

Weibull Distribution: Reliability Centered Maintenance and the Use of Bayesian networks

Roque Orlando Silva¹, Hamilton Pozo^{2*}

¹ Department of Engineering, Faculdades Metropolitanas Unidas
São Paulo, Brazil

² Department of Operations, CEETEPS/Fatec Rubens Lara
Santos, Brazil

* Corresponding author's email: [hprbrazil \[AT\] hotmail.com](mailto:hprbrazil@hotmail.com)

ABSTRACT— *This article, through the literature review, we discuss Bayesian networks and Weibull distribution to describe the processes of Reliability Centered Maintenance as well as the tools of information technology necessary for their support. To maximize asset reliability cost-effectively, maintenance should be scheduled based on the likely deterioration of an asset. Maintenance management reliability centered aims to increase the availability of the physical item in which it is applied. Since the reliability function of the quality of the program or maintenance plan is a systematic methodology that makes use of IT tools to optimize strategic actions to minimize maintenance costs. Concerning the application of reliability-centered maintenance, this paper presents considerations that allow drawing requires the use of various mathematical tools.*

Keywords— Reliability, Bayesian Networks, Maintenance, Weibull distribution

1. INTRODUCTION

The increasing complexity and integration of automation in various production systems emphasize the importance of maintenance for various systems to keep them in desired performance levels, reduce unplanned downtime and high costs.

The Reliability Centered Maintenance is strategic for any company, since, through a systematic and effective maintenance, faults can be minimized or even avoided. In addition to the intervention in the equipment or system process to occur at the most opportune or programmed, the need for care of the continuity indicators that allow companies to invest in reliability centered maintenance, seeking more effective maintenance policies that maximize the availability of their equipment and services [1].

When using Weibull, gamma or exponential distributions, has been used a model deterioration, but in many cases these do not provide adequate decision tools as the assumed data is unavailable and relevant knowledge that could be used to distinguish individual's differences in the same asset class. The main characteristics of a successful organization are the proper management of its maintenance with a view to reducing the risks inherent in their operations.

This paper, through the literature review discusses the application of Bayesian networks and distribution systems Weibull poor data in search of understanding of the conditions under which applications are best suited for the effective management of maintenance, the model in which rates failure of equipment and systems are growing with time. Thus, allowing it to be prevented lack. One hypothesis is then relating the frequency of failure events with the availability of protective barriers against its occurrence. The two parameters potentiated inverted Weibull distribution has been proposed by [2].

Maintenance management is fundamental to the definition of critical areas, which deserve greater investment, especially when working with budget constraints. Maintenance consists of a set of activities required to ensure assets are in a reliable operating condition [3].

The decision support systems, in turn, can be implemented from the Artificial Intelligence techniques such as Probabilistic Expert Systems (PES), also known as Bayesian Networks (RB). Used in this work to model the problem of defining critical areas of reliability in determining the time to failure of equipment or systems with the lack of data regarding maintenance management.

The Bayesian networks allow, under the Artificial Intelligence, reasoning under conditions of uncertainty. Become relevant when you have no absolute knowledge about the problem under consideration and no information can find data characterized by frequency of occurrence and probabilities.

Various methods can be used to optimize the availability of the equipment to obtain availability of its functions. Thus, the role of maintenance planning achieves the goals of security required by businesses. This approach leads to the acquisition of data unavailable through Bayesian analysis to meet the maintenance management and thus reduce the risk of unavailability of equipment and systems to acceptable levels. For each variable A with b_1, \dots, b_n , is associated with a table of conditional probabilities, (see equation 1).

$$P\left(\frac{A}{b_1, \dots, b_n}\right). \quad (1)$$

Concerning the application of reliability-centered maintenance, this article presents considerations that allow drawing some conclusions very important for:

- increasingly competitive;
- requires news method and management;
- and require mathematical tools.

2. PROBABILITIES UNCONDITIONAL AND CONDITIONAL

According to [4], the unconditional probability or a priori variables are associated with the network that we have no history and represent the degree to which these variables are associated with the states that characterize them.

When a variable (node) of the network has one or more nodes antecedents, the probability is said conditional or *posteriori* and is subject to the state of the variable that precedes it. One evidence is the belief that certain variable A is in the state Anopheles If the variable A has states A1, A2, A3, ..., An, and they are mutually exclusive, if this variable is in state A3, then one can say that the probability of remaining states is null. The propagation of evidence is defined by calculating the marginal probability distribution for a set of query variables.

