dot{u}_i} \right) \right) \delta \dot{u}_i]$ , as shown in figure 1:

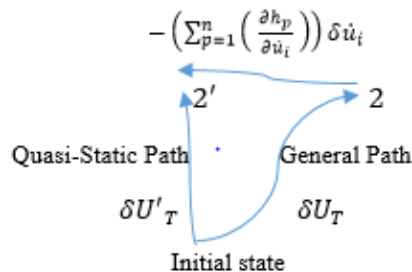


Figure 1. The variation between general and quasi-static paths

Therefore, based on the second law of thermodynamic, the term  $[- \left( \sum_{p=1}^n \left( \frac{\partial h_p}{\partial \dot{u}_i} \right) \right) \delta \dot{u}_i]$  must have the same sign with the applied energy  $\delta U_T$ . So it can be a criterion to investigate the irreversibility in the performed process. And also, by definition  $\varphi$  as follows:

$$\varphi = \frac{\left( \sum_{p=1}^n \left( \frac{\partial h_p}{\partial \dot{u}_i} \right) \right) \delta \dot{u}_i}{\delta U_T} \quad (8)$$

It will be concluded that always  $\varphi \geq 0$  (for all possible performed processes).

As expected, this quantity ( $\varphi \geq 0$ ), for a particular mechanical system depends on the amount of applied energy to the system and its applying conditions. Also, from the kinetic theory of dissipated energy, it can be concluded that relation (8) has the same base as the second law of thermodynamics [20].

#### 4. LINEAR VISCOELASTICITY

In this part, the presented equations are developed to the linear viscoelasticity problems, as a configurationally mechanical system example with the possibility of irreversibility in the performed process. The energy structure for generalized Kelvin and Maxwell models, as two main basic viscoelasticity models, is presented, and also matching the results with expected results is shown.

##### 4.1 Generalized kelvin element

Figure 2 shows the generalized Kelvin elements viscoelasticity model [21]:

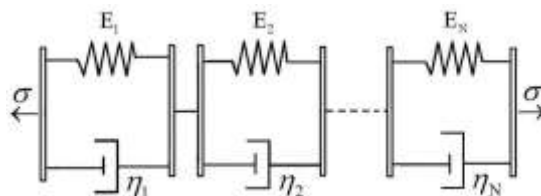


Figure 2. Generalized kelvin element

For this model, the energy structure can be rewritten as follows:

$$U_T = u_{e1} + u_{e2} + \dots + u_{eN} + c_{e1} \dot{u}_{e1} + c_{e2} \dot{u}_{e2} + \dots + c_{eN} \dot{u}_{eN} + constant \quad (9)$$

Where  $u_{ei}$  is the potential energy of  $E_i$ ,  $\dot{u}_{ei}$  is the rate of  $u_{ei}$  and the coefficient  $c_{ei}$  can be calculated as  $\frac{\eta_i}{E_i}$ .

To verify the equation (9), the following argument can be made:

For the Kelvin model, the potential and dissipated energy can be calculated from  $\frac{\partial u_{ei}}{\partial \varepsilon_i} = E_i \varepsilon_i$  and  $\frac{\partial u_{di}}{\partial \varepsilon_i} = \eta_i \dot{\varepsilon}_i$ , that  $\varepsilon_i$  is the strain. Therefore it can be concluded that  $u_{di} = \frac{\eta_i}{E_i} \dot{u}_{ei} + f(t)$ , where  $f(t)$  is a function of time-related to the initial condition.

#### 4.2 Generalized Maxwell element

Figure 3 shows the generalized Kelvin elements viscoelasticity model [12]:

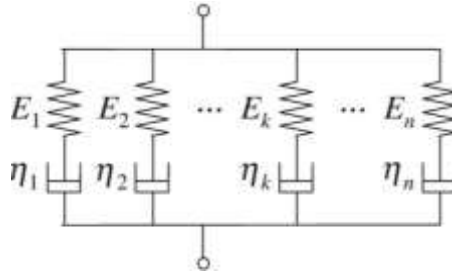


Figure 5. Generalized Maxwell element

For this model, the energy structure can be rewritten as follows:

$$U_T = u_{d1} + u_{d2} + \dots + u_{dN} + c_{d1} \dot{u}_{d1} + c_{d2} \dot{u}_{d2} + \dots + c_{dN} \dot{u}_{dN} + constant \quad (10)$$

Where  $u_{di}$  is the dissipated energy of  $\eta_i$ ,  $\dot{u}_{di}$  is the rate of  $u_{di}$  and the coefficient  $c_{di}$  can be calculated as  $\frac{\eta_i}{2E_i}$ .

To verify the equation (10), the following argument can be made:

For the Maxwell model, the potential and dissipated energy can be calculated from  $\frac{\partial u_{di}}{\partial \varepsilon_i^{OV}} = \eta_i \varepsilon_i^{OV} = E_i \varepsilon_i^e = \frac{\partial u_e}{\partial \varepsilon_i^e}$ , that  $\varepsilon_i^{OV}$  is the strain of  $\eta_i$  and  $\varepsilon_i^e$  is the strain of  $E_i$ . Therefore it can be concluded that  $u_{ei} = \frac{\eta_i}{2E_i} \dot{u}_{di}$ .

### 5. CONCLUSIONS

To investigate the performance effects of the second law of thermodynamics on the mechanical systems, innovatively their macroscopic energy components can be studied from the perspective of the energy structure. Equation (3) is considered as the energy structure equation to the system. When a particular process is performed, an energy structure equation is formed. Also, based on the energy applying conditions, the independent and dependent components can be understood.

To investigate the irreversibility, it is assumed that the same amount of energy is applied to the system in different conditions, and the variation of the energy structure is studied. A quasi-static path is used as a reference path and the variation between this path with general paths is examined. Using this approach, equation (8) is extracted that introduces a positive quantity ( $\varphi \geq 0$ ) for performed processes. This quantity can be considered as a criterion for investigating the irreversibility during performed processes. As can be concluded, for a particular mechanical system, this quantity depends on the applying energy conditions to the system.

One of the most important results of this paper, due to the equation (3) and (8), is it that the irreversibility will be occurred because of the independent components activated in performed process. In fact, when the conditions of energy applying as independent components exist in the energy structure of the system, the performed process is an irreversible process.

As viscoelastic materials can be known as materials with dissipated energy and irreversibility effects, in part 4, using the presented approach, viscoelasticity problems were investigated. Also the energy structure to the generalized Kelvin and Maxwell models (equations (9) and (10)), as two main basic viscoelasticity models, were presented.

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