Asian Journal of Applied Sciences (ISSN: 2321 – 0893)
Volume 06 – Issue 06, December 2018

Energy Transitions-Flip-Rate of Half Spin Particle by Magnetic Impurity in One Dimension

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ABSTRACT---- ***In this article, we considered the dependence of the rate of energy transition on various parameters and how the energy-transition -rate changes as a function of radius R. We observed that with increasing radius R, the energy-flip-rate decreases, which is perfectly consistent with a system approaching ferromagnetic order. Also the energy-transition-rate for different choices of the amplitude of the impurities, for a purely static potential scatter, no energy transition occurs, but for magnetic impurities, we observed a high peak in the energy-transition-rate for one particular amplitude of order 5*** ϵ_F ***. Different profiles of energy-transition-rate (ETR) against frequencies and amplitude are drawn respectively, for angle*** $\varphi_0 = 0$, $\theta = \frac{\pi}{2}$, ***with*** $\mu_B B_0$***=0.5 and*** $x = R^2 = 100$ ***and different values of frequencies and amplitude.***

Keywords---- Transitions rate, induced current, Eigenenergies, half spin particle, magnetic field and frequency

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1. INTRODUCTION

Over the last decades, spintronics become most prominent areas of research and intensive investigation has been carried out [1–3]. Moreover, the notions of the spin of electrons turn out to be an important concept in the field of quantum computing and quantum information fields [4–6]. Several research groups have designed spin-sensitive devices such as spin-qubits, spin-polarizers, spin-filters, spin-splitters, and so on, both theoretically [7–9] and experimentally [10–14]. Muhlbauer et al observed the new magnetic order in MnSi at specific temperatures and magnetic fields [15]. The physics behind the concept of an electron moving through the magnetic field can be examined from two different perspectives, namely, considering the problem in terms of emergent electric and magnetic fields; the change in spin orientation is equal to an effective Lorentz force acting on the electron, which is perpendicular to its motion [16], resulting in a magnetic field to induce a deflection of the electron, that can be determined by the topological Hall-effect [17].

As a result of the electron carrying an electric charge, a potential may be measured perpendicular to the direction of the current. Since the magnetic structure of the skyrmion lattice is very smooth, the adjustment of the spin of the electron to the magnetization of the skyrmion lattice can be considered an adiabatic process.

On contrary to that, there must be corresponding counterforce acting on the skyrmion. This force, arising from the transfer of angular momentum from the conduction electrons to the local magnetic structure (cf. [18]), can lead to drift of the domains of the lattice. Accordingly, new strategies are needed to produce spin-polarized current. Magnetic impurities embedded in nanostructures can be exploited to control the spin current and spin polarization due to the interaction between electron spin and magnetic moment of the structure [19–22].

Recently, Baedorf et al investigated the model of an electron passing through a static magnetic field [23]. Sarah et al studied the Berry Phase Physics and spin-scattering in time-dependent magnetic field [24]. Hafeez et al reported the behavior of half spin electron when passing through a magnetic field and corresponding eigenenergies were obtained [25]. Wang et al. investigated the spin flop transition of orbital ordered $KCuF_3$ under magnetic fields [26]. Hafeez et al observed that magnetic impurities as scatter can be used during spin-flip scattering of half spin electron [27].

Herein, we introduced defects (magnetic impurity) in the system as a scatterer of a half sin electron in 1D. We observed that the energy-transition rate of an incoming particle wave decreases as radius increase and increases with the decrease of frequency.

2. CONSERVATION IN SCATTERING AND TRANSITION PROCESS

Consider the continuity equation given as:

$$\nabla \cdot j = -\partial_t \rho = 0 \tag{1}$$

Where j and $\rho = I\psi I^2$ are the current and the density function. To evaluate the exact value of the current j, we apply the time dependent Schrödinger equation for ψ along with its conjugates, which is given as follows

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} - \mu_B B_o \hat{n}\sigma\psi$$

By conjugating the above equation, we obtain

$$-i\hbar\partial_t\psi^* = -\frac{\hbar^2}{2m}\frac{\partial^2\psi^*}{\partial x^2} - \mu_B B_o \hat{n}\sigma\psi^*$$

So

$$i\hbar\partial_t(\psi^*\psi) = i\hbar\partial_t I\psi I^2 = -\frac{\hbar^2}{2m}(\psi^*\partial_x^2\psi - \psi\partial_x^2\psi^*)$$