According [5], promotes the propagation of evidence fusion and propagation of new evidence by the Bayesian Network, so that, for each proposition is assigned a consistent measure, as the axioms of probability theory. In the same way [4], the relations of cause and effect between a set of nodes represent variables of interest for defining the prioritization of maintenance. For this prioritization, the end node represents the index * RPN (Risk Priority Number) to consider the aspects that characterize the methodology FMECA (Failure Mode, Effects and Criticality Analysis) for the investigation of "weaknesses" potential systems, examining the ways in which failures occur, and the effects of those nodes on the system performance, the safety hazards and severity of these events in determining maintenance priorities.

According [6], the standard criteria in a process that identifies the failure modes are:

- a. all failure modes reasonably likely to cause each functional failure must be identified;
- b. the method used to decide what constitutes a failure mode "reasonably likely" to be acceptable;
- c. the failure mode should be identified with a level of causality that makes it possible to identify a management policy;
- d. the list of failure modes includes modes of failures that occurred before the failure modes that are being prevented by existing maintenance programs, and the failure modes that have not happened yet, but that are reasonably likely in the operational context;
- e. list of failure modes shall include any form or process that is likely to cause a functional failure, including deterioration, human error, whether caused by entrants or design defects.

According [7], has added total productive maintenance managerial aspects to the previous called Reliability and Criticality Based Maintenance technique. Another maintenance strategy was introduced that this methodology was applied for creating optimum maintenance system.

Reliability searching time in which a system or equipment will be with their production function available. The time scale will feed the probability distribution equipment. Reliability centered maintenance was first used by Tom Matteson, F. Stanley; Nowlan, and other senior executives and engineers at United Airlines to describe a process used to determine maintenance requirements for their aircraft [8].

The question is: what to do and how to proceed when the historical data of the equipment or system are not available? It is in this context that the distribution and Bayesian networks provide a major contribution in the search for solution maintenance in that time between failures of equipment is not available.

To answer each of these questions in a satisfactory manner, the information will be gathered and decisions should be made taking into account historical data obtained from the records of equipment operation. All information and decisions should be documented in a way that the information and decisions are fully available, a reasonable way to making a decision as to whether or not interference in the process or equipment.

What should be done to predict or prevent each failure? This is a complex topic; your criteria are presented in two groups. The first group relates to the overall theme selection policy management failures. The second group of criteria concerns the scheduled tasks and intervals, which include proactive tasks as well as a standard action to find them.

The Weibull distribution is used to describe those instances worn in the substitution function returns the output of the equipment, if not complete, at least close to it. She has two parameters: the shape parameter β and scale parameter ρ .

An alternative approach to obtaining the model that describes the behavior of equipment failure, the dimension of time, is to assume a distribution of flaws and then estimate the parameters of the function assumed. The adoption of such a procedure often simplifies the mathematical analysis in the application of a substitution model and brings advantages in obtaining the data. According [9], treat both cases and consider only the case of the numbers of failures between replacement times in case is prescribed points in time T_1, T_2, \dots , etc. In this case the interval censored data consist of the number of failures in $[T_i - 1, T_i)$, i .

The Weibull distribution is useful in a variety of applications, particularly for the life model of devices. Widely used, the Weibull distribution assumes a very wide variety of ways and therefore is very flexible and can be used for various types of data [10].

$$F(t) = \frac{\beta}{\rho} \left[\frac{t}{\rho} \right]^{(\beta-1)} \cdot e \left[-\left(\frac{t}{\rho} \right)^\beta \right] \quad (2)$$

and the reliability function is given by:

$$R(t) = e \left[-\left(\frac{t}{\rho} \right)^\beta \right] \quad (3)$$

The exponential distribution is a particular case of the Weibull distribution (equation 2 and 3) when making $\beta = 1$, $R(t)$ is a constant proportional to the power t , then R will be a function increasing, decreasing or constant t depending on the value of β . The Weibull distribution is an appropriate model for law failure, where the system consists of several components and failure is primarily due to more serious flaw or irregularity among many imperfections in the system, therefore, can be applied the Weibull distribution and obtain failure rate increasing, decreasing, simply by choosing β .

The replacement models are a set of very important techniques in preventive maintenance, they guarantee a reduction in costs related to substitutions, minimizing them and looking for a better way to meet the needs of different systems. The strategy implemented in manufacturing industry can use one of the three approaches of Time-domain, Frequency domain and Time-Frequency domain [11].

