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Analysis for a Repairable -of *k*-out-of-*n*:*F* Systems via Order Statistics

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ABSTRACT--- ***In this paper, k-out-of-n : F repairable system is studied. The times between consecutive two machines are obtained using order statistics. Using these obtained times the system reliability is given with Laplace transform. Also the mean time to first failure is obtained.***

Keywords--- Laplace transform, order statistics, mean time to first failure, exponential distribution

1. INTRODUCTION

Reliability analysis examines system behaviors and its properties defined in various structures. The reliability study is concerned with random occurrences of undesirable events or failures during the life of a physical system. Reliability evaluation is an important, integral feature of planning design and operation of all engineering system (Arulmozhi, 2002). The probability and statistical theory are used in the study of system with randomness. Generally, whether the system is operational depends on the operation of some or all of these components. In life system, each of components systems contribute to the operation of the system ad performance of system depends not only on the operation of the system, but also on the total contribution of its components (Eryılmaz, 2013). Serial and parallel are known as two simple series types. The reliability of the series system is low. The parallel system has high reliability but tends to be very expensive (Arulmozhi, 2002).

The systems formed from a combination of parallel and series components in industry, have become more complicated with the development of technology (Gokdere and Gurcan, 2016). Therefore a new system *k*-out-of-*n* and related systems have caught the attention of may engineers.

An *n*-components system that works iff at least *k* of the *n* components work is called a *k*-out-of-*n*:*G* system (Kuo and Zuo, 2003). There is great interest in evaluating the reliability of *k*-out-of-*n*:*G* system, mainly because such systems are more general than series or parallel systems.

In literature, Boland and Proschan (1983) presented reliability of *k*-out-of-*n* system. (El-Damcese El-Sodany, 2014) examined markov models for analyzing the reliability and availability for the *k*-out-of-*n*:*G* repairable system with three failures. The distribution and expected value of the number of working components at time *t* in usual and weighted *k*-out-of-*n*:*G* systems under the condition that they are working at time *t*.

Estimation system reliability has been discussed by Chandra and Owen (1975), Bhattacharyya (1977). Most of these authors considered the strengths are independent and identically distributed random variables. Hangal (1999) obtain an estimate of system reliability for dependent and identically distributed random variables. In this paper, assumed that the mean life time of each of the parts of the system is different and the parts of the system exponentially distributed. The times between two consecutive machines are obtained using order statistics

Let $X_1, X_2, ..., X_n$ be independent and identically distribution *(i.i.d)* random variable with Exponential(λ), where λ are positive value and unknown parameter. Distribution corresponding r*th* failure for X_r random variables, using order statistics is,

$$F_{r:n}(x) = \sum_{i=r}^{n} \binom{n}{i} [1 - e^{-\lambda x}]^i [e^{-\lambda x}]^{n-i}$$

and probability density function corresponding (r-1)*th* failure for X_{r-1} random variables, using order statistics is,

$$f_{r-1:n}(x) = \frac{n!}{(r-2)!(n-r+1)!}[1-e^{-\lambda x}]^{r-2}[e^{-\lambda x}]^{n-r+1}e^{-x}$$

Then, the failure time between two machines can be calculated by the following equation.

$$P(X_r - X_{r-1} < t) = \int_0^t F_{r:n}(t+u) f_{r-1:n}(u)du \tag{1}$$

Let $X_1, X_2, ..., X_n$ be independent and non-identically distribution (*i.n.n.i.d*) random variables. For failure time between of two machines as X_{r-1} and X_r, if $\lambda_1 = 0{,}2$, $\lambda_2 = 0{,}1$, $\lambda_3 = 0{,}05$ and $\lambda_4 = 0{,}04$ are taken, results from Table 1 can be obtained.

Difference probabilities for failure times between consecutive two machines are calculated from Table 1. Because all components in the system are not identical, the failure rates of each component are obtained as $\lambda_1 = 0{,}2,\ \lambda_2 = 0{,}17,\ \lambda_3 = 0{,}06,\ \lambda_4 = 0{,}03$.