Now equation (1) becomes

$$\nabla \cdot j = -\frac{\hbar}{2mi}(\psi^*\partial_x^2\psi - \psi\partial_x^2\psi^*)$$

Then the exact value of current becomes

$$j = -\frac{\hbar}{2mi}\left(\psi^*\psi' - \psi\psi^{*'}\right) = \frac{\hbar}{m}\Im(\psi^*\psi') \tag{2}$$

Numerically, we observed that the current for a specific wave function ψ (K, n) is same as amplitude multiplied by the derivative of the energy by k evaluated for the respective K, n.

$$j_{k,\sigma,\delta} = \frac{\hbar}{m}\Im\left(\psi^*{}_{\sigma,\delta}\psi'{}_{\sigma,\delta}\right) = \frac{\hbar}{m}\psi^*{}_{\sigma,\delta}\left[\frac{\partial E}{\partial K}\right]_{k,\sigma\delta}\psi_{\sigma\delta} \tag{3}$$

Where σ = +,- and δ = l, r

Considering the direction of motion of the waves, the law of conservation is now given as

$$j_{in} = j_{refl} + j_{transm} \tag{4}$$

3. ENERGY TRANSITIONS RATE (ETR)

The rate of energy transitions is defined as the ratio of current induced by wave functions with an energy $\epsilon_n = \epsilon_o + n\omega$ different from the incoming energy (i.e $n \neq 0$) to the current induced by the incoming wave function, which has the energy ϵ_o i.e

$$ETR = \frac{\sum_{n\neq 0} j_{in}}{j_{in}} \tag{5}$$

Where

$$\sum_{n=0} j_{in} = \sum_{n=0}\left\{-\left(\left|r^n_{+l}\right|\left[\frac{\partial E}{\partial K}\right]^n_{+l} + \left|r^n_{-l}\right|\left[\frac{\partial E}{\partial K}\right]^n_{-l}\right) + \left(\left|t^n_{+r}\right|\left[\frac{\partial E}{\partial K}\right]^n_{+r} + \left|t^n_{-r}\right|\left[\frac{\partial E}{\partial K}\right]^n_{-r}\right)\right\}$$

and the exact eigenenergies for an incoming wave function is given as [25]

$$\boldsymbol{E}_{\pm} = \frac{\hbar^2}{mR^2}\left(\frac{K^2+\frac{1}{4}}{2} \pm \sqrt{\frac{\left(K-\frac{\omega m R^2}{2\hbar}\right)^2}{4} - \alpha\left(K-\frac{\omega m R^2}{2\hbar}\right)^2\cos\theta + \alpha^2}\right) = const = \boldsymbol{\epsilon_o} \qquad (6)$$

There is maximal four real solutions for K (n, σ, δ) i.e $K^{n}_{+,l}$, $K^{n}_{+,r}$, $K^{n}_{-,l}$ and $K^{n}_{-,r}$ as reported in previous literature [27]. Fig.1 revealed the eigenenergies at different eigenvalue (K).

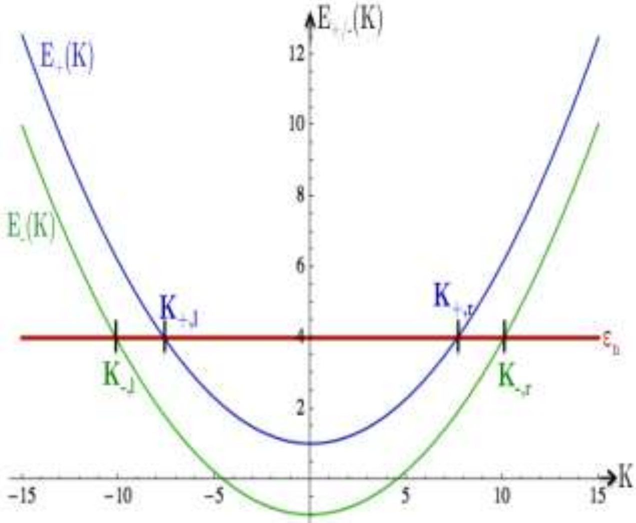

Fig. 1. Eigenenergies $E_{\pm}(K)$against the maximum eigenvalue K for α=10, ω=0.1, $\theta = \frac{\pi}{2}$ and $x = \frac{mR^2}{\hbar^2} = 10$