Therefore, [8], the age replacement policy is a procedure that involves replacing an item at the time of failure or to achieve a lifetime T (age replacement), whatever the situation that occurs first, for a charge, which will be subject to the same rules as the old. The use of this policy is only effective if the cost of replacement before failure is give provide some savings compared to replacement due to failures. The central issue, not only in this replacement policy, as in others, is what age would a unit be replaced with the lowest cost per unit of time of use. This question has an answer, in most cases, the result of optimization problems. The following is the mathematical structure of a type of replacement by age based on the minimum cost. There are two costs involved:

1. C_a is the cost of replacement after failure;
2. C_b is the cost of replacement before failure.

In this case, the assumption used is that the intervals between replacements are sufficiently short to ignore the value of money over time [8].

The expected cost for the use of a replacement policy for age is given by the equation 4.

$$C(t) = Ca \int_0^t f(x)dx + Cb \int_t^{\infty} f(x)dx \quad (4)$$

Another very important concept is the expected period of use, which expresses the average use of a particular piece of equipment, according to the equation 5.

$$T(t) = \int_0^t xf(x)dx + t \int_t^{\infty} f(x)dx \quad (5)$$

A replacement policy for age has as main objective to ensure high availability at minimal cost, through successive substitutions that precede the failure. The notion of cost does not make sense without it is linked to the time dimension. Thus, the basis for establishing the frequency of replacements is based on obtaining a time between replacements that minimize the ratio of the expected cost per unit time of use. This parameter is represented here by CM, corresponding to the expression ratio $C(t) / T(t)$ being the objective function of the problem of replacing at low cost, whose analytical, according the equation 6, is:

$$Cm(t) = \frac{CaF(t) + Cb[1 - F(t)]}{\int_0^t xf(x)dx + t \int_t^{\infty} f(x)dx} \quad (6)$$

2.1. System reliability

The question that arises at this point is: how can you assess the reliability of a system, if known reliability of its components? Surely this is one of the most difficult issues. However, since few components can be generalized for a better understanding of the matter and adequate. C1 and C2 are two components in series: this means that for the system to work, both components are to operate. If, in addition, the components operate independently, one can obtain the reliability of the components and call them as follows: R1 (t) and R2 (t).

$$R(t) = P(T > t) \quad (7)$$

Where T is the duration until the system fails. $R(t) = P(T_1 > t)$ and $P(T_2 > t)$ where T1 and T2 are the durations to failure of components C1 and C2 respectively.

$$R(t) = P(T_1 > t).P(T_2 > t) = R_1(t).R_2(t) \quad (8)$$

Thus, we find that $R(t) \leq \min [R1, R2]$. This means that a system consisting of two independent components, in series, the reliability of the system is lower than the reliability of any of its parts.

3. BAYESIAN ANALYSIS

According [12], the reliability analysis is a technique to support decision-making and control that assists managers in seeking to guarantee satisfactory performance of the functions of the items on a given system, considering its limitations, and its wear the factors that influence their performance, these items are equipment or systems.

It is common to apply techniques such as fault tree and event, the probabilistic representation of the operation of the equipment system integrators, as well as methods aimed at reliability analysis when people are part of the process. These two lines are commonly called "reliability analysis equipment" and "Human Reliability Analysis", respectively.

A major problem of such sets of these techniques is that they require adjustments in many cases become poor modeling or far from reality system. In this sense, one can cite: assumptions of independence between variables that are related; partition simplistic event as favorable or unfavorable, and difficulties for the inclusion of new knowledge or for quantifying the models built.

In this work, it is shown that the BNs for modeling methods directed to analysis of human and equipment reliability can allow greater flexibility and provide a greater fidelity as to the mechanisms that govern the probabilistic uncertainties present in the system, resulting in more accurate inferences as well as a greater understanding of the dynamic behavior of the diagrammatic trial before routine events or abnormal.

The concept of a Bayesian belief network has been found in many applications described in the literature that [13], have used dynamic Bayesian belief network on decision making process of their methodology. The probability of an event is under the Bayesian framework, a degree of belief in the likelihood that the event will take place from the point of view of any individual. According to Weber et al. [14], you should also consider the possibility of application of Bayesian networks in the maintenance planning problem. That's because several relevant factors of maintenance are known and measurable, but due to the lack of absolute knowledge and the infeasibility of measuring some of these events, evidence is certain uncertainty factors, which are perfectly measurable by Bayesian networks.