Table 1. Failure time between of two machines as X_{r-1} and X_r for *i.n.n.i.d.*

t	$P(X_{(1)} - X_{(2)} < t)$	t	$P(X_{(2)} - X_{(3)} < t)$	t	$P(X_{(3)} - X_{(4)} < t)$
1	0,025364	1	0,000031	1	$4{,}72.10^{-7}$
2	0,121255	3	0,024888	10	0,067629
3	0,256002	5	0,116774	15	0,196776
4	0,395750	7	0,254492	20	0,336310
5	0,523252	9	0,398010	25	0,456017
6	0,632666	11	0,524720	30	0,550164
7	0,723765	13	0,628045	35	0,621964
8	0,798538	15	0,709200	45	0,717609
9	0,859562	19	0,820335	60	0,792368
10	0,909306	23	0,886669	70	0,818745
11	0,949898	50	0,992748	90	0,845649
12	0,983080	100	0,999927	250	0,867390
13	1	150	1	1000	1

2. MODEL ASSUMPTIONS

1. The system under consideration is a repairable 3-out-of-4:F system.

2. Both the working time and the repair time of a component are exponentially distributed.

3. Each component after repair is as good as new.

4. The lives of the components are *i.n.n.i.d.*

5. All components are working at time *t= 0.*

3. MODEL ANALYSIS

Let $N(t)$ represents the state of the 3-out-of-4:F system at time *t*. Based on the assumption 1 and 2 we have

$$N(t) = \begin{cases} 0 & ,if\ \ at\ time\ t,\ all\ components\ work,\ the\ system\ works, \\ -1 & if\ \ at\ time\ t,\ one\ components\ \ fails,\ the\ system\ works, \\ 2 & if\ \ at\ time\ t,\ two\ components\ \ fail,\ the\ system\ works, \\ 3 & if\ \ at\ time\ t,\ three\ components\ \ fail,\ the\ system\ \ fails, \end{cases}$$

where $\lambda_i > 0,\ i = 1,2,3,4.$

Then $\{N(t), t \geq 0\}$ is a cotinuous- time homogeneous Markov process with state space $\Omega = \{0, -1, -2, 3\}$. Obviously, the set of working states is $W = \{0, -1, -2\}$ and the set of failed state is $F = \{3\}$.

According to definitions given by (Cheng and Zhang, 2010) the generalized transition probability and the key component, we can obtain the following equaions for the state transition probability in the 3-out-of-4:F system:

$$p_{00}(\Delta t) = 1 - (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)\Delta t + o(\Delta t)$$

$$p_{0-1}(\Delta t) = (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)\Delta t + o(\Delta t)$$

$$p_{0-2}(\Delta t) = p_{03}(\Delta t) = o(\Delta t)$$

$$p_{-10}(\Delta t) = \mu(\Delta t) + o(\Delta t)$$

$$p_{-1-1}(\Delta t) = 1 - \left[\frac{2(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4)}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4} + \mu\right]\Delta t + o(\Delta t)$$

$$p_{-1-2}(\Delta t) = 1 - \left[\frac{2(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4)}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}\right]\Delta t + o(\Delta t)$$

$$p_{-13}(\Delta t) = o(\Delta t)$$

$$p_{-20}(\Delta t) = o(\Delta t)$$

$$p_{-2-1}(\Delta t) = \mu(\Delta t) + o(\Delta t)$$

$$p_{-2-2}(\Delta t) = 1 - \left[\frac{3(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4)}{\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4} + \mu\right]\Delta t + o(\Delta t)$$

$$p_{-23}(\Delta t) = \left[\frac{3(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4)}{\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4}\right]\Delta t + o(\Delta t)$$

Using the equations listed above for the state transition probabilities in the 3-out-of-4:F system, we can find the transition rate matrix Q as follow.