4. THE ENERGY-TRANSITIONS-RATE (ETR) AND NUMERICAL RESULTS OF THE ETR

Firstly, we are again concerned with how the energy-transition-rate varies as a function of the radius R. We observe that with increasing radius R, the energy-flip-rate decreases, which is perfectly consistent with our expectations of a system approaching ferromagnetic order. We also observe that for decreasing $|\omega|$ the rate of energy transitions is generally higher. This is consistent with the observations we made in the previously reported [27], namely that for small the spin-flip-rate increases remarkably. In the case of energy transitions, however, the quantitative significance of these differences is only marginal. Next, we consider the energy-transition-rate at different amplitudes of the impurities. No energy transitions occur for a purely static potential scatterer. Furthermore, a high peak of energy-transition-rate was observed at amplitude of order 5 ϵ_F (Fig.3). Also, spins are about to transition into other energy states, for the magnetic potential barrier which has a similar profile as that of non-magnetic potential. Likewise, at potentials greater than the Fermi energy (ϵ_F), the energy-transition-rate reduces drastically and almost constant at value ~ 20[ϵ_F], as observed in our previous report [27].

The dependence of the energy transition rate (ETR) on the frequency bears some re- semblance to the representation of the spin-flip-rate (SFR) as a function of frequency. For large magnetic energies, the energy transition rate is nearly constant, whereas for magnetic energies much smaller than ϵ_F , the energy transition rate has a maximum for small absolute values of. What strikes us is the high peak of the ETR for small magnetic fields, which is reminiscent of a resonance peak. Unlike the maximum of the spin-flip-rate, however, the maximum peak (and the corresponding 'resonance' frequency) is shifted to the right with respect to the origin. It corresponds to the frequency where before we observed peaks at the margins of the boxes in the STR-plot [27]. In addition, one observes that for large absolute values of the frequency, the energy-transition-rate is once again almost oblivious to possible changes inflicted by higher or lower frequencies.

The numerical results of the ETR versus different parameter were plotted using different set of U and ω as depicted in Fig. 2, 3, 4 and 5.

5. DISCUSSIONS

Fig. 2 displays the ETR as a function of the radius of the ring, R, for frequencies $\omega \to 0$ (blue curve) and $\omega = 0.05$ (green curve) and $\omega = 0.15$ (red curve). Surprisingly, the energy transition rate is generally higher for $\omega \to 0$. We also observe the expected tendency of a sinking energy transition rate for a system approaching ferromagnetic order. As seen in Fig. 3, the ETR as a function of ω with which the underlying magnetic field varies in time. For very small magnetic fields, the ETR is higher. And it observed that ETR is maximum at frequency of 0.15. And we choose $x = R^2 = 100$. As seen in Fig.4, the green curve indicates the existence of both static and magnetic impurities at non-zero amplitude while the blue one reveals the SFR for pure static and magnetic impurities at $\omega = 0.05$, .Fig.5, displays the graph of the average energy distance covered by a transition into another band. The average energy covered has a maximum value for $\omega \approx 0.07$.The average distance in energy covered by a transition into another band can be received by a simple ansatz. The energy difference of a transition from a state with energy ϵ_o into a band with energy E^n equals $\hbar\omega$. Lastly, in both plots (Fig. 2, 3, 4 and 5) we set $\phi_0 = 0, \theta = \frac{\pi}{4}, U_0 = 2, U_1 = 4,$