An advantage of the Bayesian network is that it is not necessary to associate experiments to estimate the probability associated with events [15].

3.1 Bayes Theorem

The Bayesian model interprets the conditional probability. The a priori probability or unconditional, $P(A)$ is the probability existing before any evidence, and a posteriori probability or conditional, $P(B)$ is the probability after knowing the evidence. When there is some evidence in the application, the conditional probability $P(A/B)$ represents the probability of occurrence given the knowledge of B.

The conditional probability can be defined in terms of the a priori probability, denoted by the equation 9:

$$P(A \cap B) = \frac{P(A \cap B)}{P(B)} \quad (9)$$

For A and B are true, it is necessary that B is true, then A is true as B. Accordingly, the two formulas given product rule and equating them, we obtain the equation 10:

$$P\left(\frac{B_i}{A}\right) = \frac{P\left(\frac{A}{B_i}\right) \cdot P(B_i)}{\sum_{i=j}^k P\left(\frac{A}{B_j}\right) \cdot P(B_j)} \quad (10)$$

This equation is known as Bayes' Rule Act (Bayes or Bayes Theorem), which is the basis of most systems for probabilistic inference [16]. Within the context of systems that act rationally, two main approaches can be used:

deterministic logical reasoning and probabilistic reasoning. When the p-value is close to 0.5, this indicates a good agreement between the data and the model [17].

The logical ponders prior knowledge about the problem and on this knowledge base draws its conclusions. This approach, although strong may not be useful in situations where it is not known beforehand all data from the scope of the problem. For these cases, probabilistic reasoning emerges as the most appropriate option [18].

A system that can act in situations of uncertainty should be able to assign reliability levels for all statements in their knowledge base, also establish relationships between sentences. The Bayesian Networks-RBs offer an approach to probabilistic reasoning which includes graph theory, for the establishment of relations between sentences, also probability theory, for assigning levels of reliability [18].

The process of building expert systems includes knowledge acquisition, knowledge representation, verification and validation of prototypes. Rule-based expert systems are useful in encapsulating explicit knowledge from experts [19]. Mathematically, a Bayesian Network is an acyclic graph representation of the probability in the universe of discourse of the problem, which is the quantitative part of the RB. Modelling is a common physics-based model. In such critical systems as aircraft and industrial and manufacturing processes, defect initiation and propagation must be estimated for effective fault prognostic [20].

On the other hand, from the viewpoint of an expert RBs is a graphical model that represents a simple way, the causality relationships of the variables of a system.

The case does not have a parent, and a direct dependency between a graph node and another node, the table of probabilities is reduced for an unconditional probability $P(A)$ Once set the network topology, just specify the probabilities of the nodes that participate in direct dependencies, and use these to compute the other probabilities that want [18].

There are many tools that provide the functionality of probabilistic inference, which can be used to optimize replacement operations preventive maintenance to increase the reliability and functionality of the provision of equipment or system. For this purpose, these tools, some free and other professionals, were developed with the junction tree algorithm, in which nodes represent a set of random variables that are sub graphs whose nodes are all adjacent to each other, among them the tool called [21].

This was one of the first tools that have developed algorithms to perform exact inference in RB. Another feature provided by Hugin analysis is the type most likely explanation, that is, the configuration likely that more variables can assume at a given time according to the evidence available. For each inference performed Hugin allows to analyze the join tree and secondary structures generated [22].

The tool implements a probabilistic network in Java. Consists of an inference engine, a code editor, an API and a learning environment. The algorithms used are based on the method and extent of the junction tree and search fields.

4. FINAL THOUGHTS

Preventive maintenance merited more research needs to denote their applications, on the replacement policies and the problems they carry. Concerning the application of reliability-centered maintenance, this article presents considerations that allow drawing some conclusions such as:

- In an increasingly competitive and globalized, it becomes imperative for companies to maintain a development system to support decision-making that provides agility and competitiveness, security, economy systems maintenance.
- The new technological environment requires new management because the current models have attempted to evolve and solve current problems by increasing the existing paradigm rather than provoke a rupture with them.
- Although the system is seemingly simple, requires the use of various mathematical tools, which, in a business environment, it will enable only using Information Technology tools and their validation should occur through a case study in future work.

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