$$Q = \begin{pmatrix} -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) & (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) & 0 & 0 \\ \mu & -a & b & 0 \\ 0 & \mu & -c & d \\ 0 & 0 & \mu & -\mu \end{pmatrix}$$

where,

$$a = \frac{2(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4)}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4} + \mu, \quad b = \frac{2(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4)}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}$$

,

$$c = \frac{3(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4)}{\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4} + \mu,$$

$$d = \frac{3(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4)}{\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4}.$$

In the following we study the reliability of the 3-out-of-4:F sysytem with independent non-identically distributed components.

To determine system reliability $R(t)$, we consider $\{N(t), t \geq 0\}$ as a $\{\tilde{N}(t), t \geq 0\}$. Actually, $\{\tilde{N}(t), t \geq 0\}$ is a continuos-time homogeneous Markov Process with state space $\tilde{\Omega} = \{0, -1, -2,\}$ (Zhang et.al., 2000).

Now, let

$$p_j(t) = P\{\tilde{N}(t) = j\}, \quad j \in \tilde{\Omega}$$

Then, the system reliability is

$$R(t) = p_0(t) + p_{-1}(t) + p_{-2}(t)$$

According to the Fokker-Planck equation (Cao and Cheng, 1986), it is easy to derive the following syaytem of differential equations;

$$p'_{\tilde{\Omega}}(t) = p_{\tilde{\Omega}}(t).B \tag{2}$$

where

$$p_{\tilde{\Omega}}(t) = (p_0(t) + p_{-1}(t) + p_{-2}(t))$$

$p_{\tilde{\Omega}}(0) = (1,0\,0)$ and

$$B = \begin{pmatrix} -(\lambda_1+\lambda_2+\lambda_3+\lambda_4) & (\lambda_1+\lambda_2+\lambda_3+\lambda_4) & 0 \\ \mu & -a & b \\ 0 & \mu & -c \end{pmatrix} = \begin{pmatrix} -0{,}46 & 0{,}46 & 0 \\ 0{,}5 & -0{,}8 & 0{,}3 \\ 0 & 0{,}5 & -0{,}662 \end{pmatrix}$$

The laplace transform of eq. (2), considering the initial conditions are

$$sp_0^*(s) = -0{,}46\,p_0^*(s) + 0{,}5\,p_{-1}^*(s) + 1$$

$$sp_{-1}^*(s) = 0{,}46\,p_0^*(s) - 0{,}8\,p_{-1}^*(s) + 0{,}5\,p_{-2}^*(s)$$

$$sp_{-2}^*(s) = 0{,}3\,p_{-1}^*(s) - 0{,}66\,p_{-2}^*(s)$$

Simplifying these equations, we have

$$p_{-2}^*(s) = \frac{0{,}3}{s+0{,}66}\,p_{-1}^*(s)$$

$$p_0^*(s) = \frac{(s+0{,}8)(s+0{,}66) - 0{,}15}{0{,}46(s+0{,}66)}\,p_{-1}^*(s)$$

$$p_{-1}^*(s) = \frac{0{,}46(s+0{,}66)}{(s+0{,}46)(s+0{,}8)(s+0{,}66) - 0{,}15(s+0{,}46) - 0{,}23(s+0{,}66)}$$

The laplace transform at the sysytem's reliability $R(t)$ is given by,

$$R^*(s) = p_0^*(s) + p_{-1}^*(s) + p_{-2}^*(s)$$

The system mean time to first failure is given by,

$$MTTFF = \lim_{s \to 0} R^*(s) = 37{,}1196$$

4. CONCLUSION

Assumed that the mean life time of each of the parts of the system is different and the parts of the system exponentially distributed. The times between two consecutive machines are obtained using order statistics. Using these obtained times the system reliability is given with Laplace transform. Also the mean time to first failure (MTTFF) is obtained.

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