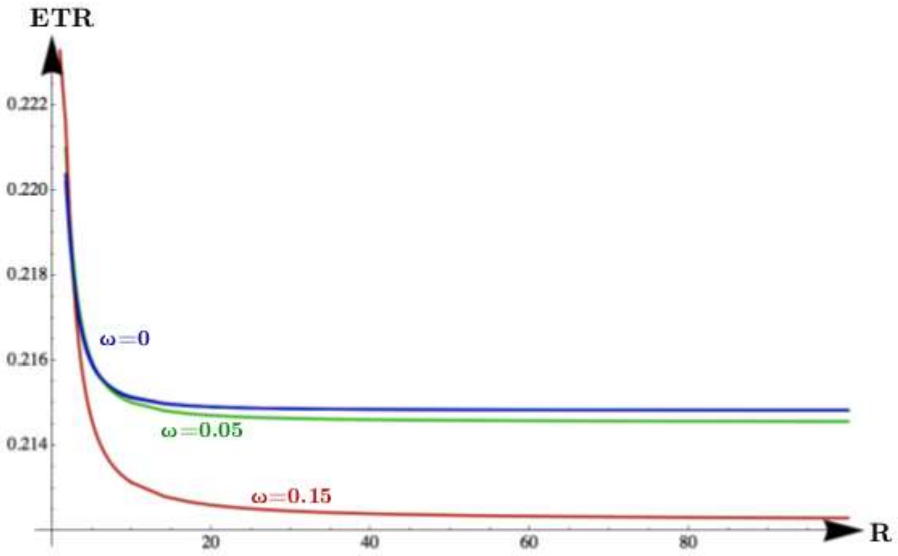

Fig. 2: Energy-transition-rate (ETR) against radius of the ring, R, at different frequencies

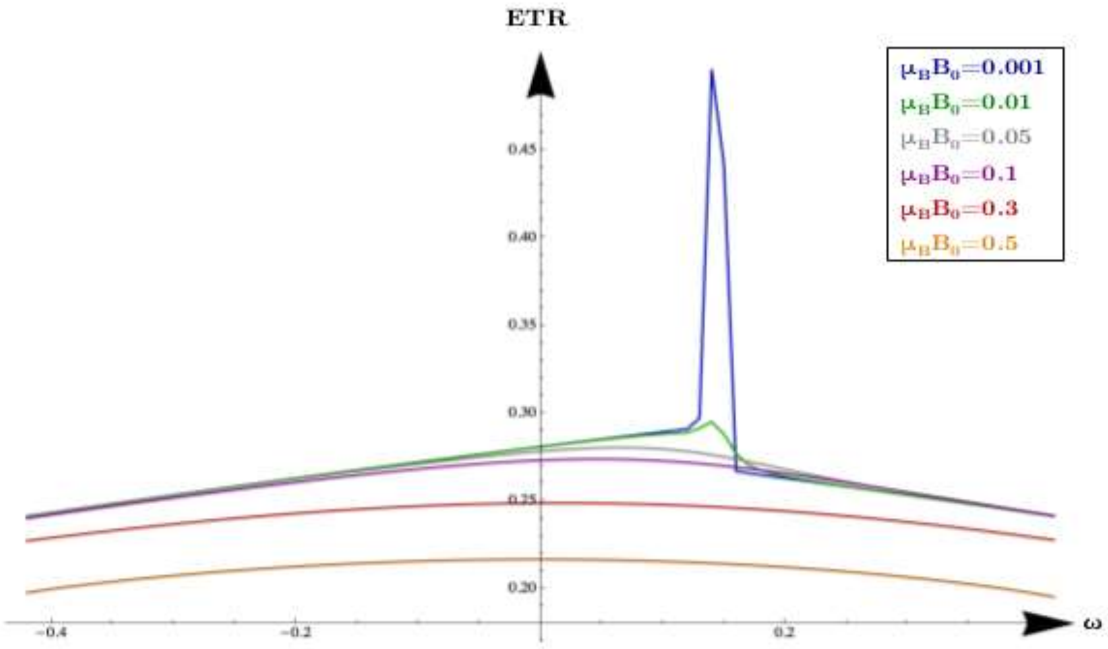

Fig. 3: Energy-transition-rate (ETR) against frequency ω at different magnetic field

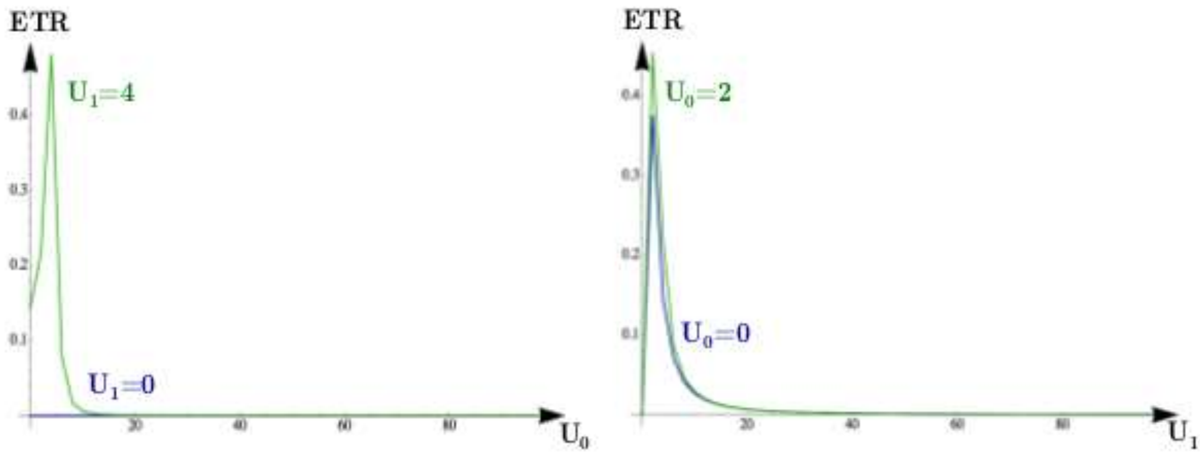

Fig. 4: Energy-transition-rate (ETR) against amplitudes of the static and magnetic impurities U_0, U_1

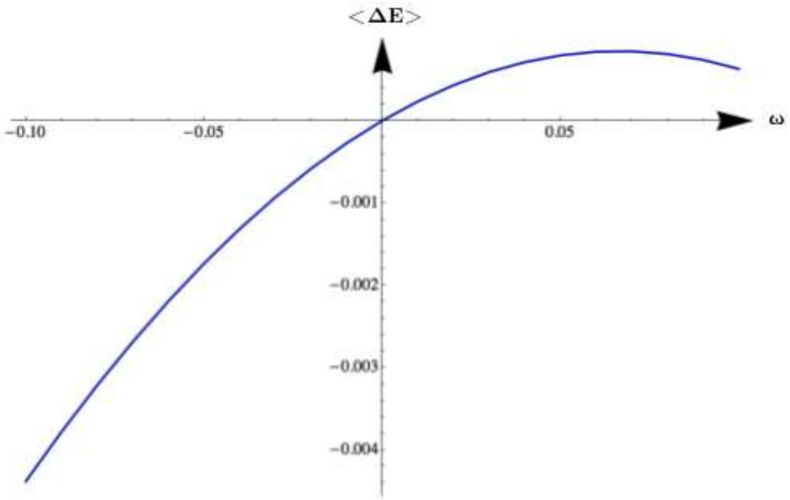

Fig. 5: Plot of the average energy distance covered by a transition into another band

From this we may conclude that the average energy is nothing less than the sum over all n of $\hbar\omega$ weighted with the probability of an energy transition into a respective band $\epsilon_n = \epsilon_o + n\hbar\omega$, which corresponds to the sum of all reflection and transmission coefficients with index n. The expectation value of the energy is then given by equation (7)

$$\langle \Delta \rangle_n = \sum_n \left(\sum_{\rho=+,-} \sum_{\delta=l,r} (n.\hbar\omega) \left(\left| t^n_{\sigma\delta} \right|^2 + \left| r^n_{\sigma\delta} \right|^2 \right) \right) \qquad (7)$$

where the index n indicates an averaging over all n. We find that for increasing ω, the average energy distance covered by a transition increases. For the chosen parameters, the curve finds its maximum for $\omega \approx 0.07$. It seems surprising that the average energy does not show symmetry with respect to ω, but steadily decreases for decreasing values of the frequency .However, as we have mentioned before, the system is generally not symmetric, as we chose an incoming wave with a specific direction of propagation and a specific spin state.

6. CONCLUSIONS

It has been observe that, the dependence of the energy transition rate (ETR) on the frequency ω bears some resemblance to the representation of the spin-flip-rate as a function of ω . For large magnetic energies, the energy transition rate is nearly constant which is perfectly consistent with our expectations of a system approaching ferromagnetic order. Also, for decreasing ω the rate of energy transition is generally higher. For a purely static potential scatterer, no energy transitions occur, and for large magnetic energies, the energy transition rate is nearly constant, whereas for magnetic energies much smaller than the Fermi energy, the energy transition rate has a maximum for the small absolute value of ω . We found that for increasing ω , the average energy distance covered by a transition increases. For the chosen parameters, the curve found its maximum for for $\omega \approx 0.07$. The rate of energy transitions of an incoming particle wave resulting from the scattering by the potential decrease for increasing adiabaticity of the problem.

7. ACKNOWLEDGEMENTS

We are grateful to Sarah Maria Schroeter for her kind support and fruitful discussion.

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